Soft Pions at High Energy as an Origin of Flavor Asymmetry of the Light Sea Quarks in the Nucleon

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By using the soft pion theorem in inclusive reactions, the soft pion contribution to the structure function F_2 in the nucleon is estimated. It is shown that this contribution produces such a large flavor asymmetry in the light sea quark distributions that it gives about 30–50% of the NMC deficit in the Gottfried sum.

§1. Introduction

The modified Gottfried sum rule $^{1)}$ has accounted for the NMC deficit in the Gottfried sum $^{2)}$ almost model independently. It has shown that the deficit is the reflection of the hadronic vacuum originating from the spontaneous chiral symmetry breakings. In this sense the physics underlining this algebraic approach has a feature common to that of the mesonic models reviewed in Ref. 3). However, in the algebraic approach, importance of the high energy region not only in the theoretical understanding but also in numerical analysis has been made clear. Further, the numerical prediction based on this sum rule agrees precisely with the recent experimental value from the E866/NuSea collaboration.⁴⁾ This experiment also gives us the light antiquark difference $(\bar{d}(x) - \bar{u}(x))$ and the ratio $\bar{d}(x)/\bar{u}(x)$ in the range $0.02 \le x \le 0.345$. An unexpected finding is that the asymmetry seems to disappear at large x. On the other hand, a typical calculation using the mesonic models based on the πNN and the $\pi N\Delta$ processes account for about half of the NMC deficit.³⁾ According to the E866 experiment, it should be possible to account for the remaining half of the NMC deficit by contributions in the medium and small x regions. Unfortunately, the approach using the mesonic models cannot account for the magnitude from these regions definitely. In fact, the $\pi N\Delta$ process partly cancels the positive contribution to $(\bar{d}(x) - \bar{u}(x))$ from the πNN process. The contributions from the higher resonances and from the multiparticle states are obscure. Hence the best we can say is that the mesonic models explain the flavor asymmetry of the light sea quarks qualitatively. These facts suggest that there may exist a dynamical mechanism so far overlooked to produce the flavor asymmetry at medium and high energy, and that it may compensate for the above-mentioned flaw of the mesonic models. In this paper, it is shown that the soft pion theorem in the inclusive reaction at high energy $^{5)}$ can account for about 30-50% of the NMC deficit, where we take its magnitude as 0.07, following the E866 experiment $^{4)}$ for definiteness.

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§2. Soft pions at high energy

Since the soft pion theorem in the inclusive reaction at high energy is not well known, let us first explain it briefly. Usually, the soft pion theorem has been considered to be applicable only in low energy regions. However in Ref. 5), it has been found that this theorem can be used in inclusive reactions at high energy if the Feynman scaling hypothesis holds. In the inclusive reaction $\pi + p \rightarrow \pi_s(k) + anything$, with π_s being the soft pion, it states that the differential cross-section in the center of mass (CM) frame defined as

$$f(k^{3}, \vec{k}^{\perp}, p^{0}) = k^{0} \frac{d\sigma}{d^{3}k},$$
(1)

where p^0 is the CM frame energy, scales as

$$f \sim f^F\left(\frac{k^3}{p^0}, \vec{k}^{\perp}\right) + \frac{g(k^3, \vec{k}^{\perp})}{p^0}.$$
 (2)

If $g(k^3, \vec{k}^{\perp})$ is not singular at $k^3 = 0$, we obtain

$$\lim_{p^0 \to \infty} f^F\left(\frac{k^3}{p^0}, \vec{k}^{\perp} = 0\right) = f^F(0, 0) = \lim_{p^0 \to \infty} f(0, 0, p^0).$$
(3)

This means that the π mesons with the momenta $k^3 < O(p^0)$ and $\vec{k}^{\perp} = 0$ in the CM frame can be interpreted as the soft pions. This fact holds even when a scaling violation effect exists, since we can replace the exact scaling by the approximate one in this discussion. In Weinberg's language, these soft pions correspond to semi-soft pions.⁶ The important point of this soft pion theorem is that the soft pion limit cannot be interchanged with the manipulation to obtain the discontinuity of the reaction $a + b + \bar{\pi}_s \rightarrow a + b + \bar{\pi}_s$. We must first take the soft pion limit in the reaction $a + b \rightarrow \pi_s + anything$. This is because the soft pion attached to the nucleon (anti-nucleon) in the final state is missed in the discontinuity of the soft pion limit of the reaction $a + b + \bar{\pi}_s \rightarrow a + b + \bar{\pi}_s$.⁵

Now, based on the null-plane formalism, this soft pion theorem has been extensively studied in Ref. 7). In the usual equal-time formalism, the contribution where the soft pion is attached to the nucleon (anti-nucleon) depends on its velocity, because the one particle helicity matrix element of the axial vector current takes the form $\langle p, h | J_a^{50}(0) | p, h' \rangle = 2hp^0 g_A(0)v\delta_{hh'}$, where h and h' denote the helicity and $v = |\vec{p}|/p^0$. On the other hand, in the null-plane formalism it takes the form $\langle p, h | J_a^{5+}(0) | p, h' \rangle = 2hp^+g_A(0)\delta_{hh'}$, where the light-like helicity base is used in this case. Hence in the null-plane formalism, the velocity factor is always 1, and the ambiguity from this part disappears. By using light-cone current algebra⁸) in the inclusive lepton-hadron scatterings, the theoretical prediction in the case of the soft π^- has been compared with the data for π^- production in the central region, and it has been suggested that the mechanism proposed in Ref. 5) should be applied to directly produced pions, i.e., pions not produced through the decay from the resonance.⁹ In Ref. 10), the cut vertex formalism¹¹ is used instead of the light-cone current algebra, and the charge asymmetry in the central region in the inclusive lepton-hadron scatterings is considered. This is because the pions from the resonance decay product due to the strong interaction cancel out in the asymmetry in the central region, and hence the experimentally measured asymmetry is mainly due to directly produced pions. It has been found that the experimental value roughly agrees with the theoretical prediction based on the soft pion theorem in the inclusive reactions. Several years ago, the photoproduction version of the modified Gottfried sum rule was studied, and it was found that the soft pion contribution at high energy plays an important role in satisfying the sum rules.¹²

§3. Contribution to the Gottfried sum

Let us now consider the reaction $\gamma_V(q)$ + nucleon $(p) \rightarrow \pi_s(k)$ + anythings, where γ_V represents the virtual photon. We take the soft pion limit $k^{\mu} \to 0$ by first setting $\vec{k}^{\perp} = 0, \ k^+ = 0$ and then taking $k^- \to 0$ in the scattering amplitude. In this limit we can classify it into three kinds of terms. The type (a) term is the amplitude in which the proper part of the axial-vector current attaches to the initial nucleon. The type (b) term is the amplitude in which the proper part of the axial-vector current attaches to the final nucleon or anti-nucleon. The type (c) term is the amplitude which comes from the commutation relation on the null-plane. Then by taking the square of the amplitude in the soft pion limit, we construct the hadronic tensor. Following Ref. 7), we classify the contribution to the hadronic tensor as follows: The term coming from the type $a^{\dagger}a$ is $A_1^{\mu\nu}$, that from the type $a^{\dagger}c + c^{\dagger}a$ is $A_2^{\mu\nu} + A_3^{\mu\nu}$, that from the type $b^{\dagger}b$ is $B_1^{\mu\nu}$, that from the type $b^{\dagger}c + c^{\dagger}b$ is $B_2^{\mu\nu} + B_3^{\mu\nu}$, that from the type $a^{\dagger}b + b^{\dagger}a$ is $C_1^{\mu\nu}$, and that from the type $c^{\dagger}c$ is $D_4^{\mu\nu}$, where a, b and c denote the type $c^{\dagger}c$ is $D_4^{\mu\nu}$. of the amplitude in the soft pion limit. Now, in inclusive reactions, the kinematic variables in the initial state are unconstrained in the soft pion limit. We can take the usual deep-inelastic limit. The hadronic tensor is light-cone dominated in the deep-inelastic limit. Hence we can use the light-cone current algebra and determine how the soft pion piece is related to the structure functions in the total inclusive reactions. In the perturbative analysis in QCD the Q^2 dependence can be taken into account by the cut vertex formalism suitable for light-cone dominated processes. This is because in our case the hadronic tensor is not short distance dominated in the short distance limit as in the hadonic tensor in the total inclusive reaction which is expressed by the matrix element of the commutation relation of the currents. In the soft pion limit, surviving pole terms are restricted by the pion's charge. For example, in the π_s^- case, the proper part of the axial-vector current attached to the initial proton is prohibited by charge conservation. Because of the asymmetry of this kind we encounter terms which cannot be expressed by the commutation relation. This prevents us from demonstrating the short distance dominance in the short distance limit. Thus the usual method, which makes the short distance expansion first and then continues it analytically to the light-cone with use of the dispersion relation, cannot be applied. The asymmetry discussed above, together with the fact that nucleon charge is changed when the proper part of the axial-vector current corresponding to the charged pion is attached to the nucleon, is the origin of the

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charge asymmetry in the soft pion limit. Now the contribution from $B_2^{\mu\nu} + B_3^{\mu\nu}$ and $C_1^{\mu\nu}$ can be neglected in the deep-inelastic region. In these terms, the positive helicity state of the final nucleon (anti-nucleon) and the negative one contribute oppositely in sign, and hence their contribution at high energy can be expected to be very small, while the contribution in the low energy region is suppressed by the form factor effect in the deep-inelastic region. Thus we consider the contribution only from $A_1^{\mu\nu}$, $A_2^{\mu\nu} + A_3^{\mu\nu}$, $B_1^{\mu\nu}$ and $D_4^{\mu\nu}$. Since the detailed expressions are given in Refs. 7) and 10), it is straightforward to obtain the soft pion contribution to the structure function F_2 . Adding the contributions from the soft π_s^+, π_s^- and π_s^0 , and subtracting the contributions to F_2^{en} from those to F_2^{ep} , we obtain

$$(F_2^{ep} - F_2^{en})|_{\text{soft}} = \frac{I_\pi}{4f_\pi^2} [g_A^2(0)(F_2^{ep} - F_2^{en})(3\langle n \rangle - 1) - 16xg_A(0)(g_1^{ep} - g_1^{en})], \qquad (4)$$

where I_{π} is the phase space factor for the soft pion defined as

$$I_{\pi} = \int \frac{d^2 \vec{k}^{\perp} dk^+}{(2\pi)^3 2k^+} \,. \tag{5}$$

Here $\langle n \rangle$ is the sum of the nucleon and anti-nucleon multiplicity defined as $\langle n \rangle = \langle n \rangle_p + \langle n \rangle_{\bar{n}} + \langle n \rangle_{\bar{p}} + \langle n \rangle_{\bar{n}}$. In Eq. (4), the contribution coming from $D_4^{\mu\nu}$ cancels. Among the terms proportional to $g_A^2(0)$, that which has a factor $\langle n \rangle$ comes from $B_1^{\mu\nu}$ and the other one comes from $A_1^{\mu\nu}$, and the term proportional to the spin-dependent function $(g_1^{ep} - g_1^{en})$ comes from $A_2^{\mu\nu} + A_3^{\mu\nu}$. Note that this spin-dependent term is obtained in the approximation in which the sea quark contribution to $(g_1^{ep} - g_1^{en})$ is ignored. Without this approximation, $16(g_1^{ep} - g_1^{en})$ in Eq. (4) should be replaced by $24(g_1^{ep} - g_1^{en}) - \frac{4}{3}(g_1^{\bar{\nu}p} - g_1^{\nu p})$.

Now, as explained, the soft pion contribution in the inclusive reaction cannot be obtained from the discontinuity formula in the sense that the interchanging the order of the manipulation to obtain the discontinuity formula and taking the soft pion limit is impossible. Because of this fact, we must revise the structure function $(F_2^{ep} - F_2^{en})$ as $(F_2^{ep} - F_2^{en}) = (F_2^{ep} - F_2^{en})_u + (F_2^{ep} - F_2^{en})|_{\text{soft}}$, where the suffix *u* specifies the usual one, which satisfies the generalized unitarity. In the parton model, using the impulse approximation, the structure function is obtained as the imaginary part of the incoherent elastic scattering of the virtual photon off quarks. Thus the soft pion piece is not included in the parton model in general. However, in the deep inelastic region we parametrize the structure function by the quark distribution functions. Hence we should revise them to include the soft pion contributions. Now the soft pion contributes to the structure function $(F_2^{\nu p} - F_2^{\bar{\nu}p})$, and the Adler sum rule fixes the valence quark distribution as $\int_0^1 dx(u_v - d_v) = 1$. Hence the phenomenologically determined valence quark distribution $(u_v - d_v)$, which already satisfies the constraint effectively, takes the contribution from the soft pion piece, since the Adler sum rule is satisfied only if this contribution is taken into account. Then we use these valence quark distributions to fit the structure function $(F_2^{ep} - F_2^{en})$. Therefore the soft pion piece $(F_2^{ep} - F_2^{en})|_{\text{soft}}$ should be effectively taken into account in the

phenomenologically determined sea quark distributions. Thus, by assuming that the light sea quark distribution is equal to its antiquark distribution for simplicity, we can express $(F_2^{ep} - F_2^{en})|_{\text{soft}}$ as the asymmetry of the antiquark distribution as

$$(F_2^{ep} - F_2^{en})|_{\text{soft}} = -\frac{2}{3}x(\bar{d} - \bar{u})|_{\text{soft}}.$$
(6)

To estimate the magnitude of this asymmetry, we approximate F_2^{ep} , F_2^{en} , g_1^{ep} and g_1^{en} on the right-hand side of Eq. (4) by the valence quarks distribution functions at $Q_0^2 = 4 \text{ GeV}^2$.¹³⁾ As the multiplicity of the nucleon and antinucleon, we set

$$\langle n \rangle = a \log_e s + 1,\tag{7}$$

where $s = (p+q)^2$. The parameter *a* is fixed to 0.2 in consideration of the proton and the anti-proton multiplicity in the e^+e^- annihilation such that $a \log_e \sqrt{s}$ with \sqrt{s} replaced by the CM energy of that reaction agrees with the multiplicity of that reaction.¹⁴ We estimate the pion phase space factor I_{π} as follows. We assume approximate Feynman scaling. Then, we regard the directly produced pions in the virtual-photon and the target-nucleon center of mass (CM) frame which satisfy the two conditions as soft pions.

(1) The transverse momentum satisfies $|\vec{k}^{\perp}| \leq bm_{\pi}$.

(2) The Feynman scaling variable $x_F = 2k^3/\sqrt{s}$ satisfies $|x_F| \le c$.

Here we take the momentum k in the CM frame and assume the constant b is near 1, and c is near 0.1. These values are fixed based on the previous works^{9), 10)} which showed that the consideration from directly produced pions in the central region in the CM frame as deduced from the experimentally measured quantity, is of the same order as the soft pion contribution. The experimentally expected values were always larger, but the differences were within a factor of 2. The upper and the lower limit of the integral with respect to k^+ in the phase space factor I_{π} are restricted by the condition (2). The lower limit of k^+ behaves as $O(1/\sqrt{s})$ at high energy. Because the soft pion contribution. Performing the explicit integration, we obtain the phase space factor I_{π} as

$$I_{\pi} = \frac{1}{16\pi^2} \left((b^2 + 1)m_{\pi}^2 \log_e \left(\frac{\sqrt{(1+b^2)m_{\pi}^2 + \frac{c^2s}{4}} + \frac{c\sqrt{s}}{2}}{\sqrt{(1+b^2)m_{\pi}^2 + \frac{c^2s}{4}} - \frac{c\sqrt{s}}{2}} \right) - m_{\pi}^2 \log_e \left(\frac{\sqrt{m_{\pi}^2 + \frac{c^2s}{4}} + \frac{c\sqrt{s}}{2}}{\sqrt{m_{\pi}^2 + \frac{c^2s}{4}} - \frac{c\sqrt{s}}{2}} \right) + c\sqrt{s} \left(\sqrt{(1+b^2)m_{\pi}^2 + \frac{c^2s}{4}} - \sqrt{m_{\pi}^2 + \frac{c^2s}{4}} \right) \right).$$
(8)

A typical example of the antiquark asymmetry $(\bar{d} - \bar{u})|_{\text{soft}}$ given by Eqs. (4) and (6) is given in Fig. 1 for a = 0.2, b = 1, and c = 0.1 in the range $0.05 \le x \le 0.6$. Further, to see the soft pion contribution to the asymmetry $(\bar{d} - \bar{u})$ qualitatively, we plot the value of $x(\bar{d} - \bar{u})$ of the CTEQ4M¹⁵ fit at $Q^2 = 4$ GeV² in Fig. 2. From Fig. 2 we can recognize that the soft pion contribution to the Gottfried sum

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Fig. 2. The soft pion contribution to $x(\bar{d}-\bar{u})$.

is large because the small x tail is slowly decreasing. However extrapolation of the theoretical curve to the very small x region cannot be trusted, because the input distribution cannot be trusted in the very small x region. Hence, we should cut the integral somewhere in the very small x region. While, theoretical predictions indicate that the contribution above x = 0.3 may be large, the contribution from this region to the Gottfried sum is small. Further, the phase space constraint from this region may become more stringent. In any case, it rapidly becomes zero as we go to large x. Now, in the medium x region, the small bump in Fig. 2 may be related to a small excess of the E866 data, compared with the contribution predicted by the meson cloud model,⁴⁾ since the soft pion contribution should be added to the contribution predicted by the meson cloud model as a background contribution. By taking these facts into consideration, we investigate $J(\alpha, \beta) = \int_{\alpha}^{\beta} \frac{dx}{x} (F_2^{ep} - F_2^{en})|_{\text{soft}}$ for various values of a, b and c. For a = 0.18, b = 1, and c = 0.1 we have $J(10^{-4}, 0.2) = -0.019, J(10^{-5}, 0.2) = -0.022, \text{ and } J(10^{-6}, 0.2) = -0.024.$ For $a = 0.20, b = 1, and c = 0.1, we have J(10^{-4}, 0.2) = -0.018, J(10^{-5}, 0.2) = -0.021,$ and $J(10^{-6}, 0.2) = -0.023$. For a = 0.22, b = 1, and c = 0.1, we have $J(10^{-4}, 0.2) =$ -0.017, $J(10^{-5}, 0.2) = -0.019$, and $J(10^{-6}, 0.2) = -0.021$. Thus the effect of the change of a consistent with the experimental value of the multiplicity data for the e^+e^- experiment is small. The extrapolation of the integral to smaller values of x make the value of J smaller, but the magnitude to be added to the value of J due to this extrapolation is not so large. For example for a = 0.20, b = 1, and c = 0.1, we have $J(10^{-9}, 1) = -0.030$ and J(0.2, 1) = -0.005. To understand the effect of the change of c, we consider the case a = 0.2, b = 1, and c = 0.05 and obtain $J(10^{-4}, 0.2) = -0.012, J(10^{-5}, 0.2) = -0.015, \text{ and } J(10^{-6}, 0.2) = -0.017.$ Thus the effect of this change is a 25% reduction compared with the case c = 0.1. We assume that b takes a value near 1, except in the large x region at low energy, where the allowed phase space becomes a ball rather than a cylinder defined by the conditions (1) and (2). This causes a more rapid decrease at large x than that shown in Fig. 2. However, the change in this region does not give a sizable effect to the value of J. Though we cannot say the exact magnitude, we see that the soft pion contribution has a sizable effect on the NMC defect. Based on the above analysis, we estimate that J(0,1) takes a value in the range -0.04 - -0.02.

§4. Conclusion

The soft pion theorem in the inclusive reaction is very general, and it is useful if an approximate scaling, such as the Feynman scaling, holds. The magnitude of the contribution is non-negligible, as shown in Refs. 9) and 10) and also in the present example. In fact, it can reach about 30 - 50% of the NMC deficit. This is just the amount missing in typical calculations using the mesonic models.³⁾ The main contribution from the soft pion comes from the medium and high energy regions, where the mesonic model lacks predictive ability and where the algebraic approach has been found to be important.

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