

## Superconductivity of Quark Matter at Finite Temperature

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Superconductivity of quark matter at finite temperature is discussed within the mean-field (BCS) approximation. The energy gap decreases very slowly at high density due to asymptotic freedom and at high-temperature due to thermal motion. It is shown that the phase transition to the normal state occurs at high density,  $\rho \gtrsim 20 \text{ fm}^{-3}$ , and/or high-temperature,  $T \gtrsim 50 \text{ MeV}$ .

### §1. Introduction

It is well known that superconductivity is realized in many fermi systems, such as electron, Helium 3 and nucleon systems. Quarks in hadrons are also fermions, so one can imagine quark matter composed of an infinite number of quarks being in a superconducting state.<sup>1),2)</sup> Although the existence of such quark matter has not been verified, it is expected to exist in situations of very high pressure, such as that in the cores of neutron stars, in high-energy heavy ion collisions, and in the early universe.<sup>3)-7)</sup> In fact the superconductivity of quark matter and its physical properties have been discussed by Bailin and Love,<sup>1),2)</sup> and later by other authors.<sup>8)</sup>

Recently Iwado and the present author proposed a microscopic (BCS) theory for quark matter<sup>9)</sup> in parallel with the usual BCS theory for electrons. This model was, however, restricted to the case of cold quark matter (at zero temperature), so that it could not be applied to actual problems. *It is the purpose of this paper to generalize the theory to finite temperature.* Since the thermal motion is random, it tends to destroy Cooper pairs, so that the superconducting state will break down and become a normal state at some temperature. This is called the critical temperature. It is important to estimate the critical temperature when we apply the theory to the actual physical problems mentioned above. In this paper we calculate the critical temperature at various quark densities  $\rho$ . Consequently, the phase diagram in the  $(T - \rho)$  plane is obtained for quark matter. This is our main result.

It is, however, very difficult to investigate such systems within full QCD. There are two main problems, the appearance of gluon degrees of freedom at finite temperature and the non-perturbative nature of QCD. Since many real gluons are created in quark matter at finite temperature, we have a quark-gluon plasma. In this paper we consider only the quark sector and do not discuss the gluons, except for virtual gluons exchanged in quark-quark interactions, because we are interested in the superconductivity of the quark sector. This view is also taken in the usual electron-phonon system in superconductors, where the phonon degrees of freedom are not considered explicitly. Moreover, we do not consider the low-density region of quark matter since quark matter is unstable at low density and transformed into hadronic matter.

Since the density of nuclear matter is about  $\rho \approx 0.17 \text{ fm}^{-3}$ , we restrict ourselves to densities higher than about  $\rho \approx 1 \text{ fm}^{-3}$ , where the quark matter is expected to be in the deconfinement phase. Consequently we neglect confinement and consider the “running” coupling constant used by Higashijima<sup>10)</sup> and Miransky<sup>11)</sup> in the study of chiral symmetry. In other words, our main interest is to investigate the phase transition to superconductivity taking into account the asymptotic freedom of QCD.

In the next section the BCS theory for quark matter is generalized to finite temperature and the gap equation is derived. In §3, the gap equation is solved numerically and the critical temperature is calculated. Section 4 is devoted to discussion of concluding remarks.

## §2. Gap equation at finite temperature

We first describe the mean field approximation for quark matter at finite temperature, following Ref. 12). Let us consider quark matter with a single flavor whose Hamiltonian is written<sup>13),14)</sup>

$$\hat{H} = \sum_{\alpha} \epsilon_{\alpha} c_{\alpha}^{\dagger} c_{\alpha} + \frac{1}{2} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta,\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\delta} c_{\gamma}, \quad (1)$$

where  $c_{\alpha}$  and  $c_{\alpha}^{\dagger}$  denote the annihilation and creation operators for quarks. The single particle state is represented by  $\alpha = \{\mathbf{k}, s, i\}$ , with the momentum  $\mathbf{k}$  (and kinetic energy  $\epsilon_{\mathbf{k}} \equiv \sqrt{\mathbf{k}^2 + m^2}$ ), the helicity  $s$  ( $s = \pm 1$ ), and the color suffix  $i$  ( $i = 1, 2, 3$ ). The potential energy is assumed to result from the one-gluon-exchange (OGE) interaction.<sup>1)</sup>

Now let us consider quark matter at temperature  $T$ , which is described by the grand canonical density matrix with the chemical potential  $\mu$ ,

$$\rho = \frac{\exp[-\beta(\hat{H} - \mu\hat{N})]}{Z} = \frac{\exp[-\beta\hat{H}']}{Z}, \quad (2)$$

where  $\beta \equiv 1/kT$  (with  $k$  the Boltzmann constant) and  $Z \equiv \text{Tr}[-\beta\hat{H}']$ . The mean-field approximation amounts to replacing  $\hat{H}'$  on the right-hand side by the mean-field Hamiltonian. Following a previous paper,<sup>9)</sup> we know that a Cooper pair is realized in an *s-wave, color-triplet and spin-parallel state*. The corresponding pairing field has the following form:

$$\langle c_{-\mathbf{k}tj} c_{\mathbf{k}si} \rangle = \hat{\epsilon}_{i,j} \delta_{s,-t} F(k), \quad (3)$$

where the brackets represent the average of the grand canonical ensemble:  $\langle O \rangle \equiv \text{Tr}[O\rho]$ . The Kronecker delta  $\delta_{s,-t}$  and the skew-symmetric tensor  $\hat{\epsilon}_{i,j}$  ( $\hat{\epsilon}_{i,j} = -\hat{\epsilon}_{j,i}$  and  $\hat{\epsilon}_{1,2} = \hat{\epsilon}_{2,3} = \hat{\epsilon}_{3,1} \equiv 1$ ) on the right-hand side of Eq. (3) ensure the spin-parallel and color-antisymmetric state, respectively. The unknown function  $F(k)$  is dependent only on  $k \equiv |\mathbf{k}|$  because of the assumption of the *s-wave* pairing. We obtain the following pairing field by using Eq. (3),

$$\frac{1}{2} \sum_{\gamma\delta} V_{\alpha\beta,\gamma\delta} \langle c_{\delta} c_{\gamma} \rangle \equiv \frac{i}{2} \delta_{i,j} \delta_{s,-t} \Delta_{\mathbf{k}}. \quad (4)$$

Like  $F(k)$ , the gap parameter  $\Delta_k$  on the right-hand side is dependent only on  $k \equiv |\mathbf{k}|$ . It will be determined later.

In order to diagonalize the linearized Hamiltonian, we introduce the annihilation and creation operators for quasi-particles,  $a_\alpha$  and  $a_\alpha^+$ , by the unitary (Bogoliubov) transformation,<sup>9)</sup>

$$\begin{pmatrix} c_{\mathbf{k}1} \\ c_{-\mathbf{k}-1}^+ \end{pmatrix} = \begin{pmatrix} A_k & B_k \\ B_k^* & A_k^* \end{pmatrix} \begin{pmatrix} a_{\mathbf{k}1} \\ a_{-\mathbf{k}-1}^+ \end{pmatrix}. \quad (5)$$

Then the Hamiltonian can be written as

$$\hat{H}' \equiv \hat{H} - \mu \hat{N} \approx \sum_{\mathbf{k}si} E_{\mathbf{k}si} a_{\mathbf{k}si}^+ a_{\mathbf{k}si} + \text{const.}, \quad (6)$$

and  $F_k = -i\Delta_k/2E_k$ . Here we have three kinds of quasi-particles with momentum  $\mathbf{k}$  and helicity  $s$  ( $\pm 1$ ). Their energies are given by  $E_{k1} = E_{k2} = \sqrt{(\epsilon_k - \mu)^2 + 3\Delta_k^2} \equiv E_k$  (finite gap) and  $E_{k3} = |\epsilon_k - \mu|$  (gapless). The Bogoliubov transformation with definite momentum  $k$  is given by

$$\begin{aligned} \text{(a)} \quad & A_k = S_k, \quad B_k = -ix_k \hat{e} S_k \quad \text{for } \epsilon_k > \mu, \\ \text{(b)} \quad & A_k = -ix_k \hat{e} S_k, \quad B_k = S_k \quad \text{for } \epsilon_k < \mu, \end{aligned} \quad (7)$$

where the number  $x_k$  and the matrix  $S_k$  are defined by

$$x_k \equiv \frac{\Delta_k}{|\epsilon_k - \mu| + E_k} \quad \text{and} \quad S_k \equiv \begin{pmatrix} a/\sqrt{2} & a/\sqrt{6} & 1/\sqrt{3} \\ -a/\sqrt{2} & a/\sqrt{6} & 1/\sqrt{3} \\ 0 & a/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \quad (8)$$

with the definition  $a \equiv \sqrt{(|\epsilon_k - \mu| + E_k)/(2E_k)}$ . One of the quasi-particles has *no gap*. This results from the fact that the unitary transformation is improper.<sup>15)</sup> In order to understand the physical meaning of the existence of such a gapless quasi-particle, let us introduce (Cooper) pair creation operators as follows:

$$P_{ij} = c_{\mathbf{k}si}^+ c_{-\mathbf{k}-sj}^+ - c_{\mathbf{k}sj}^+ c_{-\mathbf{k}-si}^+. \quad (i, j = 1, 2, 3) \quad (9)$$

There are six single-particle states having either quantum numbers  $(\mathbf{k}, s)$  or  $(-\mathbf{k}, -s)$ . Suppose the momentum is located so deep under the Fermi surface that the six states are all occupied. Then the six-quark state  $|\Psi\rangle = \prod_{i=1}^3 c_{\mathbf{k}si}^+ c_{-\mathbf{k}-si}^+ |0\rangle$  cannot be expressed by pair operators, because  $P_{12}P_{23}P_{31} = 0$ . In fact we have  $|\Psi\rangle \propto (P_{12})^2 c_{\mathbf{k}s3}^+ c_{-\mathbf{k}-s3}^+ |0\rangle \propto (P_{12}P_{23}) c_{\mathbf{k}s3}^+ c_{-\mathbf{k}-s1}^+ |0\rangle$ . Even if two Cooper pairs are made, the last pair cannot be written by means of only the pair operators. This is the reason the gapless quasi-particle appears.

At finite temperature, such quasi-particles are excited from the BCS state, and their distribution function is given by the Fermi distribution,

$$n_{ki} = \langle a_{\mathbf{k}si}^+ a_{\mathbf{k}si} \rangle = (1 + \exp \beta E_{ki})^{-1}. \quad (i = 1, 2, 3) \quad (10)$$

Note that the distribution function  $n_{k3}$  of the gapless quasi-particle is different from the others ( $n_{k3} \neq n_{k1} = n_{k2} \equiv n_k$ ), due to the different quasi-particle energy. In

the high-temperature limit  $T \rightarrow \infty$ , the occupation number of all the quasi-particle states becomes one half ( $n_{ki} \rightarrow 1/2$ ). Thus here the system has maximal entropy.

The energy gap  $\Delta_k$  should be determined self-consistently from the definition of the mean field, Eq. (4). To this end, let us calculate the average value of the pair operator in terms of the grand canonical ensemble. Substituting the transformation (7) into  $\langle c_{-ktj} c_{k si} \rangle$  and using the Fermi distribution (10), we obtain

$$\langle c_{-ktj} c_{k si} \rangle = (B_k A_k^t)_{i,j} (1 - 2n_{k1}) + (A_{i,3} B_{j,3} - B_{i,3} A_{j,3}) (n_{k3} - n_{k1}). \quad (11)$$

If we substitute Eqs. (7) and (8) into this equation, the last term on the right-hand side disappears. Therefore Eq. (11) is reduced to

$$\langle c_{-ktj} c_{k si} \rangle = \frac{-i\Delta_k}{2E_k} \hat{\epsilon}_{i,j} (1 - 2n_k). \quad (12)$$

This expression implies that *the gapless quasi-particle does not contribute the pairing field*, as expected. In other words, the gapless quasi-particle is nothing but the usual particle. In the high-temperature limit, the occupation probability of the quasi-particles becomes one half of Eq. (10), so that the pairing field vanishes from Eq. (12) (normal phase). Substituting Eq. (12) into Eq. (4), we obtain the modified gap equation for finite temperature,

$$\Delta_k = \frac{4\pi}{3V} \sum_{\mathbf{l}} \frac{\alpha_s \Delta_{\mathbf{l}}}{E_{\mathbf{l}} |\mathbf{k} - \mathbf{l}|^2} \frac{\epsilon_{k\epsilon_{\mathbf{l}}} + \mathbf{k}\mathbf{l}}{\epsilon_{k\epsilon_{\mathbf{l}}}} (1 - 2n_k), \quad (13)$$

where  $V$  is the volume of the system. The coupling ‘‘constant’’  $\alpha_s$  should be dependent on the relevant momenta  $\mathbf{k}$  and  $\mathbf{l}$  (i.e. a ‘‘running’’ coupling constant). This point will be discussed later. It is well known that this equation has two types of solutions if the coupling constant  $\alpha_s$  is sufficiently large, a trivial solution ( $\Delta_k = 0$ ) and a non-trivial one ( $\Delta_k \neq 0$ ). The former corresponds to the Fermi gas state (normal phase) and the latter the BCS state (super phase). It can be shown that if the nontrivial solution exists, the ground state becomes the BCS state.<sup>9)</sup>

The gap equation (13) determines an energy gap that is a function of temperature through the quantity  $n_k$ . The effect of the temperature on the gap equation is the appearance of the last factor on the right-hand side of Eq. (13). This factor inhibits the formation of Cooper pairs, so that the magnitude of the energy gap decreases with increasing temperature. In other words, the thermal motion works against the formation of Cooper pairs. Consequently, *superconductivity will disappear at some critical temperature*. This is discussed in the next section.

To end this section, let us consider the determination of the chemical potential  $\mu$ . To this end, we must calculate the average value of the total quark number, which is given by

$$N \equiv \sum_{k si} \langle c_{k si}^+ c_{k si} \rangle. \quad (14)$$

This equation determines the value of the chemical potential as a function of the quark density  $\rho = N/V$  and the temperature  $T$ .

### §3. Phase diagram for quark matter

In this section we solve the gap equation for various values of the density and temperature and seek conditions for the stability of the superconducting state. Namely, we wish to construct a kind of phase diagram for quark matter, excluding the low-density region ( $\rho < 1 \text{ fm}^{-3}$ ), where it is unstable and transformed into hadronic matter.

Here we make some simplifications which do not alter the essential physical points. We consider quark matter with a single flavor and neglect the change of the chemical potential due to the pairing correlation. Our main interest is to discuss the phase transition to superconductivity in quark matter and to investigate the role of the temperature and the asymptotic freedom of QCD on the superconductivity, as stressed in the Introduction. Consequently, we use the “running” coupling constant of Higashijima<sup>10)</sup> and Miransky<sup>11)</sup> which ignores the confinement of quarks but incorporates the asymptotic freedom correctly. The coupling constant employed in this paper is written

$$\alpha_s(\bar{q}^2) = \frac{12\pi}{(11N_c - 2N_f) \ln[(\bar{q}^2 + p_c^2)/\Lambda^2]}, \quad (15)$$

where  $\bar{q} \equiv \bar{k} - \bar{l}$  is the Euclidean momentum transfer, and  $N_c = 3$  and  $N_f = 3$  denote the numbers of color and flavor of the quark, respectively. The quantity  $\Lambda$  is the cutoff parameter of QCD, and the parameter  $p_c$  has been introduced to make the coupling constant finite in the infrared limit,  $\bar{q}^2 \rightarrow 0$ . This fact corresponds to a lack of confinement. We approximate the momentum dependence of  $\alpha_s$  by the separable form<sup>10), 11)</sup>

$$\alpha_s(\bar{q}^2) \rightarrow \begin{cases} \alpha_s(\bar{k}^2) & \text{for } \bar{k}^2 > \bar{l}^2, \\ \alpha_s(\bar{l}^2) & \text{for } \bar{k}^2 < \bar{l}^2, \end{cases} \quad (16)$$

where  $\bar{k}$  and  $\bar{l}$  on the right-hand side denote the momenta attached to the vertex. In this paper, we assume  $\bar{q}^2 \approx (\mathbf{k} - \mathbf{l})^2$ , so that the coupling constant in our gap equation (13) may be replaced by

$$\alpha_s \rightarrow \begin{cases} \alpha_s(\mathbf{k}^2) & \text{for } |\mathbf{k}| > |\mathbf{l}|, \\ \alpha_s(\mathbf{l}^2) & \text{for } |\mathbf{k}| < |\mathbf{l}|. \end{cases} \quad (17)$$

This expression of the coupling constant ensures that the asymptotic behavior of the QCD coupling constant<sup>10)</sup> but does not lead to the confinement of quarks. The parameters of our “running” coupling constant are taken as

$$\Lambda = 0.4 \text{ GeV} \quad \text{and} \quad p_c^2 = 1.5\Lambda^2 \text{ GeV}^2. \quad (18)$$

These are the same as those in Ref. 10) and are consistent with the data of low-energy hadron physics.

Next let us discuss the problem of gluon degrees of freedom, which has been neglected in our discussion to this point. Many gluons would be created at finite temperature, according to the laws of statistical physics. Thus our system is a quark-gluon plasma which is composed of quarks and gluons. Of course the quarks and

gluons in the medium are different from those in the vacuum. It is, however, very difficult to study the quark-gluon plasma within full QCD.<sup>17), 18)</sup> For this reason, consider the following simple picture: the gluons in the medium interact with surrounding quarks and gluons so as to acquire a finite mass (become quasi-particles). Therefore it is natural to assume that the interaction between quarks is described by exchange of the modified gluon. This two-body interaction leads to superconductivity, and the quark acquires the gap discussed in the previous sections. Our interest is to investigate the superconductivity of quarks in the quark-gluon plasma, so that we do not consider the degrees of freedom of real gluons, except for the virtual gluons exchanged between two quarks. This effect is accounted for by making the following replacement in the gap equation (10):

$$\frac{\epsilon_k \epsilon_l + \mathbf{k} \cdot \mathbf{l}}{\epsilon_k \epsilon_l |\mathbf{k} - \mathbf{l}|^2} \rightarrow \frac{\epsilon_k \epsilon_l + \mathbf{k} \cdot \mathbf{l} + m^2}{2\epsilon_k \epsilon_l (|\mathbf{k} - \mathbf{l}|^2 + M_e^2)} + \frac{\epsilon_k \epsilon_l + \mathbf{k} \cdot \mathbf{l} - m^2}{2\epsilon_k \epsilon_l (|\mathbf{k} - \mathbf{l}|^2 + M_m^2)}. \quad (19)$$

Here the screening effect is taken into account by the gluon electric mass  $M_e$  and gluon magnetic mass  $M_m$ . These masses can be calculated by perturbative approximation<sup>17), 18)</sup> and are given by

$$M_e = 4\pi\alpha_s(\bar{q}^2) \left[ \left( \frac{1}{3}N_c + \frac{1}{6}N_f \right) T^2 + \frac{1}{2\pi^2} \sum_f \mu_f^2 \right] \quad \text{and} \quad M_m = 0, \quad (20)$$

respectively.

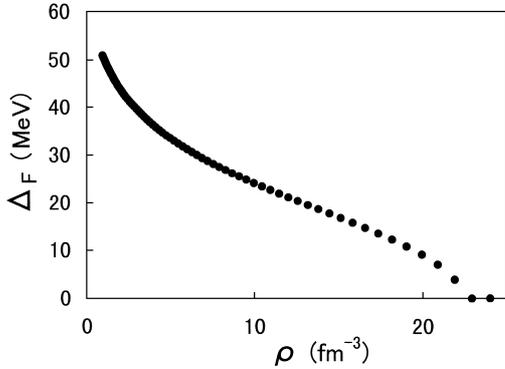


Fig. 1. The energy gap  $\Delta(k_F)$  as a function of the quark density  $\rho$  ( $T = 0$ ).

Numerical calculations of the gap equation (10) have been carried out taking into account the above-mentioned modifications. First let us investigate the quark density dependence of the energy gap at zero-temperature.<sup>9)</sup> The calculated energy gap at the Fermi momentum,  $\Delta_F \equiv \Delta(k_F)$ , is shown in Fig. 1 as a function of the quark density. It decreases with increasing quark density and vanishes at about  $\rho \sim 20 \text{ fm}^{-3}$ , which is caused by the asymptotic freedom of QCD. The reason for this behavior is the following: Only the momentum near the Fermi level contributes significantly to the integral of the gap equation.

On the other hand, the “running” coupling constant at the Fermi level approaches zero when the Fermi momentum  $k_F$  goes to infinity. Noting the relation  $k_F \propto \rho^{1/3}$ , we can realize that the vanishing gap at high density is due to asymptotic freedom. But it should be noted that *the energy gap decreases very slowly due to the slow decreasing of the “running” coupling constant (15).*

Next we discuss the temperature dependence of the energy gap  $\Delta_F$  at a certain quark density, choosing  $\rho = 1 \text{ fm}^{-3}$ . The result appears in Fig. 2. The energy gap decreases with increasing temperature and vanishes at a certain temperature (the critical temperature  $T_c \lesssim 50 \text{ MeV}$ ) just as in the electron-phonon system. If the value of the density is varied, we obtain the critical temperature as a function of the quark density, or a phase diagram for the quark matter. This behavior is described in Fig. 3, which is viewed as a phase diagram in the  $(T-\rho)$  plane. Of course this figure is valid only for  $\rho > 1 \text{ fm}^{-3}$ , since we do not consider the hadron phase ( $\rho < 1 \text{ fm}^{-3}$ ). The slight zigzag undulation in this figure is due to the fact that the variation step  $\Delta T$  of the temperature in our calculations is taken to be  $\Delta T = 1 \text{ MeV}$ . If we use a smaller  $\Delta T$ , we would obtain a smoother curve. The point under (above) the curve corresponds to the superconducting (normal) state. It is concluded that *the quark matter goes to the Fermi gas state under high density and/or high-temperature, as expected*. The phase transition at high density originates from the asymptotic freedom of QCD.

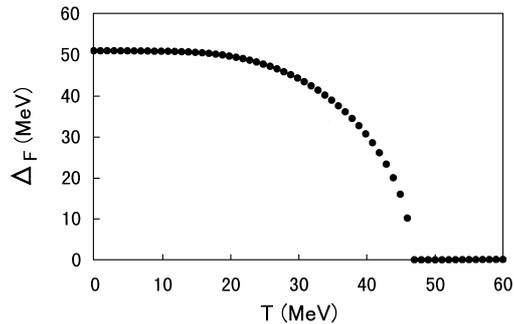


Fig. 2. The energy gap  $\Delta(k_F)$  as a function of the temperature  $T$  ( $\rho = 1 \text{ fm}^{-3}$ ).

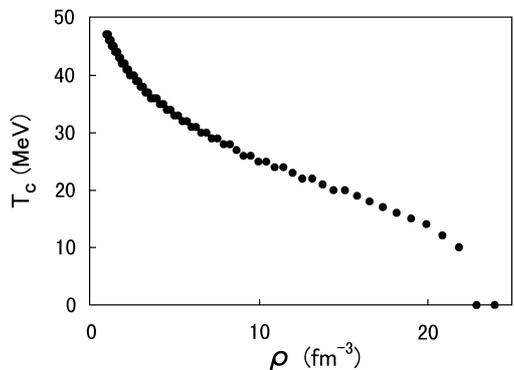


Fig. 3. The critical temperature  $T_c$  as a function of the quark density  $\rho$ .

#### §4. Concluding remarks

We have discussed the superconductivity of quark matter at finite temperature. The BCS theory has been extended to finite temperature and the gap equation has been obtained at finite temperature. The numerical calculations show that the energy gap decreases very slowly, owing to the logarithmic behavior of the “running” coupling constant. As a result, the superconductivity disappears for high density  $\rho \gtrsim 20 \text{ fm}^{-3}$  due to the asymptotic freedom of QCD and/or high-temperature  $T \gtrsim 50 \text{ MeV}$ . Thus we have obtained the phase diagram (Fig. 3) of quark matter in the  $(\rho-T)$  plane. In the following, some remarks are given.

It is most probable that quark matter would exist in the core of a compact star. For example, the core of a neutron star may be composed of  $u$ - and  $d$ -quark matter. Such quark matter may be in a super phase if the conditions for superconductivity discussed in this paper are satisfied. It would be very interesting to look for evidence that quark matter in the core of neutron stars exists in the super phase. Supercon-

ductivity would appear mainly in transport phenomena, such as the cooling,<sup>20) - 22)</sup> explosion and radiation of stars. Thus it is interesting to investigate the influence of the superconductivity on such physical quantities. In neutron stars we have coupled (*u*- and *d*-) superconductors which interact with each other through the weak interaction, etc. It is especially interesting to study transport phenomena between them.

Another possibility for the existence of quark matter is in high-energy heavy ion collisions. It is, however, doubtful if we could find superconducting quark matter in such a hot state. Even if quark (gluon) matter may be produced in the central region of high-energy heavy ion collisions, the high temperature of the matter would break down Cooper pairs, as discussed in this paper.

One of the characteristic features of our BCS theory is that our Cooper pair is spin-parallel (spin-triplet). This implies that a Cooper pair is more stable than an electron pair (spin-singlet) under a magnetic field. Since a strong magnetic field would exist inside neutron stars, this property may play an important role in neutron star physics.

Finally, let us comment on the possibility of another Cooper pair in quark matter. Our discussion has been restricted to Cooper pairs composed of quarks of the same flavor. Recently, many authors<sup>23)</sup> have discussed the existence of Cooper pairs composed of *u* and *d* quarks which interact through an instanton-induced interaction.

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