Progress of Theoretical Physics, Vol. 104, No. 3, September 2000

Detailed Study of the Ground State Properties of the Newly Found Nuclide ^{48}Ni in Relativistic Mean-Field Theory

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(Received April 11, 2000)

We investigate in detail the ground state properties of the newly discovered doubly magic nucleus ^{48}Ni [1] in the framework of relativistic mean-field (RMF) theory with various parameter sets. Calculations show that this nucleus is a bound nucleus, because both of its proton separation energies are positive and its single particle levels are bound. This agrees with an experimental result.[1] The calculated two-proton separation energy for ^{48}Ni is found to be very close to the estimated value from the same experiment.[1] The RMF calculations also show that the proton-rich nucleus ^{48}Ni is spherical and $Z = 28$ is a good magic number. The variation of the spin-orbit splittings with proton excess is discussed in the context of RMF theory.

For many years it had been believed that the proton-rich nucleus ^{48}Ni is unstable with respect to proton emissions.[1] However, a new experimental result from GANIL in France demonstrated that ^{48}Ni is a bound nucleus.[1] This is the first discovered proton-rich bound nucleus whose proton excess is as large as 8. The physicists in GANIL have pointed out that this nucleus is probably the last doubly-magic nucleus to be observed.[1] The existence of the bound nucleus ^{48}Ni provides a good opportunity to investigate the nuclear shell structure in proton-drip-line nuclei and to observe the variation of the spin-orbit splittings with the proton excess. In this paper we present a detailed relativistic mean-field (RMF) calculation for the newly found nuclide ^{48}Ni and its neighboring nuclei.

Although there are a few spherical RMF calculations for Ni isotopes,[2)−4)] only the binding energy and the radii of ^{48}Ni were reported in these calculations. It is still not clear whether ^{48}Ni is stable with respect to proton emissions. This is because the boundedness of a nucleus depends on two main factors. One is the sign of its nucleon separation energy, and the other is the boundedness of its single-particle levels. Therefore it is interesting to make a detailed RMF study of ^{48}Ni in order to determine whether or not ^{48}Ni is bounded. It is also interesting to determine if $Z = 28$ is a good magic number and to compute the variation of the spin-orbit splittings with the proton excess. Here we carry out both deformed and spherical RMF calculations with the currently accepted force parameters for ^{48}Ni. A comparison of the spin-orbit splittings with its mirror nuclei ^{48}Ca is also made.

In the RMF approach, we start from the Lagrangian density for interacting nucleons, σ, ω and ρ mesons, and photons:[5)-13)]

$$\mathcal{L} = \overline{\Psi}(i\gamma^\mu\partial_\mu - M)\Psi - g_\sigma\overline{\Psi}\sigma\Psi - g_\omega\overline{\Psi}\gamma^\mu\omega_\mu\Psi - g_\rho\overline{\Psi}\gamma^\mu\rho^a_\mu\tau^a\Psi$$

$$
\begin{aligned}
&+\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma-\frac{1}{2}m_\sigma^2\sigma^2-\frac{1}{3}g_2\sigma^3-\frac{1}{4}g_3\sigma^4\\
&+\frac{1}{4}c_3(\omega_\mu\omega^\mu)^2-\frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu}+\frac{1}{2}m_\omega^2\omega^\mu\omega_\mu\\
&-\frac{1}{4}R^{a\mu\nu}R^a_{\mu\nu}+\frac{1}{2}m_\rho^2\rho^{a\mu}\rho^a_\mu-\frac{1}{4}F^{\mu\nu}F_{\mu\nu}-e\overline{\Psi}\gamma^\mu A^\mu\frac{1}{2}(1-\tau^3)\Psi
\end{aligned}
\tag{1}
$$

with

$$\Omega^{\mu\nu}=\partial^\mu\omega^\nu-\partial^\nu\omega^\mu\ , \tag{2}$$

$$R^{a\mu\nu}=\partial^\mu\rho^{a\nu}-\partial^\nu\rho^{a\mu}\ , \tag{3}$$

$$F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu\ . \tag{4}$$

Here the meson fields are denoted by σ, ω_μ and ρ^a_μ, and their masses are denoted by m_σ, m_ω and m_ρ. The nucleon field and its rest mass are denoted by Ψ and M. A_μ is the photon field, which is responsible for the electromagnetic interaction, $e^2/4\pi=1/137$. The effective strengths of the couplings between the mesons and nucleons are, respectively, g_σ, g_ω and g_ρ. The quantities g_2 and g_3 are the nonlinear coupling strengths of the σ meson, and c_3 is the self-coupling of the ω field.[12] The isospin Pauli matrices are written τ^a, with τ^3 being the third component of τ^a.

The equations of motion for the fields are easily obtained from the variational principle.[5)-8)] In order to describe the ground state properties of nuclei we need a static solution of the above Lagrangian. For this case the meson field and photon fields are assumed to be classical fields and they are time independent (c-numbers). The nucleons move in classical fields as independent particles (mean-field approximation). The Dirac field operator can be expanded in terms of single-particle wave functions as $\Psi=\sum_i\ \phi_i a_i^+$, where a_i^+ is the particle creation operator [5)] and ϕ_i is the single-particle wave function. For actual calculations, we omit the contribution of the Fermi sea under the no-sea approximation. The sum over single particle states runs over physical bound states, i.e. the occupied shell model states. Symmetries can be used to simplify the calculations considerably. First, time reversal symmetry can be applied and this implies that the spatial vector components for ω_μ, ρ^a_μ and A_μ are zero. Charge conservation guarantees that only the third component of the isovector field, ρ^0_0, survives.[5)-13)] For simplicity, we denote ρ^0_0 as ρ_0. Finally, we have the following Dirac equation for nucleons and the Klein-Gordon equation for the meson fields:[5)-8)]

$$[-i\boldsymbol{\alpha}\bigtriangledown\ +\ \beta M^*(\boldsymbol{r})+V(\boldsymbol{r})]\phi_i(\boldsymbol{r})=\epsilon_i\phi_i(\boldsymbol{r})\ , \tag{5}$$

where the effective mass $M^*(\boldsymbol{r})=M+g_\sigma\sigma(\boldsymbol{r})$. The potential $V(\boldsymbol{r})$ is a time-like component of a Lorentz vector:

$$V(\boldsymbol{r})=g_\omega\omega_0(\boldsymbol{r})+g_\rho\,\tau^a\rho^a_0(\boldsymbol{r})+e\,((1-\tau^3)/2)A_0(\boldsymbol{r})\ . \tag{6}$$

$$(-\triangle + m_\sigma^2)\sigma(\boldsymbol{r}) = -g_\sigma \rho_s(\boldsymbol{r}) - g_2\sigma^2(\boldsymbol{r}) - g_3\sigma^3(\boldsymbol{r}) \ , \tag{7}$$

$$(-\triangle + m_\omega^2)\omega_0(\boldsymbol{r}) = g_\omega \rho_v(\boldsymbol{r}) - c_3\omega_0^3(\boldsymbol{r}) \ , \tag{8}$$

$$(-\triangle + m_\rho^2)\rho_0(\boldsymbol{r}) = g_\rho \rho_3(\boldsymbol{r}) \ , \tag{9}$$

$$-\triangle A_0(\boldsymbol{r}) = e\rho_p(\boldsymbol{r}) \ , \tag{10}$$

where ρ_s, ρ_v and ρ_p are, respectively, the scalar, baryon and proton densities. The quantity ρ_3 is the difference between the neutron and proton densities. They are c-numbers obtained by taking the ground state expectation values. Their expressions are as follows,

$$\rho_s(\boldsymbol{r}) = \sum_{i=1}^{A} \overline{\phi_i}(\boldsymbol{r})\phi_i(\boldsymbol{r}) \ , \tag{11}$$

$$\rho_v(\boldsymbol{r}) = \sum_{i=1}^{A} \phi_i^+(\boldsymbol{r})\phi_i(\boldsymbol{r}) \ , \tag{12}$$

$$\rho_3(\boldsymbol{r}) = \sum_{i=1}^{A} \phi_i^+(\boldsymbol{r})\tau^3\phi_i(\boldsymbol{r}) \ , \tag{13}$$

$$\rho_p(\boldsymbol{r}) = \sum_{i=1}^{A} \phi_i^+(\boldsymbol{r})((1-\tau^3)/2)\,\phi_i(\boldsymbol{r}) \ . \tag{14}$$

Now we have a set of coupled equation for meson fields and Dirac fields. It is easy to solve these equations both for spherical nuclei and deformed nuclei with a standard code.[5)-13)] First we apply the deformed RMF code with oscillator bases to ^{48}Ni in order to investigate its deformation.[7),13)] We find that ^{48}Ni is a spherical nucleus, as its deformation is found to be almost zero. Then we use the spherical RMF code in coordinate space to calculate its ground state properties in detail.[9)] The standard force parameters NL-SH, NL3, NL-Z2 and TMA are used in our calculations. These forces are typical forces from the Munich group,[14),15)] the Frankfurt group,[16)] and the Osaka group.[17)] Their values are given in Table I. The pairing correlations are included from the BCS theory in the code. Actually, the pairing energy is zero for ^{48}Ni due to its doubly magic properties. For this reason, we switch off the pairing force for ^{48}Ni in numerical calculations. This agrees with a recent spherical Hartree-Bogoliubov result that suggests that the pairing strength is zero for ^{48}Ni.[4)] The correction of the center-of-mass motion to the binding energy is done in the same way as the force parameter set is defined.[14)-17)]

The ground state properties of ^{48}Ni calculated with the deformed code are listed in Table II. Here β_n and β_p are the quadrupole deformation parameters for neutrons

Table I. The values of force parameters.

	NL-SH	NL3	NL-Z2	TMA
m_N [MeV]	939.0	939.0	938.9	938.900
m_σ [MeV]	508.194	526.059	493.150	519.151
m_ω [MeV]	782.501	783.0	780.0	781.950
m_ρ [MeV]	763.0	763.0	763.0	768.100
g_σ	10.217	10.4436	10.1369	$10.055 + 3.050/A^{0.4}$
g_ω	12.868	12.9451	12.9084	$12.842 + 3.191/A^{0.4}$
g_ρ	4.474	4.3828	4.55627	$3.800 + 4.644/A^{0.4}$
g_2	−10.431	−6.9099	−13.7561	$-0.328 - 27.879/A^{0.4}$
g_3	−28.885	−15.8337	−41.4013	$38.862 - 184.191/A^{0.4}$
c_3	0.0	0.0	0.0	$151.590 - 378.004/A^{0.4}$

Table II. The binding energy and radii of ^{48}Ni calculated using the deformed code.

	NL-SH	NL3	NL-Z2	TMA
B (MeV)	348.91	349.27	350.99	351.17
R_p (fm)	3.66	3.69	3.75	3.72
R_n (fm)	3.34	3.36	3.40	3.39
R_m (fm)	3.53	3.56	3.61	3.58
R_c (fm)	3.75	3.78	3.84	3.80
β_p	0.00003	0.00077	0.00113	0.00025
β_n	−0.00014	−0.00025	0.00033	−0.00005
Q_p(fm^2)	0.0041	0.0979	0.1441	0.0321
Q_n(fm^2)	−0.0123	−0.0231	0.0300	−0.0049

and protons, respectively. Q_n (fm^2) and Q_p (fm^2) are the corresponding quadrupole moments. It is seen from Table I that the deformation parameters and quadrupole moments of ^{48}Ni are very small. This clearly shows that ^{48}Ni is truly a spherical nucleus.

In Table II, B (MeV) is the total binding energy and R_p, R_n, R_m, R_c are, respectively, the root-mean-square radii of the proton distribution, neutron distribution, matter distribution, and charge density distribution.

In view of the above result that ^{48}Ni is a spherical nucleus, it is interesting to use the spherical RMF code to investigate its ground state properties in detail. The binding energy, radii and single-particle levels of ^{48}Ni are given in Table III. By comparing the binding energy and radii in Table III with those in Table II, we see that the deformed RMF code and the spherical RMF code lead to almost identical values for these quantities. This suggests that both numerical calculations are reliable here. For the RMF results with different force parameters, the maximum difference of the binding energy is approximately 2.3 MeV. This is very small as compared with the total binding energy, which is about 350 MeV. Therefore the relative difference of the total binding energy is less than 0.7% among the different force parameters. This implies that the RMF model is very stable in this mass range.

Now let us consider the single-particle levels in ^{48}Ni. The nuclear shell gaps are evident for magic numbers 2, 8, 20 and 28. Therefore the RMF model strongly supports the experimental conclusion that ^{48}Ni is a doubly magic nucleus.[1)] All the

Table III. The ground state properties of ^{48}Ni calculated using the spherical code.

	NL-SH	NL3	NL-Z2	TMA
B (MeV)	348.91	349.24	350.99	351.17
R_p (fm)	3.67	3.70	3.75	3.71
R_n (fm)	3.34	3.36	3.39	3.37
R_m (fm)	3.54	3.56	3.60	3.57
R_c (fm)	3.76	3.79	3.83	3.80
$\epsilon(1s_{1/2})(p)$ (MeV)	−42.86	−42.74	−42.67	−41.56
$\epsilon(1p_{3/2})(p)$ (MeV)	−28.77	−28.25	−27.67	−27.78
$\epsilon(1p_{1/2})(p)$ (MeV)	−25.60	−24.99	−24.24	−24.84
$\epsilon(1d_{5/2})(p)$ (MeV)	−14.34	−13.83	−13.34	−13.67
$\epsilon(2s_{1/2})(p)$ (MeV)	−7.49	−7.97	−8.58	−7.25
$\epsilon(1d_{3/2})(p)$ (MeV)	−8.32	−7.98	−7.63	−8.00
$\epsilon(1f_{7/2})(p)$ (MeV)	−0.57	−0.37	−0.28	−0.28
$\epsilon(1s_{1/2})(n)$ (MeV)	−58.95	−58.69	−58.31	−58.06
$\epsilon(1p_{3/2})(n)$ (MeV)	−45.62	−44.98	−44.13	−45.13
$\epsilon(1p_{1/2})(n)$ (MeV)	−42.32	−41.61	−40.61	−42.05
$\epsilon(1d_{5/2})(n)$ (MeV)	−31.14	−30.53	−29.79	−31.03
$\epsilon(2s_{1/2})(n)$ (MeV)	−22.24	−22.77	−23.32	−22.44
$\epsilon(1d_{3/2})(n)$ (MeV)	−24.90	−24.44	−23.82	−25.14
S_p (MeV)	0.58	0.39	0.31	0.30
S_{2p} (MeV)	1.31	0.93	0.79	0.79

single-particle levels in Table III are bound levels. This suggests that ^{48}Ni is a bound nucleus. In order to confirm further that ^{48}Ni is bound, we also calculate the binding energies of its neighboring nuclei, ^{47}Co and ^{46}Fe. From the binding energy differences, we obtain the one-proton separation energy and two-proton separation energy of ^{48}Ni. These values are in the last two columns of Table III. Because the proton separation energy is greater than zero, it is concluded that ^{48}Ni is stable with respect to to proton emissions. Hence, we conclude that ^{48}Ni is a bound spherical nucleus in the RMF model. From an experimental result, Blank et al.[1)] determined an upper limit of the two-proton separation energy for ^{48}Ni of $Q_{2p} = 1.5$ MeV. It is interesting to note that our theoretical values are within this upper limit.

It is noted that we have omitted the effect of the breaking of time reversal symmetry for odd particles when we calculated the binding energy of ^{47}Co. In principle, an odd particle can induce polarization currents and time-odd components in the meson fields (the spatial vector currents).[24)-26)] Warrier and Gambhir discussed this in detail and concluded that its influence on binding energies and single-particle levels can be neglected.[24)-26)] Lalazissis et al. also concluded that its effect on deformations and binding energies is very small and can be neglected to a good approximation [25)] for proton drip-line nuclei (proton emitters). Rutz et al. considered the possibility that symmetry breaking may have an effect on separation energy,[26)] but they also concluded that energy differences within occupied or unoccupied states can be safely estimated from only the single-nucleon spectra of the even-even reference nucleus.[26)] For the nuclei considered here, we can to good approximation omit this effect, because these are spherical nuclei near the doubly magic number nucleus ^{48}Ni. For

Table IV. The spin-orbit splittings of ^{40}Ca and ^{48}Ni with the four parameter sets. In the brackets, we also list the spin-orbit splittings of ^{48}Ca in the RMF theory for comparison. The experimental data for spin-orbit splittings of ^{40}Ca are, respectively, 7.2 MeV and 6.3 MeV for protons and neutrons in $1d$ levels. The experimental data for spin-orbit splittings of ^{48}Ca are, respectively, 4.7 MeV and 3.6 MeV for protons and neutrons in $1d$ levels.

parameter	nucleus	$1p(p)$[MeV]	$1d(p)$ [MeV]	$1p(n)$ [MeV]	$1d(n)$ [MeV]
NL-SH	^{40}Ca	4.33	6.87	4.32	6.93
NL-SH	^{48}Ni	3.17	6.02	3.30	6.24
NL-SH	^{48}Ca	3.43	6.47	3.19	6.24
NL3	^{40}Ca	4.47	6.63	4.48	6.71
NL3	^{48}Ni	3.26	5.85	3.37	6.09
NL3	^{48}Ca	3.33	6.38	3.31	6.16
NLZ2	^{40}Ca	5.12	6.59	5.15	6.70
NLZ2	^{48}Ni	3.43	5.71	3.52	5.97
NLZ2	^{48}Ca	3.81	6.44	3.64	6.24
TMA	^{40}Ca	4.11	6.61	4.10	6.67
TMA	^{48}Ni	2.94	5.67	3.08	5.89
TMA	^{48}Ca	3.19	6.11	2.94	5.85

^{48}Ni, the proton separation energy is almost equal to the single-particle energy of protons in the $1f_{7/2}$ level, and this fact supports this conclusion. The inclusion of the polarization of odd nucleon does not change the conclusion of this discussion.

It is known from previous RMF calculations [18)−21)] that the spin-orbit splittings of neutron-rich nuclei decrease with neutron excess. Because ^{48}Ni is a bound nucleus, it is interesting to consider the variation of the spin-orbit splitting with the proton excess for proton-rich nuclei. We list the spin-orbit splittings of two doubly magic nuclei, ^{40}Ca and ^{48}Ni, in Table IV. From ^{40}Ca to ^{48}Ni, it is close to the proton-drip-line by adding protons. It is seen from Table IV that the spin-orbit splittings decrease with the increase of the proton excess. This behavior is seen for all force parameters. In order to compare the variation of spin-orbit splittings, we list the spin-orbit splittings of ^{48}Ca in brackets. It is known from the experimental data for spin-orbit splittings of ^{40,48}Ca (the heading of Table IV) that the spin-orbit splittings of $1d$ levels decrease from ^{40}Ca to ^{48}Ca. The result of the RMF theory agrees qualitatively with this. We expect that the change of the spin-orbit splitting will be larger for proton-rich nuclei such as ^{48}Ni, than for neutron-rich nuclei such as ^{48}Ca. It would be interesting to check this prediction experimentally.

We note here that there are theoretical studies by several groups on the spin-orbit splittings for various nuclei.[22),23)] In these studies, it has been found that the Fock terms with a pionic interaction greatly reduce the spin-orbit splitting. This allows for accurate reproduction of the d-state spin-orbit splitting in ^{48}Ca. However, for some heavier nuclei such as ^{208}Pb, Shi et al.[22)] observed that for some neutron-rich states, a flip of the spin-orbit splitting occurs in the Hartree-Fock calculations. We need to carefully investigate the Fock effect.

In conclusion, we have studied the ground state properties of the recently discovered doubly magic nucleus ^{48}Ni in the context of the RMF theory with various parameter sets. We have found that it is a spherical, bound nucleus. This result

agrees with an experimental result.[1)] The spin-orbit splitting of ^{48}Ni decreases due to its proton excess in comparison with that of ^{40}Ca. This is true for all values of the force parameters in the RMF model.

Acknowledgements

This work is supported by a COE Professorship Program of Monbusho in Japan. Z. R. thanks RCNP of Osaka University for the hospitality extended to him.

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