

## **G-Matrix Equation in the Quark-Model Resonating-Group Method for Baryon-Baryon Interaction**

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The  $G$ -matrix equation is most straightforwardly formulated in the resonating-group method if the quark-exchange kernel is directly used as the driving term for the infinite sum of all the ladder diagrams. The inherent energy dependence involved in the exchange term of the normalization kernel plays an essential role in defining the off-shell  $T$ -matrix when the complete Pauli-forbidden state exists. We analyze this using a simple solvable model with no quark-quark interaction, and calculating the most general  $T$ -matrix in the formulation developed by Noyes and Kowalski. This formulation gives a certain condition for the existence of the solution in the Lippmann-Schwinger resonating-group method. A new procedure to deal with the corrections for the reduced masses and the internal-energy terms in the  $\Lambda N$ - $\Sigma N$  coupled-channel resonating-group equation is proposed.

### **§1. Introduction**

The  $G$ -matrix formalism is one of the best known frameworks to study the effective baryon-baryon interaction in nuclear media.<sup>1),2)</sup> When free baryon-baryon scattering is described by the non-relativistic Schrödinger equation with a simple energy-independent local potential, the derivation of the  $G$ -matrix equation is straightforward, even for the most general off-shell  $G$ -matrix used in many-body calculations. This is not the case if the basic baryon-baryon interaction is formulated as a composite-particle interaction in the framework of the resonating-group method (RGM).<sup>3),4)</sup> It is well known that the RGM relative wavefunction between clusters has a normalization property that differs from the wavefunction of the ordinary Schrödinger equation, owing to the antisymmetrization of constituent quarks ( $q$ ).<sup>5)</sup> It is, therefore, claimed that the quark-model potential derived from the  $(3q)$ - $(3q)$  RGM<sup>6)</sup> should be defined by rewriting the original RGM equation as a Schrödinger-type equation. One of the merits of this method is that it is possible to eliminate the explicit energy dependence of the quark-exchange kernel, which inherently appears in the RGM equation. The essential point of this procedure is to renormalize the RGM relative wavefunction,  $\chi(\mathbf{r})$  by the exchange normalization kernel  $K$  as  $\psi(\mathbf{r}) = \sqrt{1-K}\chi(\mathbf{r})$ . This prescription, however, requires a special care when a complete Pauli-forbidden state exists.

In this paper, we discuss the roles of the Pauli-forbidden state in the off-shell  $T$ -matrix derived from the RGM equation. It is shown that direct use of the quark-

exchange kernel for the driving term of the  $T$ -matrix (or  $G$ -matrix) equation is the simplest and most straightforward procedure in the sense that the orthogonality condition<sup>7)</sup> to the complete Pauli-forbidden state is automatically incorporated in the structure of the exchange kernel. The concept of the orthogonality need not be applied solely to the relative wavefunction. Rather, it can also be applied to the correlation function for the off-shell scattering amplitudes. The energy dependence involved in the normalization exchange kernel is not an unfavorable feature. On the contrary, it is essential to represent the effect of the compositeness of the nucleon clusters. To demonstrate these points, we use the simplest version of Saito's orthogonality condition model (OCM)<sup>7),5)</sup> with no  $q$ - $q$  interaction. In this case, the exact solution of the complete off-shell  $T$ -matrix is analytically given. If we solve this problem using the Noyes<sup>8)</sup> and Kowalski<sup>9)</sup> method, we find what kind of condition is necessary to guarantee the existence of the solution of the basic Lippmann-Schwinger-type equation. In the general RGM equation, this condition is automatically satisfied. When we need to modify the exchange kernel, the modification should be made in such a way that this condition is still satisfied. An example of this kind of modification is the correction of the reduced masses and a small readjustment of the threshold energies in the coupled-channel RGM (CCRGM). One can preserve the realistic kinematics of the baryon-baryon scatterings even in the very rigorous framework of the CCRGM.

In the next section, we first briefly illustrate the RGM formalism to introduce the notation used in this paper. An orthogonality condition model is used as an example of a simplified version of the RGM. The analytic solution of the  $T$ -matrix equation is first given in a heuristic way. Next, we consider the basic equations for the most general  $T$ -matrix in the Noyes and Kowalski method. The orthogonality relations for the Pauli-forbidden state are analyzed in §3 with respect to the simplest version of the OCM. The modification of the exchange kernel of the RGM equation is discussed in §4. The final section is devoted to discussion and a brief summary.

## §2. Formulation

### 2.1. RGM equation

The RGM equation for the relative-motion wavefunction  $\chi(\mathbf{r})$  is usually formulated from the variational equation<sup>4)</sup>

$$\langle \phi^{\text{int}} | E - H | \mathcal{A} \{ \phi^{\text{int}} \chi \} \rangle = 0, \quad (2.1)$$

where  $\phi^{\text{int}}$  is an appropriate internal cluster function and the total Hamiltonian consists of

$$H = \sum_{i=1}^6 t_i - T_G + \sum_{i<j}^6 v_{ij}. \quad (2.2)$$

Here we particularly consider (3 $q$ )-(3 $q$ ) RGM<sup>6)</sup> for the baryon-baryon interaction. The normalization kernel  $N$  represents

$$N = \langle \phi^{\text{int}} | \mathcal{A} | \phi^{\text{int}} \rangle = 1 - K, \quad (2.3)$$

where the exchange normalization kernel  $K$  sometimes allows a complete Pauli-forbidden state  $|u\rangle$  satisfying

$$K|u\rangle = |u\rangle \quad \text{and} \quad \mathcal{A}\{\phi^{\text{int}}u\} = 0 . \quad (2.4)$$

We use the common symbol  $\Lambda = 1 - |u\rangle\langle u|$  to denote the projection operator on the Pauli-allowed model space.<sup>5)</sup> For the direct term, we express  $H$  as

$$H = H_{\text{int}} + H_0 + \sum_{i=1}^3 \sum_{j=4}^6 v_{ij} \quad (2.5)$$

and define the internal-energy term  $E_{\text{int}}$  and the direct potential  $V_D$  by

$$\begin{aligned} \langle \phi^{\text{int}} | H_{\text{int}} | \phi^{\text{int}} \rangle &= E_{\text{int}} = E_{\text{int}}^{\text{K}} + E_{\text{int}}^{\text{V}} , \\ \langle \phi^{\text{int}} | \sum_{i=1}^3 \sum_{j=4}^6 v_{ij} | \phi^{\text{int}} \rangle &= V_D . \end{aligned} \quad (2.6)$$

For the exchange term, we use the definitions

$$\begin{aligned} G &= G^{\text{K}} + G^{\text{V}} , \\ G^{\text{K}} &= \langle \phi^{\text{int}} | \left( \sum_{i=1}^6 t_i - T_G \right) (\mathcal{A} - 1) | \phi^{\text{int}} \rangle + E_{\text{int}}^{\text{K}} K , \\ G^{\text{V}} &= \langle \phi^{\text{int}} | \left( \sum_{i<j}^6 v_{ij} \right) (\mathcal{A} - 1) | \phi^{\text{int}} \rangle + E_{\text{int}}^{\text{V}} K . \end{aligned} \quad (2.7)$$

The exchange kinetic-energy kernel  $G^{\text{K}}$  is symmetric, since  $E_{\text{int}}^{\text{K}}$  is common for all channels,<sup>\*)</sup> while  $G^{\text{V}} - E_{\text{int}}^{\text{V}} K$  is symmetric for the exchange interaction kernel. Using these definitions, Eq. (2.1) can be written as a usual RGM equation,

$$(\varepsilon - H_0 - V_{\text{RGM}})\chi = 0 , \quad (2.8)$$

with  $\varepsilon = E - E_{\text{int}}$  and  $V_{\text{RGM}} = V_D + G + \varepsilon K$ .

The essential feature of the RGM equation, Eq. (2.8), is the existence of the trivial solution  $\chi = u$ ; namely, we can write Eq. (2.8) as

$$\Lambda(\varepsilon - H_0 - V_{\text{RGM}})\Lambda\chi = 0 . \quad (2.9)$$

This equation can be rewritten as

$$(\varepsilon - H_0 - V)\chi = 0 , \quad (2.10)$$

with

$$\begin{aligned} V &= V(\varepsilon) + v , \\ V(\varepsilon) &= (\varepsilon - H_0) - \Lambda(\varepsilon - H_0)\Lambda , \\ v &= \Lambda V_{\text{RGM}}\Lambda = \Lambda(V_D + G + \varepsilon K)\Lambda . \end{aligned} \quad (2.11)$$

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<sup>\*)</sup> Here we use the simplest center-of-mass coordinate  $\mathbf{X} = (\mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3)/3$  for the (3q) clusters and assume that the effect of the flavor symmetry breaking is realized only in the original Hamiltonian  $H$ .

We note the simple relationships  $\Lambda V(\varepsilon)\Lambda = 0$ ,  $v = \Lambda v\Lambda$ , and  $V = V(\varepsilon) + v = V_{\text{RGM}}$ . Actually,  $v$  also has a weak  $\varepsilon$  dependence. If the effect of the Pauli principle has a simple structure, as in a deuteron-deuteron system, the  $\Lambda K\Lambda$  term exactly vanishes. In this particular case, we can also show that  $G^{\text{K}} \sim \Lambda H_0\Lambda - H_0$  and  $\Lambda G^{\text{K}}\Lambda \sim 0$ . This implies that the  $V(\varepsilon)$  term in Eq. (2.11) represents the dominant part of  $G^{\text{K}} + \varepsilon K$  in the original RGM kernel  $V_{\text{RGM}}$ , and  $v \sim \Lambda(V_{\text{D}} + G^{\text{V}})\Lambda$ .

From these observations, we can conclude that the most essential part of the Pauli principle of the RGM equation is already contained in the simple OCM-type equation

$$(\varepsilon - H_0)\psi = V(\varepsilon)\psi, \quad (2.12)$$

and the general RGM equation, Eq. (2.8), can be obtained by adding to  $V(\varepsilon)$  the potential term  $v$  having the property  $v = \Lambda v\Lambda$ . We consider the  $T$ -matrix of Eq. (2.12) in the next subsection.

## 2.2. $T$ -matrix of the simple OCM equation

In this subsection we derive a complete off-shell  $T$ -matrix for the simple OCM equation<sup>7), 5)</sup>

$$\Lambda(\varepsilon - H_0)\Lambda\psi = 0, \quad (2.13)$$

which is equivalent to Eq. (2.12).\*) Since Eq. (2.13) has a trivial solution  $u$ , we first consider a more general equation,

$$(\omega - H_0)\psi = V(\varepsilon)\psi \quad \text{with} \quad \omega \neq \varepsilon, \quad (2.14)$$

and take the limit  $\omega \rightarrow \varepsilon$  in the final expression. One can formulate this in two ways. The first method is to consider the solution of

$$(\omega - H_0)\psi = V(\varepsilon)\psi \quad \text{with} \quad (\omega - H_0)|\omega\rangle = 0, \quad (2.15)$$

and the second method is to use

$$(\omega - H_0)(\psi - \phi) = V(\varepsilon)\psi \quad \text{with} \quad (\varepsilon - H_0)|\phi\rangle = 0. \quad (2.16)$$

The latter equation is motivated by the correlation function technique for the  $G$ -matrix, for which  $\chi = \phi - \psi$  corresponds to the so-called defect function.<sup>14)</sup> In this case, the starting energy  $\omega$  is usually negative. For  $\omega \neq \varepsilon$ , one can easily prove  $\langle u|\psi\rangle = 0$  for the solution of Eq. (2.15) and  $\langle u|\chi\rangle = 0$  for the solution of Eq. (2.16).

Both Eqs. (2.15) and (2.16) correspond to the same definition for the  $T$ -matrix, given by

$$\begin{aligned} T(\omega, \varepsilon) &= V(\varepsilon) + V(\varepsilon)G_0^{(+)}(\omega)T(\omega, \varepsilon) \\ \text{with} \quad G_0^{(+)}(\omega) &= \frac{1}{\omega - H_0 + i0}. \end{aligned} \quad (2.17)$$

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\*) The on-shell  $T$ -matrix for OCM equations with local potentials, more general than Eq. (2.13), has been extensively studied by many authors. (See, for example, Refs. 10) – 13).)

The solution of Eq. (2·17) is derived as follows.\*) We first define the free Green function in the allowed model space by

$$G_{\Lambda}(\omega) = G_0^{(+)}(\omega) - G_0^{(+)}(\omega)|u\rangle \frac{1}{\langle u|G_0^{(+)}(\omega)|u\rangle} \langle u|G_0^{(+)}(\omega) , \quad (2\cdot18)$$

which satisfies

$$\Lambda(\omega - H_0)\Lambda G_{\Lambda}(\omega) = G_{\Lambda}(\omega)\Lambda(\omega - H_0)\Lambda = \Lambda . \quad (2\cdot19)$$

The Green function for  $\omega - H_0 - V(\varepsilon) + i0$  can be easily found to be

$$G^{(+)}(\omega, \varepsilon) = \frac{1}{\omega - H_0 - V(\varepsilon) + i0} = G_{\Lambda}(\omega) + |u\rangle \frac{1}{\omega - \varepsilon} \langle u| . \quad (2\cdot20)$$

Since this  $G^{(+)}(\omega, \varepsilon)$  is the formal solution of

$$G^{(+)}(\omega, \varepsilon) = G_0^{(+)}(\omega) + G_0^{(+)}(\omega)V(\varepsilon)G^{(+)}(\omega, \varepsilon) , \quad (2\cdot21)$$

the  $T$ -matrix solution of Eq. (2·17) can be derived from

$$G^{(+)}(\omega, \varepsilon) = G_0^{(+)}(\omega) + G_0^{(+)}(\omega)T(\omega, \varepsilon)G_0^{(+)}(\omega) . \quad (2\cdot22)$$

We find

$$T(\omega, \varepsilon) = -\frac{|u\rangle\langle u|}{\langle u|G_0^{(+)}(\omega)|u\rangle} + (\omega - H_0)|u\rangle \frac{1}{\omega - \varepsilon} \langle u|(\omega - H_0) . \quad (2\cdot23)$$

This expression seemingly has a singularity at  $\omega = \varepsilon$  in the second term. There is, however, no such singularity for the initial state  $|\omega\rangle$  or  $|\phi\rangle$ :

$$\begin{aligned} T(\omega, \varepsilon)|\omega\rangle &= -\frac{|u\rangle\langle u|\omega\rangle}{\langle u|G_0^{(+)}(\omega)|u\rangle} , \\ T(\omega, \varepsilon)|\phi\rangle &= -\frac{|u\rangle\langle u|\phi\rangle}{\langle u|G_0^{(+)}(\omega)|u\rangle} + (\omega - H_0)|u\rangle\langle u|\phi\rangle . \end{aligned} \quad (2\cdot24)$$

In particular, the on-shell  $T$ -matrix given by

$$\langle\omega|T(\omega, \varepsilon)|\omega\rangle = -\frac{\langle\omega|u\rangle\langle u|\omega\rangle}{\langle u|G_0^{(+)}(\omega)|u\rangle} \quad (2\cdot25)$$

has no  $\varepsilon$  dependence. The  $T$ -matrix element of the type  $\langle\phi|T(\omega, \varepsilon)|\phi\rangle$  with initial energy  $\varepsilon$  is given by

$$\langle\phi|T(\omega, \varepsilon)|\phi\rangle = -\frac{\langle\phi|u\rangle\langle u|\phi\rangle}{\langle u|G_0^{(+)}(\omega)|u\rangle} + (\omega - \varepsilon)\langle\phi|u\rangle\langle u|\phi\rangle . \quad (2\cdot26)$$

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\*) Here we assume a single-channel problem for simplicity.

Note that the second term in Eq. (2·26) is necessary to guarantee

$$\lim_{\omega \rightarrow \pm\infty} \langle \phi | T(\omega, \varepsilon) | \phi \rangle = \langle \phi | V(\varepsilon) | \phi \rangle . \quad (2\cdot27)$$

Summarizing this subsection, we find that, to define the complete off-shell  $T$ -matrix, it is convenient to start with Eq. (2·16), which is similar to a Schrödinger-type equation for the defect function in the  $G$ -matrix formalism.<sup>14)</sup> The energy  $\varepsilon$  in  $V(\varepsilon)$  is the relative energy for the initial two-particle state, and it should not be confused with the energy  $\omega$  in the free Green function. The energy  $\omega$  is usually negative and is sometimes referred to as the “starting energy” in  $G$ -matrix calculations for ground state properties. In the former equation, Eq. (2·15), the orthogonality is imposed on  $\psi$ , while in Eq. (2·16) the defect function  $\chi = \psi - \phi$  respects the orthogonality to the Pauli-forbidden state  $u$ .

### 2.3. The Noyes-Kowalski equation

Lippmann-Schwinger-type equations in the momentum representation can be nicely solved using the techniques developed by Noyes<sup>8)</sup> and Kowalski.<sup>9)</sup> (For details of this method, the original papers should be referred to.) Here we give only the equations necessary for the following discussion.

Let us consider the Lippmann-Schwinger equation for the wavefunction

$$|\psi^{(+)}\rangle = |\phi\rangle + G_0^{(+)}(E)V|\psi^{(+)}\rangle \quad \text{with} \quad (E - H_0)|\phi\rangle = 0 , \quad (2\cdot28)$$

and the  $T$ -matrix equation

$$T(E) = V + VG_0^{(+)}(E)T(E) . \quad (2\cdot29)$$

We assume that the partial-wave decomposition has already been made, and we write the free Green function as<sup>\*)</sup>

$$G_0^{(+)}(E) = \mathcal{P} \frac{1}{E - H_0} - i\pi\delta(E - H_0) = \mathcal{P}G_0(E) - i\pi|\phi\rangle\langle\phi| . \quad (2\cdot30)$$

Then, it is convenient to deal with the  $K$ -matrix equation,

$$R(E)|\phi\rangle = V|\phi\rangle + V\mathcal{P}G_0^{(+)}(E)R(E)|\phi\rangle , \quad (2\cdot31)$$

instead of  $T(E)|\phi\rangle$ , which is obtained from

$$R(E)|\phi\rangle = T(E)|\phi\rangle[1 - i\pi\langle\phi|T(E)|\phi\rangle]^{-1} . \quad (2\cdot32)$$

The essential point of the Noyes-Kowalski formalism is to use

$$W = V - V|\phi\rangle \frac{1}{\langle\phi|V|\phi\rangle} \langle\phi|V , \quad (2\cdot33)$$

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<sup>\*)</sup> For the  $S$ -wave forbidden state  $|u\rangle$ ,  $|\phi\rangle$  is the plane wave normalized as  $|\phi\rangle = (4\pi\mu k/\hbar^2)^{1/2}|\mathbf{k}\rangle$  with  $E = \hbar^2 k^2/2\mu$ . This corresponds to the energy normalization.

satisfying  $\langle\phi|W = 0$  and  $W|\phi\rangle = 0$ . The solution of Eq. (2.31) is factorized as

$$R(E)|\phi\rangle = f|\phi\rangle\langle\phi|R(E)|\phi\rangle . \tag{2.34}$$

Then the square integrable function  $|f\rangle = f|\phi\rangle\langle\phi|V|\phi\rangle$  should satisfy the basic equation

$$|f\rangle = V|\phi\rangle + WG_0(E)|f\rangle . \tag{2.35}$$

Note that  $WG_0(E)$  with  $G_0(z) = 1/(z - H_0)$  is the Hilbert-Schmidt kernel, and the solution of Eq. (2.35) satisfies the trivial relationship  $\langle\phi|f|\phi\rangle = 1$ . The on-shell  $T$ -matrix is calculated from

$$\langle\phi|R(E)|\phi\rangle = [1 - \langle\phi|V\mathcal{P}G_0(E)f|\phi\rangle]^{-1}\langle\phi|V|\phi\rangle \tag{2.36}$$

and

$$\langle\phi|T(E)|\phi\rangle = [1 + i\pi\langle\phi|R(E)|\phi\rangle]^{-1}\langle\phi|R(E)|\phi\rangle . \tag{2.37}$$

The half off-shell  $T$ -matrix is given by

$$T(E)|\phi\rangle = f|\phi\rangle\langle\phi|T(E)|\phi\rangle . \tag{2.38}$$

In order to obtain the complete off-shell  $T$ -matrix, we should generalize Eq. (2.35) as

$$f = V\langle\phi|V|\phi\rangle^{-1} + WG_0(E)f \tag{2.39}$$

and its transpose,<sup>\*)</sup>

$$\tilde{f} = \langle\phi|V|\phi\rangle^{-1}V + \tilde{f}G_0(E)W . \tag{2.40}$$

The full  $T$ -matrix is given by

$$T(E) = f|\phi\rangle\langle\phi|T(E)|\phi\rangle\langle\phi|\tilde{f} + \left\{ \begin{array}{l} f\langle\phi|V|\phi\rangle - f|\phi\rangle\langle\phi|V \\ \langle\phi|V|\phi\rangle\tilde{f} - V|\phi\rangle\langle\phi|\tilde{f} \end{array} \right\} . \tag{2.41}$$

Let us discuss the condition for which the solution of Eq. (2.35) exists. From the Fredholm alternative theorem, it is essential to examine if there exist square integrable solutions  $\langle\psi|$  to the conjugate homogeneous equation

$$\langle\psi|(1 - WG_0(E)) = 0 . \tag{2.42}$$

If there exist such solutions, the necessary and sufficient condition for the unique solution of Eq. (2.35) is

$$\langle\psi|V|\phi\rangle = 0 , \tag{2.43}$$

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<sup>\*)</sup> Here we have simply assumed a symmetric potential  $V$ , i.e.,  ${}^tV = V$ . Extension to the unsymmetrical case (with  ${}^tV \neq V$ ) is discussed fully in Ref. 15).

for all  $\psi$ . Suppose there exists only one  $\psi$  for Eq. (2.42) and that Eq. (2.35) has a special solution  $|f_0\rangle$ . Then the general solution of Eq. (2.35) is given by

$$|f\rangle = |f_0\rangle + C(E - H_0)|\psi\rangle \quad (2.44)$$

for any arbitrary constant  $C$ . If  $\psi$  does not satisfy Eq. (2.43), Eq. (2.35) is unsolvable. We can also rewrite the integral equation (2.42) into the form of the differential equation

$$(E - H_0 - W)|\psi\rangle = 0, \quad (2.45)$$

with the proper boundary condition. The plane wave  $|\phi\rangle$  satisfies Eq. (2.45), but it is not a solution of Eq. (2.42), since it is not square integrable. Unfortunately, we cannot find any simple criterion on the potential  $V$ , that determines whether the positive-energy bound state for  $W$  exists. In the following, we assume that there is no such solution that satisfies Eq. (2.42) for standard potential  $V$ .

When we apply the present formalism to the simple OCM equation (2.12) or to the RGM equation (2.8), we find an apparent solution  $|\psi\rangle = |u\rangle$ . In fact, for  $(\varepsilon - H_0)|\phi\rangle = 0$ , we find that  $W = W(\varepsilon)$  for  $V = V(\varepsilon)$  is given by

$$W(\varepsilon) = \frac{(\varepsilon - H_0)|u\rangle\langle u|(\varepsilon - H_0)}{\varepsilon - \varepsilon_0} \quad \text{with} \quad \varepsilon_0 = \langle u|H_0|u\rangle. \quad (2.46)$$

This  $W(\varepsilon)$  satisfies

$$W(\varepsilon)|u\rangle = (\varepsilon - H_0)|u\rangle \quad \text{and} \quad \langle u|W(\varepsilon) = \langle u|(\varepsilon - H_0), \quad (2.47)$$

which is similar to

$$V(\varepsilon)|u\rangle = (\varepsilon - H_0)|u\rangle \quad \text{and} \quad \langle u|V(\varepsilon) = \langle u|(\varepsilon - H_0). \quad (2.48)$$

Thus we can easily see that  $|\psi\rangle = |u\rangle$  is a solution of Eq. (2.45); i.e.,  $(\varepsilon - H_0 - W(\varepsilon))|u\rangle = 0$ , and  $|u\rangle$  satisfies  $\langle u|V(\varepsilon)|\phi\rangle = 0$ . When we apply the present formulation to the RGM equation (2.10),  $W$  for  $V = V(\varepsilon) + v$  is no longer simple like Eq. (2.46), but the basic relationship Eq. (2.47) for  $W(\varepsilon) \rightarrow W$  and  $V(\varepsilon) \rightarrow V$  is still valid, owing to the property  $v = \Lambda v \Lambda$ . We assume that there is no other solution  $\psi$  to Eq. (2.42).

Let us rederive the  $T$ -matrix of §2.2 for the simple OCM. For the on-shell and the half off-shell  $T$ -matrix, direct use of Eq. (2.46) in Eq. (2.35) yields the solution

$$|f_0\rangle = (\varepsilon_0 - H_0)|u\rangle\langle u|\phi\rangle, \quad C = \frac{1}{\varepsilon - \varepsilon_0}\langle u|f\rangle. \quad (2.49)$$

Here  $f_0$  is a special solution which satisfies  $\langle u|f_0\rangle = 0$ . Using this solution, we can easily derive

$$\langle\phi|R(\varepsilon)|\phi\rangle = -\frac{\langle\phi|u\rangle\langle u|\phi\rangle}{\langle u|\mathcal{P}G_0(\varepsilon)|u\rangle}, \quad \langle\phi|T(\varepsilon)|\phi\rangle = -\frac{\langle\phi|u\rangle\langle u|\phi\rangle}{\langle u|G_0^{(+)}(\varepsilon)|u\rangle}. \quad (2.50)$$

For the complete off-shell  $T$ -matrix, we again need to assume  $\omega \neq \varepsilon$ , since otherwise Eq. (2.46) applied to Eq. (2.39) leads to the condition  $\langle u | (\varepsilon - H_0) = 0$ , which is apparently not satisfied for general  $\varepsilon$ . The decomposition of  $V(\varepsilon)$  with respect to  $(\omega - H_0)|\omega\rangle = 0$  leads to the result

$$\begin{aligned} W(\omega, \varepsilon) &= V(\varepsilon) - V(\varepsilon)|\omega\rangle \frac{1}{\langle \omega | V | \omega \rangle} \langle \omega | V(\varepsilon) \\ &= \frac{(\omega - H_0)|u\rangle \langle u | (\omega - H_0)}{2\omega - \varepsilon - \varepsilon_0} . \end{aligned} \tag{2.51}$$

After some calculations, the solution of Eq. (2.39) with  $E$ ,  $|\phi\rangle$ ,  $V$  and  $W$  replaced with  $\omega$ ,  $|\omega\rangle$ ,  $V(\varepsilon)$  and  $W(\omega, \varepsilon)$ , respectively, is found to be

$$f = \frac{1}{\langle \omega | u \rangle \langle u | \omega \rangle} \left\{ |u\rangle \langle u| - \frac{(2\omega - \varepsilon - H_0)|u\rangle \langle u | (\omega - H_0)}{(2\omega - \varepsilon - \varepsilon_0)(\omega - \varepsilon)} \right\} . \tag{2.52}$$

This expression leads to the simple relations

$$\begin{aligned} f|\omega\rangle &= \frac{|u\rangle}{\langle \omega | u \rangle} , & \langle \omega | f | \omega \rangle &= 1 , \\ \langle u | f &= \frac{1}{\langle \omega | u \rangle \langle u | \omega \rangle} \left\{ \langle u | - \frac{1}{\omega - \varepsilon} \langle u | (\omega - H_0) \right\} , \\ \langle u | f | \omega \rangle &= \frac{1}{\langle \omega | u \rangle} . \end{aligned} \tag{2.53}$$

Similarly, the solution of Eq. (2.40) is given by

$$\tilde{f} = \frac{1}{\langle \omega | u \rangle \langle u | \omega \rangle} \left\{ |u\rangle \langle u| - \frac{(\omega - H_0)|u\rangle \langle u | (2\omega - \varepsilon - H_0)}{(\omega - \varepsilon)(2\omega - \varepsilon - \varepsilon_0)} \right\} , \tag{2.54}$$

and

$$\begin{aligned} \langle \omega | \tilde{f} &= \frac{\langle u |}{\langle u | \omega \rangle} , & \langle \omega | \tilde{f} | \omega \rangle &= 1 , \\ \tilde{f}|u\rangle &= \frac{1}{\langle \omega | u \rangle \langle u | \omega \rangle} \left\{ |u\rangle - \frac{1}{\omega - \varepsilon} (\omega - H_0)|u\rangle \right\} , \\ \langle \omega | \tilde{f} | u \rangle &= \frac{1}{\langle u | \omega \rangle} . \end{aligned} \tag{2.55}$$

Finally, we use Eqs. (2.41) with  $|\phi\rangle \rightarrow |\omega\rangle$ , (2.25), and (2.52) – (2.55) to reconstruct  $T(\omega, \varepsilon)$ . The final result is, of course, Eq. (2.23).

### §3. Orthogonality in the $T$ -matrix for the simple OCM equation

We can use the full expression of the  $T$ -matrix derived in the preceding section to investigate how the idea of orthogonality to the Pauli-forbidden state is preserved in the simple OCM. Let us first consider the  $\omega \rightarrow \varepsilon$  limit in the two expressions in

Eq. (2·24). These two expressions correspond to the wavefunction of the ordinary scattering problem Eq. (2·15) (with  $\varepsilon$  a simple parameter) and to the off-shell  $T$ -matrix in Eq. (2·26), respectively. We find

$$\lim_{\omega \rightarrow \varepsilon} T(\omega, \varepsilon)|\omega\rangle \neq \lim_{\omega \rightarrow \varepsilon} T(\omega, \varepsilon)|\phi\rangle . \quad (3\cdot1)$$

On the left-hand side of Eq. (3·1), the wavefunction is orthogonal to the Pauli-forbidden state  $|u\rangle$ , while on the right-hand side,  $\chi = \phi - \psi$  in Eq. (2·16) is orthogonal. When  $\omega = \varepsilon$ , the solution of Eq. (2·13) is not uniquely determined due to an arbitrary admixture of the redundant component, which resolves the discrepancy of the two different half off-shell  $T$ -matrices defined in Eq. (3·1). In order to see this, we assume  $\omega = \varepsilon$  and derive the wavefunction, preserving this undetermined redundant component related to  $|u\rangle$ . The solution Eq. (2·44) with Eq. (2·49) immediately gives

$$\begin{aligned} f|\phi\rangle &= [(\varepsilon_0 - H_0)|u\rangle\langle u|\phi\rangle + (\varepsilon - H_0)|u\rangle C] \frac{1}{(\varepsilon_0 - \varepsilon)\langle\phi|u\rangle\langle u|\phi\rangle} \\ &= \frac{|u\rangle}{\langle\phi|u\rangle} + (\varepsilon - H_0)|u\rangle c . \end{aligned} \quad (3\cdot2)$$

The new parameter  $c$  is related to  $C$  through

$$c = \frac{1}{(\varepsilon_0 - \varepsilon)\langle\phi|u\rangle\langle u|\phi\rangle} [\langle u|\phi\rangle + C] . \quad (3\cdot3)$$

On the other hand, the wavefunction is derived from

$$\begin{aligned} |\psi^{(+)}\rangle - |\phi\rangle &= G_0^{(+)}(\varepsilon)T(\varepsilon)|\phi\rangle = G_0^{(+)}(\varepsilon)f|\phi\rangle\langle\phi|T(\varepsilon)|\phi\rangle , \\ |\psi_R\rangle - |\phi\rangle &= \mathcal{P}G_0(\varepsilon)R(\varepsilon)|\phi\rangle = \mathcal{P}G_0(\varepsilon)f|\phi\rangle\langle\phi|R(\varepsilon)|\phi\rangle , \end{aligned} \quad (3\cdot4)$$

where  $|\psi_R\rangle$  is the standing-wave solution for  $\mathcal{P}G_0(\varepsilon)$ . For simplicity, we use the shorthand notation

$$\begin{aligned} D(\varepsilon) &= \langle u|\mathcal{P}G_0(\varepsilon)|u\rangle , \\ D^{(+)}(\varepsilon) &= \langle u|G_0^{(+)}(\varepsilon)|u\rangle = D(\varepsilon) - i\pi\langle u|\phi\rangle\langle\phi|u\rangle \end{aligned} \quad (3\cdot5)$$

and express the on-shell  $T$ -matrix as

$$\langle\phi|R(\varepsilon)|\phi\rangle = -\frac{\langle\phi|u\rangle\langle u|\phi\rangle}{D(\varepsilon)} , \quad \langle\phi|T(\varepsilon)|\phi\rangle = -\frac{\langle\phi|u\rangle\langle u|\phi\rangle}{D^{(+)}(\varepsilon)} . \quad (3\cdot6)$$

From Eq. (3·2),  $T(\varepsilon)|\phi\rangle$  is obtained as

$$T(\varepsilon)|\phi\rangle = f|\phi\rangle\langle\phi|T(\varepsilon)|\phi\rangle = -\frac{|u\rangle\langle u|\phi\rangle}{D^{(+)}(\varepsilon)} + (\varepsilon - H_0)|u\rangle\langle u|\phi\rangle B , \quad (3\cdot7)$$

where the third parameterization of the  $|u\rangle$  component,  $B$ , is given by

$$B = -c\frac{\langle\phi|u\rangle}{D^{(+)}(\varepsilon)} = -\frac{1}{(\varepsilon_0 - \varepsilon)D^{(+)}(\varepsilon)\langle u|\phi\rangle} [\langle u|\phi\rangle + C] . \quad (3\cdot8)$$

Table I. The orthogonality properties for the simple OCM in Eqs. (2·12) and (2·13).

orthogonality	$C$	$c$	$B$
$\langle u \psi^{(+)}\rangle = 0$	$-\langle u \phi\rangle$	0	0
$\langle u \psi^{(+)} - \phi\rangle = 0$	$-[1 + (\varepsilon_0 - \varepsilon)D^{(+)}(\varepsilon)] \langle u \phi\rangle$	$-\frac{D^{(+)}(\varepsilon)}{\langle\phi u\rangle}$	1
$\langle u \psi_R - \phi\rangle = 0$	$-[1 + (\varepsilon_0 - \varepsilon)D(\varepsilon)] \langle u \phi\rangle$	$-\frac{D(\varepsilon)}{\langle\phi u\rangle}$	$\frac{D(\varepsilon)}{D^{(+)}(\varepsilon)}$
$\langle u f\rangle = 0$	0	$\frac{1}{(\varepsilon_0 - \varepsilon)\langle\phi u\rangle}$	$-\frac{1}{(\varepsilon_0 - \varepsilon)D^{(+)}(\varepsilon)}$

We have various orthogonalities, as shown in Table I, depending on what values we use for the arbitrary  $C$ ,  $c$  and  $B$ . All of these half off-shell  $T$ -matrices have equal qualifications for the solution of Eq. (2·12).

Summarizing this section, we have found that the Lippmann-Schwinger equation Eq. (2·17) is a very general equation describing not only free scattering but also the correlation for the scattering in nuclear media. For the simple OCM Eq. (2·13), the solution of the half off-shell  $T$ -matrix depends on how the model is formulated from the more general equation in which the Pauli-forbidden state does not exist. The second method given in Eq. (2·16) seems to be more natural than that given in Eq. (2·15), since it has the direct physical meaning of summing up all the ladder diagrams in the  $G$ -matrix formulation. In this case, the orthogonality to the Pauli-forbidden state is represented with respect to the correlation function, instead of the wavefunction itself.

#### §4. Modification of the RGM kernel

The uniqueness of the solution for the Noyes-Kowalski equation Eq. (2·35) is derived from the structure  $V(\varepsilon) = (\varepsilon - H_0) - \Lambda(\varepsilon - H_0)\Lambda$  for  $(\varepsilon - H_0)|\phi\rangle = 0$  and the relationship  $v = \Lambda v \Lambda$ . It does not depend on the explicit form  $v = \Lambda V_{\text{RGM}} \Lambda = \Lambda(V_D + G + \varepsilon K)\Lambda$ . We can modify the internal energy part and the reduced masses of the CCRGM kernel by using this property. Namely, we leave  $\varepsilon$  in  $v$  as it is, and set up the RGM equation as

$$\begin{aligned}
 (\varepsilon^{\text{exp}} - H_0^{\text{exp}}) \chi &= (V^{\text{exp}}(\varepsilon) + v) \chi, \\
 V^{\text{exp}}(\varepsilon) &= (\varepsilon^{\text{exp}} - H_0^{\text{exp}}) - \Lambda (\varepsilon^{\text{exp}} - H_0^{\text{exp}}) \Lambda,
 \end{aligned}
 \tag{4.1}$$

where

$$\varepsilon^{\text{exp}} = E - E_{\text{int}}^{\text{exp}}, \quad H_0^{\text{exp}} = -\frac{\hbar^2}{2\mu^{\text{exp}}} \left( \frac{\partial}{\partial \mathbf{r}} \right)^2 = \frac{\mu}{\mu^{\text{exp}}} H_0
 \tag{4.2}$$

are the empirical relative energies and free kinetic-energy operators, respectively. We denote the modification of these quantities by

$$\Delta E_{\text{int}} = E_{\text{int}}^{\text{exp}} - E_{\text{int}} , \quad \Delta H_0 = H_0^{\text{exp}} - H_0 = \left( \frac{\mu}{\mu^{\text{exp}}} - 1 \right) H_0 \quad (4.3)$$

and use the notation

$$\Delta G = \Lambda (\Delta E_{\text{int}} + \Delta H_0) \Lambda - (\Delta E_{\text{int}} + \Delta H_0) . \quad (4.4)$$

The new RGM equation (4.1) is equivalent to

$$\Lambda (\varepsilon^{\text{exp}} - H_0^{\text{exp}} - V_{\text{RGM}}) \Lambda \chi = 0 \quad (4.5)$$

if we use  $v = \Lambda V_{\text{RGM}} \Lambda$ . This implies that we have replaced  $\varepsilon$  and  $H_0$  in the direct term as

$$\varepsilon \rightarrow \varepsilon^{\text{exp}} , \quad H_0 \rightarrow H_0^{\text{exp}} \quad (4.6)$$

in the allowed model space, without changing  $\varepsilon$  in  $v = \Lambda V_{\text{RGM}} \Lambda = \Lambda (V_{\text{D}} + G + \varepsilon K) \Lambda$ . In the original form of the RGM equation (2.8), it is easy to see that Eq. (2.8) should be modified to

$$(\varepsilon^{\text{exp}} - H_0^{\text{exp}} - V_{\text{RGM}} - \Delta G) \chi = 0 . \quad (4.7)$$

This equation indicates that the extra modification of adding  $\Delta G$  is required, in addition to the modification Eq. (4.6) of the direct term.

As an example, let us consider  $\Lambda N$ - $\Sigma N$  ( $I = 1/2$ ) CCRGM. The scattering problem of this system is solved in Ref. 16), using the Lippmann-Schwinger RGM (LS-RGM) formalism. Since this system involves a complete Pauli-forbidden state in the  $(11)_s$   $SU_3$  representation for the  $^1S_0$  state,<sup>17)</sup> we must take a special care in the treatment of the Pauli principle. In our previous publications, the realistic treatment of the reduced mass in the direct term, using the empirical  $\Lambda$  and  $\Sigma$  masses, required some modification of the exchange kinetic energy kernel. The procedure adopted in Ref. 17), multiplying the exchange kinetic energy kernel  $G^{\text{K}}$  by the factor  $\sqrt{\mu_\alpha/\mu_\alpha^{\text{exp}}}$  from the bra and ket sides, as  $\sqrt{\mu_\alpha/\mu_\alpha^{\text{exp}}} G_{\alpha\alpha'}^{\text{K}} \sqrt{\mu_{\alpha'}/\mu_{\alpha'}^{\text{exp}}}$ , is not accurate for coupled-channel problems. As a result, the previous calculation yields catastrophic resonance behavior for the  $\Lambda N$   $^1S_0$  phase shift in the low-momentum region around  $p_\Lambda \sim 100$  MeV/ $c$  if our momentum discretization is too fine. We therefore use the following two-step modification for the  $\Lambda N$  and  $\Sigma N$  reduced masses. We first multiply all the channels by the common  $\mu_{\alpha=1}/\mu_{\alpha=1}^{\text{exp}}$  factor for the incident baryon channel, just as is done for the single-channel problem (see Ref. 18)). This process is necessary to reduce the too strong effect of the momentum-dependent Darwin term involved in the Fermi-Breit interaction. The Pauli principle is exactly preserved at this stage with respect to the kinetic energy term. Next, we introduce a small modification of the reduced mass of the second baryon channel with respect to the direct term. The modification of the exchange term is carried out by using  $\Delta G$  in Eq. (4.4). Since the magnitude of the  $\Delta H_0$  term is at most a few MeV, the error caused by this approximate treatment of the exchange term should be more than

one order smaller than the “exact” value of the exchange kernel. In practice, we augment the exchange kernel  $G(\mathbf{q}_f, \mathbf{q}_i)$  with the  $S$ -wave Born kernel

$$\begin{aligned} \Delta G^K(\mathbf{q}_f, \mathbf{q}_i) &= \left( \Delta \frac{\hbar^2}{2\mu} \right) f \frac{9}{100} \begin{pmatrix} \frac{9}{4b^2} & -\frac{10}{3}q_i^2 + \frac{27}{4b^2} \\ -\frac{10}{3}q_f^2 + \frac{27}{4b^2} & -10(q_f^2 + q_i^2) + \frac{81}{4b^2} \end{pmatrix}, \\ \Delta G^K(\mathbf{q}_f, \mathbf{q}_i) &= \left( \Delta \frac{\hbar^2}{2\mu} \right) f \frac{9}{100} \begin{pmatrix} \frac{9}{4b^2} & -\frac{10}{3}q_i^2 + \frac{3}{4b^2} \\ -\frac{10}{3}q_f^2 + \frac{3}{4b^2} & -\frac{10}{9}(q_f^2 + q_i^2) + \frac{1}{4b^2} \end{pmatrix}, \end{aligned} \tag{4.8}$$

for  $\Lambda N$ -incident and  $\Sigma N$ -incident scatterings, respectively. Here we have defined  $\Delta \hbar^2/2\mu = \hbar^2/2\mu_{\Sigma N}^{\text{exp}} - \hbar^2/2\mu_{\Lambda N}^{\text{exp}}$ , the quantity  $b$  is the harmonic oscillator width parameter, and the  $S$ -wave spatial function  $f$  is given by

$$f = f(q_f, q_i) = \left( \sqrt{\frac{8\pi}{3}} b \right)^3 \exp \left\{ -\frac{1}{3} b^2 (q_f^2 + q_i^2) \right\}. \tag{4.9}$$

The condition  $\langle u | V_{\text{RGM}} + \Delta G | \phi \rangle = 0$  for the Pauli-forbidden state  $|u\rangle$ , and  $|\phi\rangle$  satisfying  $(\varepsilon^{\text{exp}} - H_0^{\text{exp}}) |\phi\rangle = 0$ , guarantees the existence of the solution of Eq. (4.7). If we assume  $\Delta G = 0$  in Eq. (4.7), we obtain the expression

$$\langle u | V_{\text{RGM}} | \phi \rangle = -\langle u | \Delta G | \phi \rangle = \langle u | \Delta(E_{\text{int}} + H_0) | \phi \rangle. \tag{4.10}$$

This quantity is actually 0 if the incident momentum of  $\Lambda$  is below the  $\Sigma N$  threshold. This is because the plane wave solution  $|\phi\rangle$  has a non-zero component only for the  $\Lambda N$  channel, and the  $2 \times 2$  matrix  $\Delta(E_{\text{int}} + H_0)$  has a non-zero component only for the second diagonal channel. In this particular case, neglecting  $\Delta G$  in the second step is harmless, even if we modify only the direct term.

The modification of the internal energy term is not necessary in the  $\Lambda N$ - $\Sigma N$  coupled-channel problem in the isospin basis, since the model parameters are usually fixed to reproduce the mass difference between  $\Lambda$  and  $\Sigma$  in our previous models, FSS and RGM-H.<sup>19)-21)</sup> This is, however, no longer valid in the particle-basis calculation<sup>22)</sup> used to study the charge symmetry breaking, since a consistent description of the baryon-mass splitting in terms of the up-down quark mass difference and the Coulomb energies is not always successful. It may, therefore, be useful to present a general expression of the extra Born kernel Eq. (4.4). We have

$$\begin{aligned} \Delta G_{\alpha\alpha'}(\mathbf{q}_f, \mathbf{q}_i) &= \Delta G_{\alpha\alpha'}^{\text{int}}(\mathbf{q}_f, \mathbf{q}_i) + \Delta G_{\alpha\alpha'}^{\text{K}}(\mathbf{q}_f, \mathbf{q}_i), \\ \Delta G_{\alpha\alpha'}^{\text{int}}(\mathbf{q}_f, \mathbf{q}_i) &= c_\alpha c_{\alpha'} f \left[ -\left( \Delta E_\alpha^{\text{int}} + \Delta E_{\alpha'}^{\text{int}} \right) + \sum_{\alpha''} c_{\alpha''}^2 \Delta E_{\alpha''}^{\text{int}} \right], \\ \Delta G_{\alpha\alpha'}^{\text{K}}(\mathbf{q}_f, \mathbf{q}_i) &= c_\alpha c_{\alpha'} f \left[ -\left( q_f^2 \Delta \frac{\hbar^2}{2\mu_\alpha} + q_i^2 \Delta \frac{\hbar^2}{2\mu_{\alpha'}} \right) + \frac{9}{4b^2} \sum_{\alpha''} c_{\alpha''}^2 \Delta \frac{\hbar^2}{2\mu_{\alpha''}} \right], \end{aligned} \tag{4.11}$$

where  $f$  is given in Eq. (4.9) and

$$\Delta E_\alpha^{\text{int}} = \left(E_\alpha^{\text{int}}\right)^{\text{exp}} - \left(E_\alpha^{\text{int}}\right)^{\text{cal}}, \quad \Delta \frac{\hbar^2}{2\mu_\alpha} = \frac{\hbar^2}{2\mu_\alpha^{\text{exp}}} - \frac{\hbar^2}{2\mu_{\alpha=1}^{\text{exp}}}. \quad (4.12)$$

The normalized eigenvector  $c_\alpha$  of the Pauli-forbidden state in the flavor space is obtained from the eigenvalue equation

$$\sum_{\alpha'} (\lambda \delta_{\alpha\alpha'} + (X_N)_{\alpha\alpha'}) c_{\alpha'} = 0 \quad \text{with} \quad \lambda = 1, \quad (4.13)$$

by using the spin-flavor-color factors of the exchange normalization kernel  $X_N$ . The result of Eq. (4.8) is obtained if one uses  $c_1 = 1/\sqrt{10}$  and  $c_2 = 3/\sqrt{10}$  for the  $\Lambda N$ -incident channel and  $c_1 = 3/\sqrt{10}$  and  $c_2 = 1/\sqrt{10}$  for the  $\Sigma N$ -incident channel. This formula can also be used for the  $\Lambda\Lambda$ - $\Xi N$ - $\Sigma\Sigma$  CCRGM with strangeness  $S = -2$ .

## §5. Discussion and summary

In this paper we have discussed the question of what kind of equation we should solve for the off-shell  $T$ -matrix derived from the  $(3q)$ - $(3q)$  resonating-group method (RGM) for the baryon-baryon interaction. This is an important issue, since the present-day quark-model descriptions of the nucleon-nucleon ( $NN$ ) and the hyperon-nucleon ( $YN$ ) interactions are very accurate, and a realistic calculation of the hypertriton and baryonic matter, etc., using the quark-exchange kernel of these interactions directly is feasible to investigate the important off-shell effect and intricate behavior of short-range correlations.\*)

The off-shell  $T$ -matrix in RGM requires a rather involved formulation when the complete Pauli-forbidden state exists. Since the relative wavefunction can contain an arbitrary admixture of the redundant components, the half off-shell  $T$ -matrix should be defined as a limit of some definite  $T$ -matrix equation without this ambiguity. As a possible choice of such an equation, we have proposed a standard  $G$ -matrix equation which uses the quark-exchange kernel  $V_{\text{RGM}} = V_{\text{D}} + G + \varepsilon K$  directly as the driving term for the infinite sum of all the ladder diagrams. The relative energy  $\varepsilon$  should be evaluated with respect to the initial two-particle state.\*\*)

By using a simplified version of Saito's orthogonality condition model (OCM),<sup>7)</sup> we have argued that this inherent energy dependence of the Born kernel is essential to preserve the major role of the Pauli principle, which is sometimes represented in the form of the orthogonality condition with respect to the Pauli-forbidden state. In the ordinary formulation of OCM this orthogonality condition is applied to the relative wavefunction,<sup>7),5)</sup> while in the present formulation, as in the correlation function approach to the  $G$ -matrix, the defect function is orthogonal to the Pauli-forbidden state. One can also start from the OCM-type RGM equation  $\Lambda(\varepsilon - H_0 - v)\Lambda\psi = 0$  for the properly normalized

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\*) For the  $G$ -matrix calculations in the  $NN$  and  $YN$  systems using the model FSS and RGM-H, see Refs. 23) and 24).

\*\*) In Ref. 25), the energy-dependent term  $\varepsilon K$  in  $V_{\text{RGM}}$  is simply ignored.

relative wavefunction  $\psi = \sqrt{1-K}\chi$ . The effective potential  $v$  in this approach is given by  $v = (1/\sqrt{1-K})'(H_0 + V_D + G)(1/\sqrt{1-K})' - \Lambda H_0 \Lambda$ , where the prime in  $(1/\sqrt{1-K})'$  implies the inversion of  $\sqrt{1-K}$  in the allowed model space. The derivation of the off-shell  $T$ -matrix in this framework requires an explicit evaluation of  $V = V(\varepsilon) + v$ . On the other hand, we have discussed the structure involved in  $V(\varepsilon)$  only for a formal separation of the major effect of the Pauli principle. The present prescription to deal with  $V_{\text{RGM}} = V_D + G + \varepsilon K$  directly is the simplest and the most straightforward in the sense that the effect of the  $V(\varepsilon)$  term is already included in the exchange kernel,  $G^K + \varepsilon K$ , for the normalization and kinetic-energy terms.

We have also clarified the condition for which the the integral equation derived by Noyes<sup>8)</sup> and Kowalski<sup>9)</sup> is solvable and has a unique solution for the most general off-shell  $T$ -matrix. For the on-shell and half off-shell  $T$ -matrices, the conjugate homogeneous equation has a trivial solution if the Pauli-forbidden state  $|u\rangle$  exists. The condition  $\langle u|V_{\text{RGM}}|\phi\rangle = 0$  is automatically satisfied for the plane-wave solution  $|\phi\rangle$  in the initial channel, as long as the energy  $\varepsilon$  in  $V_{\text{RGM}}$  is fixed to the energy of  $|\phi\rangle$ . We can use this relationship to test if the exchange kernel is calculated correctly. When one needs to modify the exchange kernel, the modification should be made in such a way that this condition is still satisfied. An example of this kind of modification is the correction of the reduced masses and the small readjustment of the threshold energies in the coupled-channel RGM (CCRGM). In the rigorous framework of the RGM, it is sometimes difficult to reproduce correct reduced masses and internal energies starting from a unique Hamiltonian. In particular, the baryon masses for the inertia mass and the rest mass are sometimes inconsistent in the framework of the non-relativistic quark model, due to the residual  $qq$  interaction. On the other hand, the correct kinematics is necessary for a realistic description of the baryon-baryon interaction. We have shown that, if one uses the empirical reduced masses and threshold energies, an additional exchange term  $\Delta G(\mathbf{q}_f, \mathbf{q}_i)$  should be added in general to the Born amplitude  $V_{\text{RGM}}(\mathbf{q}_f, \mathbf{q}_i)$ . We have given the explicit expression for  $\Delta G(\mathbf{q}_f, \mathbf{q}_i)$ , which can be used for  $\Lambda N$ - $\Sigma N$  and  $\Lambda\Lambda$ - $\Xi N$ - $\Sigma\Sigma$  CCRGM.

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