

Gaugeon Formalism for Spin-3/2 Rarita-Schwinger Gauge Field

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We apply the BRST symmetric gaugeon formalism to the spin-3/2 Rarita-Schwinger gauge field. To this time, this formalism has been developed for quantum electrodynamics and the Yang-Mills gauge field. The theory admits a quantum gauge transformation by which we can shift the gauge fixing parameter. The quantum gauge transformation does not change the BRST charge. Thus, the physical Hilbert space is trivially independent of the gauge fixing parameter. By virtue of the BRST symmetry, the physical Hilbert space is still well-defined when we incorporate the interaction with the Ricci flat background gravitational field.

§1. Introduction

In the standard formalism of canonically quantized gauge theories,^{1),2)} we do not consider the gauge transformation on the quantum level. There is no quantum gauge freedom, since the quantum theory is defined only after the gauge fixing.

Yokoyama's gaugeon formalism³⁾⁻⁸⁾ provides a wider framework in which we can consider the quantum gauge transformation among a family of Lorentz covariant linear gauges. In this formalism, a set of extra fields, so-called gaugeon fields, is introduced as the quantum gauge freedom. This theory was proposed for quantum electrodynamics³⁾⁻⁵⁾ and for Yang-Mills theory.^{6),7)} Owing to the quantum gauge freedom, it becomes almost trivial to check the gauge parameter independence of the physical S -matrix.⁸⁾

We should ensure that the gaugeon modes do not contribute to physical processes. In fact, the gaugeon fields yield negative normed states.³⁾ To remove these unphysical modes, Yokoyama imposed a Gupta-Bleuler-type subsidiary condition,^{3),6),7)} which is not applicable if an interaction exists for gaugeon fields. Yokoyama's condition has been improved by introducing BRST symmetry for gaugeon fields.⁹⁾⁻¹²⁾ Unphysical gaugeon modes, as well as unphysical modes of the gauge field, are removed by a single Kugo-Ojima-type condition.²⁾ Thus, the formalism is now applicable even in the background gravitational field. The BRST symmetry is also very helpful in the analysis of the gauge structure of the Fock space in the gaugeon formalism.^{11),13)} In particular, we can define the gauge invariant physical Hilbert space.

Presently, we have the BRST symmetric gaugeon formalism of the electromagnetic gauge theory^{10),11)} and of the Yang-Mills gauge theory.^{9),12)} There are, how-

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ever, other types of gauge fields, such as the gravitational field, the gravitino field (spin-3/2 gauge field), the anti-symmetric tensor gauge fields and string theory. One might wonder whether the gaugeon formalism is applicable to these gauge fields.

The main purpose of the present paper is to show that the BRST symmetric gaugeon formalism is applicable to the spin-3/2 Rarita-Schwinger gauge field, and that the physical Hilbert space of the theory in this formalism is gauge invariant. Although we treat it mainly in the free field case, the interaction with the Ricci flat background gravity is also discussed. This will be the first step to investigate the supergravity theory in the gaugeon formalism.

The paper is organized as follows. In §2, we briefly review the theory of Hata and Kugo¹⁴⁾ as a standard formalism of the canonically quantized spin-3/2 gauge field. In §3, we propose a BRST symmetric gaugeon formalism for the spin-3/2 gauge field, where the gauge fixing parameter can be shifted by a q -number gauge transformation. Then, the gauge invariant physical Hilbert space is defined. We see in §4 how the Fock space of the standard formalism is embedded in the wider Fock space of the present formalism. Section 5 is devoted to comments and discussion, including remarks on the interaction with background gravity.

§2. Standard formalism

The classical Lagrangian of the free gravitino field ψ_μ in n (≥ 3) dimensional flat space-time is given by^{*)}

$$\mathcal{L}_{\text{RS}} = -\frac{i}{2} \bar{\psi}_\mu \gamma^{\mu\nu\lambda} \partial_\nu \psi_\lambda, \quad (2.1)$$

where $\gamma^{\mu\nu\lambda}$ is the matrix $\gamma^\mu \gamma^\nu \gamma^\lambda$ antisymmetrized with respect to μ, ν and λ :

$$\gamma^{\mu\nu\lambda} = \frac{1}{6} (\gamma^\mu \gamma^\nu \gamma^\lambda \pm 5 \text{ terms}). \quad (2.2)$$

The factor 1/2 arises in (2.1) because we assumed the field ψ_μ to be a Majorana spinor-vector.^{**)} The Lagrangian (2.1) is invariant up to total derivatives under the gauge transformation

$$\delta\psi_\mu = \partial_\mu A, \quad (2.3)$$

where A is an arbitrary spinor field.

To carry out the quantization, it is necessary to add a gauge fixing term and a corresponding Faddeev-Popov (FP) ghost term. As a standard formalism, we use the theory of Hata and Kugo.¹⁴⁾ Their quantum Lagrangian is given by

$$\mathcal{L}_{\text{HK}} = \mathcal{L}_{\text{RS}} + \bar{B} \not{\partial} (\gamma\psi) - \frac{ia}{2} \bar{B} \not{\partial} B - i\partial_\mu \bar{c}_* \partial^\mu c, \quad (2.4)$$

^{*)} We use the convention of Bjorken-Drell. For example, $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$ with $g_{\mu\nu} = \text{diag}(1, -1, -1, \dots, -1)$.

^{**)} If one needs to consider the non-Majorana case, the factor 1/2 should be omitted. In this paper, we assume that all the spinor fields are Majorana.

where $\gamma\psi = \gamma^\mu\psi_\mu$, $\not{\partial} = \gamma^\mu\partial_\mu$, B is a spinor multiplier (subject to Fermi statistics), c and c_* are the spinor FP ghosts (subject to Bose statistics), and a is a numerical gauge-fixing parameter.*) Note that the FP ghost fields satisfy a second order differential equation. Owing to this property, the FP ghosts c and c_* together with the multiplier B realize the correct ghost counting.^{15),16)}

The field equations are given by

$$\begin{aligned} -i\gamma^{\mu\nu\lambda}\partial_\nu\psi_\lambda &= \gamma^\mu\not{\partial}B, \\ \not{\partial}(\gamma\psi) &= ia\not{\partial}B, \\ \square c = \square c_* &= 0, \end{aligned} \tag{2.5}$$

from which we also have

$$\begin{aligned} \square B &= 0, \\ \square(\gamma\psi) &= 0. \end{aligned} \tag{2.6}$$

The Lagrangian (2.4) leads to the following n -dimensional (anti)commutation relations:

$$\begin{aligned} \{\psi_\mu(x), \bar{\psi}_\nu(y)\} &= \left[g_{\mu\nu}\not{\partial} + \frac{1}{n-2}\gamma_\mu\not{\partial}\gamma_\nu - \frac{2}{n-2}(\gamma_\mu\partial_\nu + \gamma_\nu\partial_\mu) \right] D(x-y) \\ &\quad + \left(\frac{4}{n-2} - a \right) \partial_\mu\partial_\nu\not{\partial}E(x-y), \\ \{B(x), \bar{\psi}_\nu(y)\} &= i\partial_\nu D(x-y), \\ \{B(x), \bar{B}(y)\} &= 0, \\ [c(x), \bar{c}_*(y)] &= -D(x-y), \end{aligned} \tag{2.7}$$

where the functions D and E are defined by

$$\begin{aligned} \square D(x) = 0, \quad D(0, \mathbf{x}) = 0, \quad \dot{D}(0, \mathbf{x}) &= -\delta^{n-1}(\mathbf{x}), \\ \square E(x) = D(x), \quad E(0, \mathbf{x}) = \dot{E}(0, \mathbf{x}) &= 0. \end{aligned} \tag{2.8}$$

From the first equation of (2.7) we have

$$\{\gamma\psi(x), \bar{\psi}(y)\gamma\} = -a\not{\partial}D(x-y). \tag{2.9}$$

There are two special gauges. One is the Landau gauge ($a = 0$), in which $\{\gamma\psi, \bar{\psi}\gamma\} = 0$, so that the $\gamma\psi$ mode has vanishing norm. The other is the Feynman gauge ($a = 4/(n-2)$), in which ψ_μ does not include dipole modes; that is, the Feynman propagator $\langle T(\psi_\mu\bar{\psi}_\nu) \rangle$ does not have a $1/p^4$ term.**)

*) Hereafter, we refer to this parameter a as the "standard gauge parameter."

**) In the Feynman gauge, it is convenient to use the field variable $\phi_\mu = \psi_\mu - \frac{1}{2}\gamma_\mu(\gamma\psi)$.¹⁷⁾ When $a = 4/(n-2)$, this variable satisfies the Dirac equation $\not{\partial}\phi_\mu = 0$, and the anticommutation relation becomes

$$\{\phi_\mu(x), \bar{\phi}_\nu(y)\} = g_{\mu\nu}\not{\partial}D(x-y).$$

The Lagrangian (2.4) is invariant up to total derivative terms under the following BRST transformation:

$$\begin{aligned} \delta_B \psi_\mu &= i\partial_\mu c, \\ \delta_B c_* &= B, \\ \delta_B B &= \delta_B c = 0. \end{aligned} \tag{2.10}$$

The corresponding conserved BRST charge is given by

$$Q_{B(\text{HK})} = -i \int \bar{B} \overleftrightarrow{\partial}_0 c \, d^{n-1} \mathbf{x}, \tag{2.11}$$

where $\overleftrightarrow{\partial}_0 = \overrightarrow{\partial}_0 - \overleftarrow{\partial}_0$. Using this charge we can define the physical subspace $\mathcal{V}_{\text{phys}}^{(\text{HK})}$ as a space of the states which satisfy the physical subsidiary condition of Kugo-Ojima,²⁾

$$Q_{B(\text{HK})} |\text{phys}\rangle = 0. \tag{2.12}$$

There are many unphysical zero-normed states in the physical subspace $\mathcal{V}_{\text{phys}}^{(\text{HK})}$. In fact, $\mathcal{V}_{\text{phys}}^{(\text{HK})}$ has the zero-normed subspace

$$\text{Im } Q_{B(\text{HK})} = \left\{ |\Phi\rangle; |\Phi\rangle = Q_{B(\text{HK})} |*\rangle \right\}.$$

Considering the quotient space of $\mathcal{V}_{\text{phys}}^{(\text{HK})}$ by this subspace, we can define the physical Hilbert space,

$$\mathcal{H}_{\text{phys}}^{(\text{HK})} = \mathcal{V}_{\text{phys}}^{(\text{HK})} / \text{Im } Q_{B(\text{HK})}, \tag{2.13}$$

which has positive definite metric.

§3. Gaugeon formalism

We start from the Lagrangian

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\text{RS}} + \bar{B} \not{\partial} (\gamma\psi) - \frac{i\varepsilon}{2} (\bar{Y}_* + \alpha\bar{B}) \not{\partial} (Y_* + \alpha B) - \partial_\mu \bar{Y}_* \partial^\mu Y \\ &\quad - i\partial_\mu \bar{c}_* \partial^\mu c - i\partial_\mu \bar{K}_* \partial^\mu K, \end{aligned} \tag{3.1}$$

where, in addition to the usual multiplier B and FP ghosts c and c_* , we have introduced the spinor gaugeon fields Y and Y_* (subject to Fermi statistics) and the corresponding spinor FP ghosts K and K_* (subject to Bose statistics). In (3.1), ε denotes a sign factor ($\varepsilon = \pm 1$), and α is a numerical gauge-fixing parameter. As seen below, the standard gauge-fixing parameter, which is denoted by a in the present paper, can be identified with

$$a = \varepsilon\alpha^2. \tag{3.2}$$

3.1. Field equations and (anti)commutation relations

The field equations that follow from (3·1) are

$$\begin{aligned}
 -i\gamma^{\mu\nu\lambda}\partial_\nu\psi_\lambda &= \gamma^\mu\rlap{-}\partial B, \\
 \rlap{-}\partial(\gamma\psi) &= i\varepsilon\alpha\rlap{-}\partial(Y_* + \alpha B), \\
 \square Y &= i\varepsilon\rlap{-}\partial(Y_* + \alpha B), \\
 \square Y_* &= 0, \\
 \square c &= \square c_* = 0, \\
 \square K &= \square K_* = 0.
 \end{aligned}
 \tag{3·3}$$

From these equations we also have

$$\begin{aligned}
 \square B &= 0, \\
 \square(\gamma\psi) &= 0, \\
 \rlap{-}\partial\square Y &= 0.
 \end{aligned}
 \tag{3·4}$$

The canonical prescription of quantization leads to the following n -dimensional (anti)commutation relations: Among the usual fields (ψ_μ, B, c, c_*), we have

$$\begin{aligned}
 \{\psi_\mu(x), \bar{\psi}_\nu(y)\} &= \left[g_{\mu\nu}\rlap{-}\partial + \frac{1}{n-2}\gamma_\mu\rlap{-}\partial\gamma_\nu - \frac{2}{n-2}(\gamma_\mu\partial_\nu + \gamma_\nu\partial_\mu) \right] D(x-y) \\
 &\quad + \left(\frac{4}{n-2} - \varepsilon\alpha^2 \right) \partial_\mu\partial_\nu\rlap{-}\partial E(x-y), \\
 \{B(x), \bar{\psi}_\nu(y)\} &= i\partial_\nu D(x-y), \\
 \{B(x), \bar{B}(y)\} &= 0, \\
 [c(x), \bar{c}_*(y)] &= -D(x-y).
 \end{aligned}
 \tag{3·5}$$

Among the gaugeons and their FP ghosts (Y, Y_*, K, K_*), we have

$$\begin{aligned}
 \{Y_*(x), \bar{Y}_*(y)\} &= 0, \\
 \{Y_*(x), \bar{Y}(y)\} &= -iD(x-y), \\
 \{Y(x), \bar{Y}(y)\} &= \varepsilon\rlap{-}\partial E(x-y), \\
 [K(x), \bar{K}_*(y)] &= -D(x-y).
 \end{aligned}
 \tag{3·6}$$

Anticommutators between the gaugeons and the usual fields are given by

$$\begin{aligned}
 \{Y_*(x), \bar{B}(y)\} &= \{Y_*(x), \bar{\psi}_\mu(y)\} = 0, \\
 \{Y(x), \bar{B}(y)\} &= 0, \\
 \{Y(x), \bar{\psi}_\mu(y)\} &= -\varepsilon\alpha\partial_\mu\rlap{-}\partial E(x-y).
 \end{aligned}
 \tag{3·7}$$

The (anti)commutation relations (3·5) are exactly the same as (2·7) if we assume (3·2). In particular, $\alpha = 0$ corresponds to the Landau gauge, and $\alpha = 2/\sqrt{n-2}$ (together with $\varepsilon = +1$) leads to the Feynman gauge. We note from (3·7) that in the Landau gauge ($\alpha = 0$), the gaugeon modes Y and Y_* completely decouple from the usual fields ψ_μ and B .

3.2. BRST symmetry

The Lagrangian (3.1) is invariant up to total derivatives under the following BRST transformation:

$$\begin{aligned}
 \delta_B \psi_\mu &= i \partial_\mu c, \\
 \delta_B c_* &= B, \\
 \delta_B B &= \delta_B c = 0, \\
 \delta_B Y &= iK, \\
 \delta_B K_* &= Y_*, \\
 \delta_B Y_* &= \delta_B K = 0.
 \end{aligned} \tag{3.8}$$

This obviously satisfies the nilpotency, $\delta_B^2 = 0$. The corresponding conserved BRST charge is given by

$$Q_B = -i \int \left(\bar{B} \overleftrightarrow{\partial}_0 c + \bar{Y}_* \overleftrightarrow{\partial}_0 K \right) d^{n-1} \mathbf{x}. \tag{3.9}$$

With the help of this charge we can define the physical subspace $\mathcal{V}_{\text{phys}}$ as the space of states satisfying

$$Q_B |\text{phys}\rangle = 0. \tag{3.10}$$

This subsidiary condition removes the gaugeon modes as well as the unphysical gravitino modes from the physical subspace; Y and Y_* together with K and K_* constitute the BRST quartet.²⁾

3.3. q -number gauge transformation

The Lagrangian (3.1) admits a q -number gauge transformation. Under the field redefinition

$$\begin{aligned}
 \hat{\psi}_\mu &= \psi_\mu + \tau \partial_\mu Y, \\
 \hat{Y}_* &= Y_* - \tau B, \\
 \hat{B} &= B, \quad \hat{Y} = Y, \\
 \hat{c} &= c + \tau K, \\
 \hat{K}_* &= K_* - \tau c_*, \\
 \hat{c}_* &= c_*, \quad \hat{K} = K,
 \end{aligned} \tag{3.11}$$

with τ being a numerical parameter, the Lagrangian (3.1) is *form invariant* (up to total derivative terms); that is, it satisfies

$$\mathcal{L}(\varphi_A, \alpha) = \mathcal{L}(\hat{\varphi}_A, \hat{\alpha}) + \text{total derivatives}, \tag{3.12}$$

where φ_A stands for any of the relevant fields and $\hat{\alpha}$ is defined by

$$\hat{\alpha} = \alpha + \tau. \tag{3.13}$$

An immediate conclusion from the form invariance (3.12) is the following: All the field equations and all the (anti)commutation relations are *gauge covariant* under

the q -number gauge transformation (3.11). $\hat{\varphi}_A$ satisfies the field equations (3.3) and (3.4) and the (anti)commutation relations (3.5) – (3.7) with α replaced by $\hat{\alpha}$.

It should be noted that the q -number gauge transformation (3.11) commutes with the BRST transformation (3.8). As a result, our BRST charge (3.9) is invariant under the q -number gauge transformation:

$$\hat{Q}_B = Q_B. \tag{3.14}$$

The physical subspace $\mathcal{V}_{\text{phys}}$ is, therefore, also invariant under the q -number gauge transformation:

$$\hat{\mathcal{V}}_{\text{phys}} = \mathcal{V}_{\text{phys}}. \tag{3.15}$$

Similarly, our physical Hilbert space $\mathcal{H}_{\text{phys}} = \mathcal{V}_{\text{phys}}/\text{Im } Q_B$ is also gauge invariant:

$$\hat{\mathcal{H}}_{\text{phys}} = \mathcal{H}_{\text{phys}}. \tag{3.16}$$

§4. Gauge structure of the Fock space

In addition to the BRST symmetry (3.8), the Lagrangian (3.1) has several other symmetries. In particular, we have the following BRST-like conserved charges:

$$\begin{aligned} Q_{B(\text{HK})} &= -i \int \bar{c} \overleftrightarrow{\partial}_0 B d^{n-1} \mathbf{x}, \\ Q_{B(Y)} &= -i \int \bar{K} \overleftrightarrow{\partial}_0 Y_* d^{n-1} \mathbf{x}, \\ Q'_{B(\text{HK})} &= -i \int \bar{K} \overleftrightarrow{\partial}_0 B d^{n-1} \mathbf{x}, \\ Q'_{B(Y)} &= -i \int \bar{c} \overleftrightarrow{\partial}_0 Y_* d^{n-1} \mathbf{x}. \end{aligned} \tag{4.1}$$

All of these satisfy the nilpotency condition. Our BRST charge Q_B can be decomposed as

$$Q_B = Q_{B(\text{HK})} + Q_{B(Y)}. \tag{4.2}$$

The charge $Q_{B(\text{HK})}$ generates the BRST transformation only for the usual fields ψ_μ , B , c and c_* , while $Q_{B(Y)}$ applies only for Y , Y_* , K and K_* . The charge $Q'_{B(\text{HK})}$ generates the BRST transformation for ψ_μ and B , but with K and K_* treated as their FP ghosts. Similarly, $Q'_{B(Y)}$ generates the BRST transformation for Y and Y_* , with c and c_* as their FP ghosts.

In the last section, we took (3.10) as a physical condition. Instead of this, however, we may choose the condition as

$$\begin{aligned} Q_{B(\text{HK})} |\text{phys}\rangle &= 0, \\ Q_{B(Y)} |\text{phys}\rangle &= 0. \end{aligned} \tag{4.3}$$

The unphysical modes of the gravitino are removed by the first equation, while the gaugeon modes are removed by the second. We express the space of states satisfying

(4.3) by $\mathcal{V}_{\text{phys}}^{(\alpha)}$. As is easily seen, this space is a subspace of $\mathcal{V}_{\text{phys}}$ defined in the last section:

$$\mathcal{V}_{\text{phys}}^{(\alpha)} \subset \mathcal{V}_{\text{phys}}. \tag{4.4}$$

We have attached the index (α) to $\mathcal{V}_{\text{phys}}^{(\alpha)}$ to emphasize that its definition depends on the gauge fixing parameter α . In fact, the BRST charges $Q_{\text{B(HK)}}$ and $Q_{\text{B(Y)}}$ transform as

$$\begin{aligned} \widehat{Q}_{\text{B(HK)}} &= Q_{\text{B(HK)}} + \tau Q'_{\text{B(HK)}}, \\ \widehat{Q}_{\text{B(Y)}} &= Q_{\text{B(Y)}} - \tau Q'_{\text{B(HK)}}, \end{aligned} \tag{4.5}$$

while their sum Q_{B} (and thus $\mathcal{V}_{\text{phys}}$) remains invariant.

Let us define the subspace $\mathcal{V}^{(\alpha)}$ of the total Fock space \mathcal{V} by

$$\mathcal{V}^{(\alpha)} = \ker Q_{\text{B(Y)}} = \{|\Phi\rangle \in \mathcal{V}; Q_{\text{B(Y)}}|\Phi\rangle = 0\} \subset \mathcal{V}, \tag{4.6}$$

which includes $\mathcal{V}_{\text{phys}}^{(\alpha)}$ as a subspace, since by definition $\mathcal{V}_{\text{phys}}^{(\alpha)}$ can be expressed as

$$\mathcal{V}_{\text{phys}}^{(\alpha)} = \{|\Phi\rangle \in \mathcal{V}^{(\alpha)}; Q_{\text{B(HK)}}|\Phi\rangle = 0\} \subset \mathcal{V}^{(\alpha)}. \tag{4.7}$$

The space $\mathcal{V}^{(\alpha)}$ corresponds to the total Fock space of the standard formalism in the $a = \varepsilon\alpha^2$ gauge. Thus, as seen from (4.7), $\mathcal{V}_{\text{phys}}^{(\alpha)}$ corresponds to the physical subspace $\mathcal{V}_{\text{phys}}^{(\text{HK})}$ of the standard formalism in the $a = \varepsilon\alpha^2$ gauge. This can be understood from the facts that

1. The modes of gaugeons and their FP ghosts are removed from the space $\mathcal{V}^{(\alpha)}$ by the condition $Q_{\text{B(Y)}}|\text{phys}\rangle = 0$.
2. The usual fields ψ_μ, B, c and c_* satisfy the (anti)commutation relations exactly, just as those of the standard formalism in the $a = \varepsilon\alpha^2$ gauge.

One may understand the first fact by expressing the Lagrangian (3.1) as

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\text{HK}}(a = \varepsilon\alpha^2) - i \left\{ Q_{\text{B(Y)}}, \partial_\mu \bar{K}_* \partial^\mu Y + i\varepsilon \bar{K}_* \not{\partial} \left(\frac{1}{2} Y_* + \alpha B \right) \right\} \\ &\quad + \text{total derivatives}, \end{aligned} \tag{4.8}$$

where $\mathcal{L}_{\text{HK}}(a = \varepsilon\alpha^2)$ denotes the Lagrangian of the $a = \varepsilon\alpha^2$ standard formalism.

We emphasize that the above arguments are also valid if we start from the q -number gauge transformed charges (4.5) rather than from $Q_{\text{B(HK)}}$ and $Q_{\text{B(Y)}}$. For example, we can define the subspaces $\mathcal{V}^{(\alpha+\tau)}$ and $\mathcal{V}_{\text{phys}}^{(\alpha+\tau)}$ by

$$\begin{aligned} \mathcal{V}^{(\alpha+\tau)} &= \ker \widehat{Q}_{\text{B(Y)}}, \\ \mathcal{V}_{\text{phys}}^{(\alpha+\tau)} &= \ker \widehat{Q}_{\text{B(HK)}} \cap \ker \widehat{Q}_{\text{B(Y)}}. \end{aligned} \tag{4.9}$$

$\mathcal{V}^{(\alpha+\tau)}$ can be identified with the Fock space of the standard formalism in the $a = \varepsilon(\alpha + \tau)^2$ gauge, and $\mathcal{V}_{\text{phys}}^{(\alpha+\tau)}$ corresponds to its physical subspace. Thus various Fock spaces of the standard formalism in different gauges are embedded in the single Fock

space \mathcal{V} of our theory.*)

§5. Comments and discussion

5.1. Type II theory

We have seen in §3 that the gauge fixing parameter α can be shifted freely by the q -number gauge transformation. However, we cannot change the sign of the standard gauge parameter $a = \varepsilon\alpha^2$. The situation is analogous to the Type I gaugeon formalism for QED. There are two types of the gaugeon theory for QED.⁴⁾ One of them is the Type I theory, where the standard gauge parameter is expressed as $a = \varepsilon\alpha^2$, and the other is the Type II theory, where $a = \alpha$. For both types of the theory, α can be shifted as $\hat{\alpha} = \alpha + \tau$ by the q -number gauge transformation. Thus, in the Type II theory, we can shift the standard gauge parameter quite freely. We comment here that the Type II theory can also be formulated for the spin-3/2 gauge field.**)

Let us consider the Lagrangian

$$\begin{aligned} \mathcal{L}_{II} = \mathcal{L}_{RS} + \bar{B}\not{\partial}(\gamma\psi) - \frac{i\alpha}{2}\bar{B}\not{\partial}B - \frac{i}{2}\bar{Y}_*\not{\partial}B - \partial_\mu\bar{Y}_*\partial^\mu Y \\ - i\partial_\mu\bar{c}_*\partial^\mu c - i\partial_\mu\bar{K}_*\partial^\mu K. \end{aligned} \tag{5.1}$$

Under the q -number gauge transformation (3.11), this Lagrangian is also form invariant (up to total derivatives):

$$\mathcal{L}_{II}(\varphi_A, \alpha) = \mathcal{L}_{II}(\hat{\varphi}_A, \hat{\alpha}) + \text{total derivatives}, \tag{5.2}$$

with $\hat{\alpha}$ defined by (3.13). As is easily seen, the Lagrangian (5.1) is also invariant (up to total derivatives) under all of the transformations corresponding to the BRST charges (4.1). Using the charge $Q_{B(Y)}$, we can express the Lagrangian as

$$\mathcal{L}_{II} = \mathcal{L}_{HK}(a = \alpha) - i \left\{ Q_{B(Y)}, \partial_\mu\bar{K}_*\partial^\mu Y + \frac{i}{2}\bar{K}_*\not{\partial}B \right\} + \text{total derivatives}, \tag{5.3}$$

which leads to the identification

$$a = \alpha. \tag{5.4}$$

This is exactly the characteristic of a Type II theory. It should be noted that all the arguments given in §4 also apply to this Type II theory.

*) Strictly speaking, we have two theories corresponding to the values of $\varepsilon = \pm 1$. Consequently, we have two Fock spaces, to which we refer as \mathcal{V}_+ and \mathcal{V}_- , corresponding to the value of ε . Thus the statement becomes the following: All of the Fock spaces of the standard formalism for all values of $a \geq 0$ [$a \leq 0$] are embedded in the single Fock space \mathcal{V}_+ [\mathcal{V}_-] of our theory.

***) Furthermore, we can see that the extended Type I theory for QED¹³⁾ is also applicable to the spin-3/2 gauge field.

5.2. Gauge invariance

We have seen in §4 that the subspace $\mathcal{V}^{(\alpha)} = \ker Q_{B(Y)} \subset \mathcal{V}$ can be identified with the total Fock space $\mathcal{V}^{(\text{HK})}$ of the standard formalism in the $a = \varepsilon\alpha^2$ gauge. This does not mean, however, that these spaces are isomorphic. Instead, we can show the following isomorphism:

$$\mathcal{V}^{(\alpha)} / \text{Im } Q_{B(Y)} \cong \mathcal{V}^{(\text{HK})}. \quad (5.5)$$

Namely, by considering the quotient space, we can ignore the $Q_{B(Y)}$ -exact states (states having the form $Q_{B(Y)}|*\rangle$), which have no corresponding states in $\mathcal{V}^{(\text{HK})}$. Equation (5.5) is a precise statement that our theory includes the standard formalism as a sub-theory. As for the Hilbert spaces, it can be shown that

$$\mathcal{H}_{\text{phys}}^{(\alpha)} \cong \mathcal{H}_{\text{phys}}^{(\text{HK})}, \quad (5.6)$$

where $\mathcal{H}_{\text{phys}}^{(\alpha)}$ is a physical Hilbert space defined by

$$\mathcal{H}_{\text{phys}}^{(\alpha)} = \mathcal{V}_{\text{phys}}^{(\alpha)} / \mathcal{N}^{(\alpha)}, \quad (5.7)$$

with $\mathcal{N}^{(\alpha)}$ being a zero-normed subspace of $\mathcal{V}_{\text{phys}}^{(\alpha)}$. Furthermore, we can also verify that our gauge invariant Hilbert space $\mathcal{H}_{\text{phys}}$ is isomorphic to $\mathcal{H}_{\text{phys}}^{(\alpha)}$. Therefore, we are lead to the gauge invariant result

$$\mathcal{H}_{\text{phys}}^{(\text{HK})} \cong \mathcal{H}_{\text{phys}}^{(\alpha)} \cong \mathcal{H}_{\text{phys}}. \quad (5.8)$$

Detailed arguments regarding (5.5) and (5.6) will be reported elsewhere. Similar discussion holds for theories of Type II and extended Type I.

5.3. Background gravitational field

We have considered to this point the theory in flat space-time. It is straightforward to incorporate the interaction with background gravity if the space-time satisfies Ricci flatness.

In the background gravitational field $g_{\mu\nu}$, the classical Lagrangian (2.1) becomes

$$\mathcal{L}_{\text{RS}} = -\frac{i}{2} \sqrt{g} \bar{\psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \psi_\lambda, \quad (5.9)$$

where $g = |\det g_{\mu\nu}|$, D_ν is the covariant derivative, and the Greek indices of $\gamma^{\mu\nu\lambda}$ are now of the world coordinate, thus having vielbein dependence. The Lagrangian (5.9) is invariant up to total derivatives under the gauge transformation

$$\delta\psi_\mu = D_\mu \Lambda \quad (5.10)$$

if the background gravitational field satisfies the vacuum Einstein equation:

$$R_{\mu\nu} = 0. \quad (5.11)$$

The quantum Lagrangian (3.1) is now given by

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{\text{RS}} + \sqrt{g}\bar{B}\overleftarrow{\mathcal{D}}(\gamma\psi) - \frac{i\varepsilon}{2}\sqrt{g}(\bar{Y}_* + \alpha\bar{B})\overleftarrow{\mathcal{D}}(Y_* + \alpha B) - \sqrt{g}\bar{Y}_*\overleftarrow{\mathcal{D}}\overleftarrow{\mathcal{D}}Y \\ - i\sqrt{g}\bar{c}_*\overleftarrow{\mathcal{D}}\overleftarrow{\mathcal{D}}c - i\sqrt{g}\bar{K}_*\overleftarrow{\mathcal{D}}\overleftarrow{\mathcal{D}}K. \end{aligned} \tag{5.12}$$

This Lagrangian is invariant up to total derivatives under the BRST transformation (3.8), with the exception of ψ_μ , which now transforms as

$$\delta_{\text{B}}\psi_\mu = iD_\mu c. \tag{5.13}$$

The corresponding BRST charge is given by

$$Q_{\text{B}} = \int \sqrt{g} J_{\text{B}}^0 d^{n-1}\mathbf{x}, \tag{5.14}$$

where J_{B}^μ is the BRST current defined by

$$J_{\text{B}}^\mu = \bar{B}\overleftrightarrow{D}^\mu c + \bar{Y}_*\overleftrightarrow{D}^\mu K. \tag{5.15}$$

[Note that this current is actually conserved, since the fields $\varphi = B, Y_*, c$ and K satisfy the Klein-Gordon equation,

$$\square\varphi = D^\mu D_\mu\varphi = 0.$$

This is due to the Ricci flatness,

$$\overleftarrow{\mathcal{D}}^2\varphi = \left(D^\mu D_\mu + \frac{1}{4}R\right)\varphi = D^\mu D_\mu\varphi,$$

where R is the scalar curvature $R = g^{\mu\nu}R_{\mu\nu} (= 0)$.] Thus, owing to the Ricci flatness, we can consistently define the physical subspace by $\mathcal{V}_{\text{phys}} = \ker Q_{\text{B}}$.

The form invariance (3.12) also holds for the Lagrangian (5.12) under the q -number gauge transformation (3.11), with the exception of ψ_μ , which now transforms as

$$\hat{\psi}_\mu = \psi_\mu + \tau D_\mu Y. \tag{5.16}$$

Note that we have again used Ricci flatness to confirm the form invariance.

All the arguments in §3 hold also in the present case. In particular, the BRST transformation commutes with the q -number gauge transformation, which leads to the gauge invariance of the physical subspace (and the physical Hilbert space).

As seen above, the interaction with the background gravity can be consistently incorporated if the gravity satisfies the vacuum Einstein equation. This fact leads us to the expectation that the gaugeon formalism should be applicable to supergravity theory.

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