

Proper Construction of the Continuum in Light-Cone QCD Sum Rules

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A proper method of subtracting the continuum contributions in light-cone QCD sum rules (LCQSR) is demonstrated. Specifically, we calculate the continuum corresponding to a typical operator product expansion (OPE) appearing in LCQSR by properly combining the double dispersion relation with QCD duality. We demonstrate how the subtraction terms can spuriously contribute to the sum rules. In the limit of zero external momentum, removal of the spurious continuum is found to yield the sum rules using the single-variable dispersion relation. The continuum factor constructed in this way differs from that appearing in usual LCQSR. The difference substantially affects the extraction of hadronic parameters from the correlation function involving baryon currents.

§1. Introduction

The QCD sum rule^{1),2)} is a framework widely used to investigate hadronic properties in terms of QCD degrees of freedom.³⁾ In this method, it is crucial to represent a correlation function through a dispersion relation. This is because QCD calculations of the correlation function through the operator product expansion (OPE) can be done only in deep space-like regions, whereas the hadronic parameters are defined by the nonanalytic structure existing in time-like regions. Through a dispersion relation, the calculated correlation function can be matched with the hadronic parameters.

Within the QCD sum rule framework, a correlation function with an external field is often considered for the purpose of calculating meson-baryon couplings,^{4)–11)} $g_{D^*D\pi}$ and $g_{B^*B\pi}$,¹²⁾ and magnetic moments of baryons.^{13),14)} At present, there are two methods to construct QCD sum rules when an external field is present: the conventional approach, relying on the short-distance expansion, and the light-cone QCD sum rule (LCQSR),^{15)–18)} based on the expansion along the light-cone. Within the conventional approach one expands over the small momentum of the external field and constructs a separate sum rule for the correlation function appearing at each order of the expansion. In this approach, however, there has been a debate over which dispersion relation to use;^{19)–21)} one can start either from the single-variable dispersion relation or from the double-variable dispersion relation, and the results seem to depend strongly on which dispersion relation is used.¹⁹⁾ Later, it was shown²²⁾ that the two dispersion relations in the conventional sum rule yield identical results provided that the spurious continuum appearing in the double dispersion relation is properly eliminated. The spurious continuum, which comes from subtraction terms, appears when QCD duality is naively imposed on the double dispersion relation. A

safer approach, therefore, is to start from the single dispersion relation.

The spurious continuum can cause more serious problems in LCQSR, in which the double-variable dispersion relation is the only choice in representing correlation functions. In LCQSR, one keeps the external momentum finite, and the correlation function contains two momenta. The double-variable dispersion relation and, subsequently, the double Borel transformations must be applied for the two finite momenta. Then, it is not entirely clear how to apply QCD duality in subtracting the continuum, similar terms that are mathematically spurious can appear.

Indeed, in the case of the $D^*D\pi$ coupling, the issue of the spurious continuum has been briefly investigated²³⁾ in an attempt to revolve the discrepancy between the existing LCQSR calculation¹²⁾ and the recent CLEO measurement.²⁴⁾ In particular, the LCQSR¹²⁾ predicts this coupling to be around 12.5, which is substantially lower than a recent measurement,²⁴⁾ 17.9. The experimental value, however, is consistent with the quark model,²⁵⁾ and lattice calculation.²⁶⁾ This clearly indicates that the existing LCQSR need to be improved. It is, however, noteworthy that the coupling calculated from the LCQSR with the continuum modification²³⁾ is comparable to the experimental value. In this work, we now generalize the argument of Ref. 23) and propose an effective method of subtracting the continuum in the LCQSR. This should provide a guideline for future applications of LCQSR. We study how this improvement changes the existing results of LCQSR.

§2. A proper method of subtracting the continuum in LCQSR

Most LCQSR consider two-point correlation functions with an external field. A typical OPE that often appears in LCQSR takes the form¹¹⁾

$$\int_0^1 dv \varphi(v) \Gamma \left(\frac{D}{2} - n \right) \left[\frac{1}{-vp_1^2 - (1-v)p_2^2} \right]^{\frac{D}{2}-n}. \quad (2.1)$$

Here D is the dimensionality of the space. The wave function of the external field is denoted by $\varphi(v)$. When the external field is a pion, $\varphi(v)$ is the pion wave function, while its argument v represents the fractional momentum carried by a quark inside the pion. The wave function is usually a polynomial in v (for the case of a pion external field, see Ref. 12)):

$$\varphi(v) = \sum_k a_k v^k. \quad (2.2)$$

Thus, it would be sufficient to consider the case $\varphi(v) \rightarrow v^k$. For notational simplicity, let us introduce

$$A \equiv vp_1^2 + (1-v)p_2^2. \quad (2.3)$$

Since we are concerned with the duality issue in subtracting the continuum in LCQSR, we focus on the OPE that is dual to the continuum. Because the dimensionality of space satisfies $D \sim 4$, the typical OPE, Eq. (2.1), has a pole at the

zeros of A for $n \leq 1$. Therefore the OPE with $n \leq 1$ should not be dual to the continuum.^{*)}

For $n \geq 2$, $\Gamma(\frac{D}{2} - n)$ is singular. In this case, one expands around $2 - D/2 \equiv \epsilon \rightarrow 0^{27)}$ to separate the regular part from the singular part,

$$\Gamma\left(\frac{D}{2} - n\right) \left[\frac{1}{-A}\right]^{\frac{D}{2} - n} \sim -A^{n-2} \ln(-A) \frac{1}{(n-2)!} + \frac{\Gamma(\epsilon)}{(n-2)!} A^{n-2}, \quad (2.4)$$

so that our trial OPE (with $\varphi(v) \rightarrow v^k$) becomes

$$\begin{aligned} \Pi^{\text{ope}}(p_1^2, p_2^2) \equiv & - \int_0^1 dv \frac{v^k}{(n-2)!} A^{n-2} \ln(-A) \\ & + \int_0^1 dv \frac{v^k}{(n-2)!} \Gamma(\epsilon) A^{n-2}. \quad (n \geq 2, k \geq 0) \end{aligned} \quad (2.5)$$

We note here that the second term contains the singular coefficient $\Gamma(\epsilon)$. However, it is a simple power of A , constituting the so called “subtraction terms” in QCD sum rules. To get the finite result, it is necessary to eliminate the second term. This is one important reason to apply the Borel transformations, $\mathcal{B}(M_1^2, -p_1^2)\mathcal{B}(M_2^2, -p_2^2)$, where

$$\mathcal{B}(M^2, Q^2) = \lim_{Q^2, n \rightarrow \infty, Q^2/n = M^2} \frac{(Q^2)^{n+1}}{n!} \left(-\frac{d}{dQ^2}\right)^n. \quad (2.6)$$

Under this operation, the subtraction terms containing $\Gamma(\epsilon)$ vanish.

For the OPE given by Eq. (2.5), let us illustrate how a sum rule is constructed on the OPE side. The first step is to obtain the spectral density $\rho^{\text{ope}}(s_1, s_2)$ corresponding to the OPE (the logarithmic part) through the double dispersion relation

$$\Pi^{\text{ope}}(p_1^2, p_2^2) = \int_0^\infty ds_1 \int_0^\infty ds_2 \frac{\rho^{\text{ope}}(s_1, s_2)}{(s_1 - p_1^2)(s_2 - p_2^2)}. \quad (2.7)$$

Then, following the duality argument, one subtracts the continuum contribution lying above the threshold S_0 simply by restricting the integral within the range $[0, S_0]$,

$$\Pi_{\text{sub}}^{\text{ope}} \equiv \int_0^{S_0} ds_1 \int_0^{S_0} ds_2 \frac{\rho^{\text{ope}}(s_1, s_2)}{(s_1 - p_1^2)(s_2 - p_2^2)}. \quad (2.8)$$

As we discuss below, however, this step is dangerous because, there might be spurious terms contributing to the sum rule. Nevertheless, the subsequent Borel transformations yield the final expression for the OPE side of the sum rule,

$$\mathcal{B}(M_2^2, -p_2^2)\mathcal{B}(M_1^2, -p_1^2)\Pi_{\text{sub}}^{\text{ope}} = \int_0^{S_0} ds_1 \int_0^{S_0} ds_2 e^{-s_1/M_1^2 - s_2/M_2^2} \rho^{\text{ope}}(s_1, s_2). \quad (2.9)$$

^{*)} Note, however, that in the case $n = 1$, the continuum contribution is accounted for in normal LCQSR but, for $D^*D\pi$ coupling, it has been discussed²³⁾ that the case $n = 1$ should not be dual to the continuum. Indeed, the LCQSR without the continuum in this case provide a coupling comparable to the experiment value.

We show how the spurious terms enter in this prescription based on a simple mathematical reasoning.

To proceed, let us first demonstrate how the spectral density is normally determined in LCQSR. A common method is to apply the following Borel transformations to Eq. (2.7):^{28)–30)}

$$\begin{aligned} & \mathcal{B}\left(\tau_2^2, \frac{1}{M_2^2}\right) \mathcal{B}\left(\tau_1^2, \frac{1}{M_1^2}\right) \mathcal{B}(M_2^2, -p_2^2) \mathcal{B}(M_1^2, -p_1^2) \Pi^{\text{ope}}(p_1^2, p_2^2) \\ &= \rho^{\text{ope}}\left(\frac{1}{\tau_1^2}, \frac{1}{\tau_2^2}\right). \end{aligned} \tag{2.10}$$

For the OPE given in Eq. (2.5), this equation yields a spectral density of the form^{12), 14), 18)}

$$\rho^{\text{ope}}(s_1, s_2) = \frac{s_2^{n+k-1}}{(n+k-1)!} \left(-\frac{\partial}{\partial s_2}\right)^k \delta(s_1 - s_2), \text{ where } n \geq 2, k \geq 0. \tag{2.11}$$

Note that the term containing the powers of A in Eq. (2.5) vanishes under the Borel transformations. A crucial point that we want to make is that this spectral density satisfies the double dispersion relation Eq. (2.7), up to *subtraction terms*. Of course, the subtraction terms do not contribute if the sum rule is constructed under the double dispersion relation with the integral taken from 0 to ∞ . However, we emphasize that, as this spectral density enters in the restricted interval of $[0, S_0]$, the subtraction terms can contribute to the sum rule. We refer to such contributions from the subtraction terms as “spurious”.

To demonstrate in detail, let us see how the spectral density Eq. (2.11) reproduces the logarithmic part of the OPE in Eq. (2.5) through the double dispersion relation Eq. (2.7). This is a natural mathematical check. We first substitute Eq. (2.11) into Eq. (2.7) to obtain

$$\begin{aligned} & \int_0^\infty ds_1 \int_0^\infty ds_2 \frac{\rho^{\text{ope}}(s_1, s_2)}{(s_1 - p_1^2)(s_2 - p_2^2)} \\ &= \frac{1}{(n+k-1)!} \int_0^\infty ds_1 \int_0^\infty ds_2 \frac{s_2^{n+k-1}}{(s_1 - p_1^2)(s_2 - p_2^2)} \left(-\frac{\partial}{\partial s_2}\right)^k \delta(s_1 - s_2). \end{aligned} \tag{2.12}$$

We integrate first over s_1 by moving the part $\int_0^\infty ds_1 / (s_1 - p_1^2)$ inside the partial derivatives. After performing the partial derivatives, the Feynman parametrization leads to

$$\int_0^1 dv v^k \frac{(k+1)!}{(n+k-1)!} \int_0^\infty ds_2 \frac{s_2^{n+k-1}}{(s_2 - A)^{k+2}}. \tag{2.13}$$

Here again, we have introduced $A = vp_1^2 + (1-v)p_2^2$ for notational simplicity. We then perform the s_2 integration by parts successively and thereby reduce the power of the denominator order by order. In this process, we eventually end up with

$$\int_0^1 dv v^k \left[\int_0^\infty ds_2 \frac{1}{(n-2)!} \frac{s_2^{n-2}}{s_2 - A} - \sum_{i=0}^k \frac{(k-i)!}{(n+k-1-i)!} \frac{s_2^{n+k-1-i}}{(s_2 - A)^{k+1-i}} \Bigg|_0^\infty \right]. \tag{2.14}$$

One can see that the first term involving the s_2 integral is sufficient for reproducing the logarithmic part of Eq. (2.5). This can be easily seen by rewriting the numerator $s_2^{n-2} = (s_2 - A + A)^{n-2}$ and making use of the binomial formula; that is, the double dispersion relation Eq. (2.7) is satisfied with *this first term only*. We do not need the second term involving the summation to reproduce the OPE with which we started.

Then, what is the nature of the second term involving the summation? We note that the power in the numerator is greater than or equal to 1 because $n \geq 2$, $k \geq 0$ and $0 \leq i \leq k$. At the lower limit, $s_2 = 0$, therefore, the second term is always 0. But at the upper limit, $s_2 = \infty$, it consists of powers of A whose coefficients are infinite. This term at the upper limit has precisely the same form as the powers of A in Eq. (2.5) constituting the so-called “subtraction terms”, and we know that this should not contribute to the sum rule. What is crucial here is that, when the double dispersion relation is naively restricted within the integration interval $[0, S_0]$, the upper limit becomes $s_2 = S_0$, and the second term at this new upper limit no longer consists of powers of A . Instead, it has a pole at $A = S_0$, which obviously does not vanish even after the Borel transformations with respect to $-p_1^2$ and $-p_2^2$. Therefore, this contribution to the sum rule is spurious as in the case of the subtraction terms.

Another indication that the second term of Eq. (2.14) is spurious can be seen in the limit that the external momentum vanishes. In this limit, we have $p_1^2 = p_2^2 \equiv q^2$, $A = q^2$ and the OPE of interest, Eq. (2.5), becomes

$$\Pi^{\text{ope}}(q^2) = -\frac{1}{(k+1)(n-2)!} (q^2)^{n-2} \ln(-q^2) + [\text{subtraction terms}] . \quad (2.15)$$

As $\Pi^{\text{ope}}(q^2)$ contains only one variable, one must use the single-variable dispersion relation

$$\Pi^{\text{ope}}(q^2) = \int_0^\infty ds \frac{\rho^{\text{ope}}(s)}{s - q^2} \quad (2.16)$$

in order to obtain the corresponding spectral density. Within this approach, we immediately find that the spectral density is given by

$$\rho^{\text{ope}}(s) = \frac{1}{(k+1)(n-2)!} s^{n-2} , \quad (2.17)$$

which yields the OPE side after subtracting the continuum

$$\int_0^{S_0} ds \frac{1}{(k+1)(n-2)!} \frac{s^{n-2}}{s - q^2} . \quad (2.18)$$

This expression should be recovered from Eq. (2.14) when we set the external momentum to 0. In fact, the first term in Eq. (2.14) can reproduce this result, but the second term in Eq. (2.14) is not necessary in this check, again showing its spurious nature.

As we have demonstrated, to construct light-cone sum rules properly for the OPE given in Eq. (2.5), *we must take only the first term in Eq. (2.14), restrict the integral below S_0 , and perform the Borel transformation*. Next, let us demonstrate

how the OPE side looks within this approach. Using the double Borel transformation formula¹²⁾

$$\mathcal{B}(M_1^2, -p_1^2)\mathcal{B}(M_2^2, -p_2^2)\frac{(r-1)!}{[s-vp_1^2-(1-v)p_2^2]^r} = (M^2)^{2-r}e^{-s/M^2}\delta(v-v_0), \tag{2.19}$$

$(r = 1, 2, 3 \dots)$

we obtain, from the first term of Eq. (2.14),

$$\begin{aligned} \mathcal{B}(M_1^2, -p_1^2)\mathcal{B}(M_2^2, -p_2^2) \int_0^1 dv v^k \int_0^{S_0} ds_2 \frac{1}{(n-2)!} \frac{s_2^{n-2}}{s_2-A} \\ = \frac{1}{\beta} \frac{v_0^k}{(n-2)!} \left(-\frac{\partial}{\partial\beta}\right)^{n-2} \left(-\frac{1}{\beta} e^{-s\beta}\right) \Big|_0^{S_0}, \end{aligned} \tag{2.20}$$

where we have defined

$$\beta = \frac{1}{M^2}; \quad \frac{1}{M^2} = \frac{1}{M_1^2} + \frac{1}{M_2^2}; \quad v_0 = \frac{M_2^2}{M_1^2 + M_2^2}. \tag{2.21}$$

The two Borel masses are often taken to be equal, i.e. $M_1^2 = M_2^2$,¹²⁾ so that $v_0 = 1/2$. This implies that in the final form of a LCQSR, QCD inputs contain the wave functions at the middle point $\varphi(1/2)$, i.e., the probability for quark and antiquark to equally share the momentum of the external field. By directly performing the derivative $\left(-\frac{\partial}{\partial\beta}\right)^{n-2}$ in Eq. (2.20), we obtain the main result of this work,

$$v_0^k (M^2)^n E_{n-2} (S_0/M^2). \quad (n \geq 2) \tag{2.22}$$

Here we have defined the continuum factor as $E_n(x \equiv S_0/M^2) = 1 - (1 + x + \dots + x^n/n!)e^{-x}$. Note, the factor $v_0^k(M^2)^n$ is just twice the Borel transform of the OPE given in Eq. (2.5) without the continuum subtraction. We stress that Eq. (2.22) is the correct expression to appear in the final sum rule when the continuum is properly subtracted for the OPE of Eq. (2.5).

In contrast, in usual light-cone sum rules, the OPE after the Borel transformation contains^{*)}

$$v_0^k (M^2)^n E_{n-1} (S_0/M^2). \quad (n \geq 2) \tag{2.23}$$

This formula can be obtained from our approach by adding to our result the term for $i = k$ with $s_2 = S_0$ in the second term involving the summation in Eq. (2.14). This additional (but spurious) term becomes

$$-\int_0^1 dv v^k \frac{1}{(n-1)!} \frac{S_0^{n-1}}{(S_0-A)}. \tag{2.24}$$

^{*)} Most works on light-cone sum rules do not show a clear derivation of this formula, but their final OPE contains this continuum factor. [See for example Eq. (5.13) in Ref. 15), the discussion on p. 162 of Ref. 16), or Eq. (27) in Ref. 18). A technical derivation can also be found in the appendix of Ref. 14).]

Under the double Borel transformation given in Eq. (2·19), this becomes

$$-v_0^k S_0^{n-1} M^2 e^{-S_0/M^2} . \quad (2\cdot25)$$

If this is added to our result Eq. (2·22), we exactly obtain Eq. (2·23). This means that the usual LCQSR formula Eq. (2·23) contains spurious continuum. Furthermore, within the common method, it is not clear why one can simply drop the continuum contribution from the terms with $i \neq k$ in Eq. (2·14).

As far as mathematical form is concerned, Eq. (2·23) differs from our formula Eq. (2·22) only slightly. For a given power of Borel mass $(M^2)^n$, Eq. (2·23) contains the continuum factor E_{n-1} while our formula Eq. (2·22) contains E_{n-2} . However, this slight difference affects the final results substantially. We discuss this in the next section.

§3. Effects on existing sum rule analysis

In this section, we discuss how the difference in the continuum factor changes the predictions of previous LCQSR calculations. As far as the $D^*D\pi$ coupling is concerned, our prescription²³⁾ yields a value comparable to experiment,²⁴⁾ whereas the previous light-cone sum rule calculation gives a much smaller value.¹²⁾ The changes resulting from implementation of our prescription will be most effective in the sum rules that use a high-dimensional current, for example, the sum rule for a nucleon magnetic moment¹⁶⁾ or a pion-nucleon coupling.^{11),16)} Also, it will probably affect the other baryon sum rules. In these sum rules, the continuum threshold is $S_0 \sim 2 \text{ GeV}^2$, which corresponds to the squared mass of the Roper resonance. The leading term in the OPE after the Borel transformation contains the Borel mass M^6 . For this term, our result Eq. (2·22) suggests that the continuum factor $E_1(x \equiv S_0/M^2) = 1 - (1+x)e^{-x}$ should be multiplied, while the conventional light-cone sum rules^{11),16)} contain the factor $E_2 = 1 - (1+x+x^2/2)e^{-x}$. At a typical Borel mass $M^2 \sim 1 \text{ GeV}^2$, we have $E_1 \sim 0.6$, while $E_2 \sim 0.32$, only about half of E_1 . For an OPE leading to M^4 , our continuum factor is $E_0 \sim 0.86$, while the usual light-cone sum rules give $E_1 \sim 0.6$, about 30% lower. Thus, Eq. (2·23) suppresses the perturbative contributions too strongly. It is precisely this kind of suppression that leads to a value of the $D^*D\pi$ coupling that is much smaller than the experimental value.

The first example to investigate the effect of the new prescription is the calculation of the nucleon magnetic moments.¹⁶⁾ For this purpose, we simply take Eq. (14) of Ref. 16) and reproduce Fig. 2 of that work denoted by the solid curves in Fig. 1 (a) here. When the sum rule is changed according to our prescription, $E_1 \rightarrow E_0$ and $E_0 \rightarrow 1$, we obtain the dashed curves in Fig. 1(a). Depending on the continuum factors, we clearly obtain quite different Borel curves. It can be seen that the Borel stability of the solid curves comes purely from the continuum factor, and, therefore, the prediction is not stable with respect to variation of the continuum threshold. To show this, we plot the leading term of the OPE *without the continuum factor* in Fig. 1(b) with the dashed curve (indicated by f_1). The magnitude of the rest of the OPE is approximately 0.7 (not shown). As a reference curve, we again plot the Borel

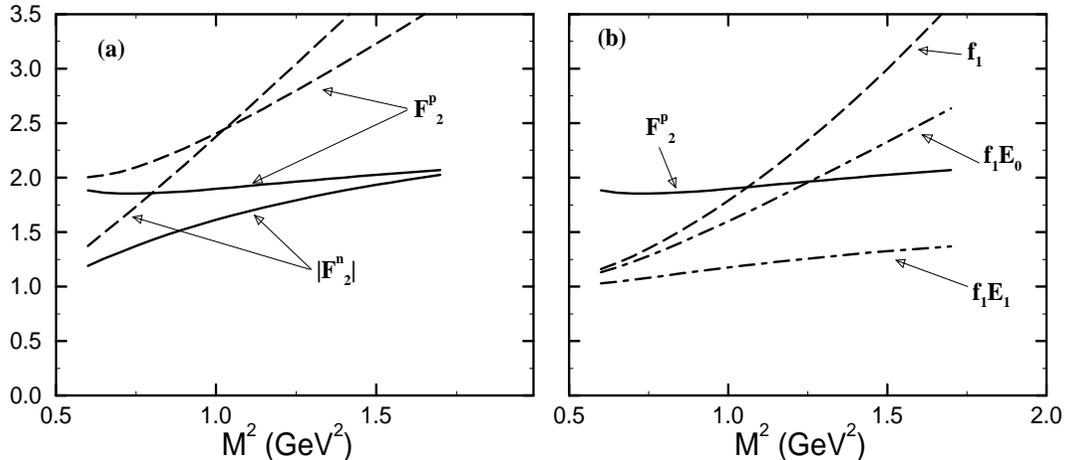


Fig. 1. The nucleon magnetic moments ($F_2^p = \mu_p - 1$ and $F_2^n = \mu_n$). In (a), the solid curves qualitatively reproduce Fig. 2. of Ref. 16). The dashed curves represent the result when the continuum factors are corrected in the manner described in the text. In (b), the leading term in the OPE (dashed curve indicated by f_1) without the continuum, with our continuum factor (dot-dashed curve indicated by $f_1 E_0$), and with the usual light-cone continuum factor (dot-dashed curve indicated by $f_1 E_1$) are plotted. Also shown by the solid curve is the total OPE from Ref. 16).

curve (solid curve) for F_2^p . The dashed curve is already above the total OPE (the solid curve indicated by F_2^p) for $M^2 \geq 1.1$. The usual continuum factor containing the spurious contribution lowers the curve substantially, as shown by the dot-dashed curve [denoted as $f_1 E_1$ in Fig. 1(b)], which suppresses the perturbative part too strongly. The degree of suppression depends on the Borel mass, but there is more than 50% reduction for $M^2 \geq 1$ GeV², which results from only a simple modeling of higher resonance contributions. Even if we restrict the continuum contribution to be less than 50%, we still cannot obtain a Borel window around 1 GeV², indicating that the result is extremely sensitive to the continuum threshold. On the other hand, as shown by the other dot-dashed curve (denoted by $f_1 E_0$), our prescription does not suppress the perturbative part too strongly.

Another example to investigate is the pion-nucleon coupling calculation using the nucleon two-point correlation function with a pion within the light-cone sum rule approach,^{11),16)}

$$\Pi(q, p) = i \int d^4x e^{iq \cdot x} \langle 0 | T [J_N(x) \bar{J}_N(0)] | \pi(p) \rangle. \quad (3.1)$$

In Ref. 16), light-cone sum rules are constructed for the $i\gamma_5 \not{p}$ and $i\gamma_5 \not{q} \not{p}$ Dirac structures^{*)} from the correlation function Eq. (3.1). They compared the two sum rules and extracted the twist-2 pion wave function at the symmetric point $\varphi_\pi(1/2)$, using

^{*)} It should be noted that the $i\gamma_5 \not{q} \not{p}$ structure sum rule is not independent of the $i\gamma_5$ Dirac structure. Since $i\gamma_5 \not{q} \not{p} = i\gamma_5 p \cdot q + \gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$, one actually needs to construct a sum rule for the structure $\gamma_5 \sigma_{\mu\nu} q^\mu p^\nu$.

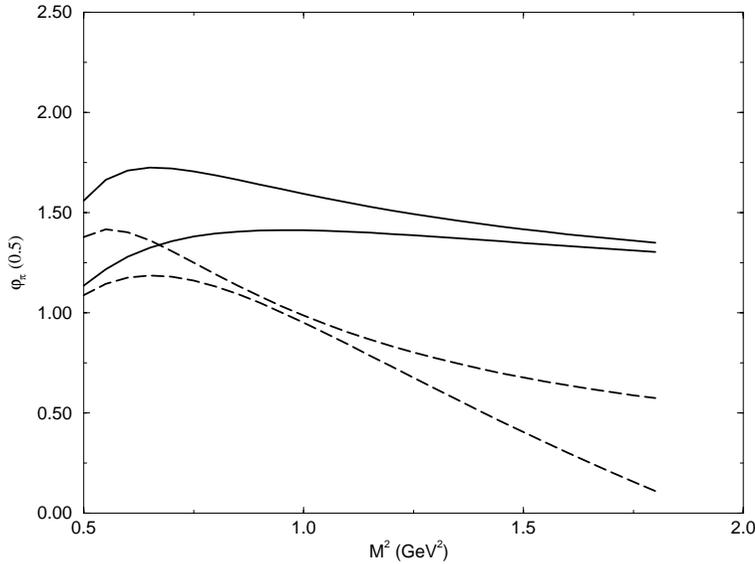


Fig. 2. The twist-2 pion wave function at the middle point $\varphi_\pi(1/2)$ corresponding to the result of Ref. 16). Again, the solid curves reproduce Fig. 4 of Ref. 16), and the dashed curves are those obtained when the continuum is corrected.

the experimental pion-nucleon coupling $g_{\pi N} \sim 13.5$ as an input. The solid curves in Fig. 2 qualitatively reproduce the result of Ref. 16). When the continuum factors are changed according to our prescription given in Eq. (2·22), we obtain the dashed curves, which are substantially lower than the solid curves, again showing very large modifications of the sum rule results.

The OPE calculation for the $i\gamma_5\not{p}$ sum rule of Ref. 16) was improved by Zhu et al.,¹¹⁾ who claimed that there are missing OPE terms in Ref. 16). Even in Ref. 11), however, the spurious continuum is very large. We simply take the formula Eq. (23) of Ref. 11) and reproduce Fig. 2 of Ref. 11) for $\varphi_\pi(1/2) \sim 1.5$ as shown by the thick solid curve in Fig. 3 ($S_0 = 2.25 \text{ GeV}^2$). However, if we simply take a slightly higher threshold, $S_0 = 2.75 \text{ GeV}^2$, we obtain the thin solid curve. As expected, there is very strong sensitivity to the continuum, and therefore the result of Ref. 11) is not conclusive. When the continuum factors are corrected according to our formula given in Eq. (2·22), we obtain the thick dashed curve with which we cannot conclude that their result $\varphi_\pi(1/2) \sim 1.5$ is consistent with the pion-nucleon coupling $g_{\pi N} \sim 13.5$.

As we have shown in these examples, correcting continuum factors substantially change the results of previous light-cone sum rule calculations. The failure to reproduce the known phenomenological parameters may indicate that the approach adopted in Refs. 16) and 11) is not optimal for the investigation of the baryon properties of interest. In particular, the $i\gamma_5\not{p}$ sum rule is known to be very sensitive to the continuum threshold.^{6),9)} Higher resonances with different parities add up to constitute a very large continuum. Thus, it is not realistic to predict the coupling after subtracting more than 50% from the total strength by a simple modeling of

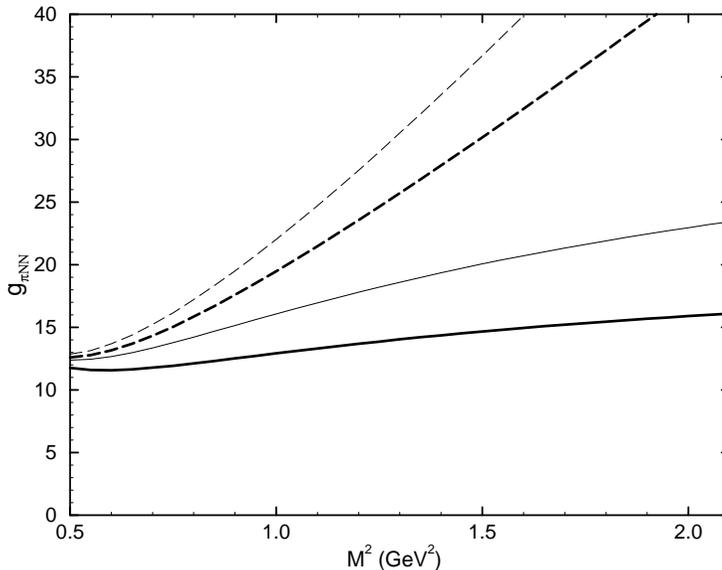


Fig. 3. The Borel curves of the pion-nucleon coupling corresponding to the result in Ref. 11). The thick solid curve reproduces the result of Ref. 11) while the thick dashed curve is obtained when the continuum factor is corrected according to our prescription. The corresponding thin curves are obtained when the continuum threshold is shifted by only 0.5 GeV^2 , showing that the result is very sensitive to the continuum threshold.

the continuum. Instead, one may need to consider the $\gamma_5 \sigma_{\mu\nu}$ structure in the investigation of the πNN coupling. In this case, higher resonances with different parities cancel, and the resulting sum rules are less sensitive to the continuum threshold. Similar changes can be expected from other light-cone sum rules. Therefore, it is important to re-analyze previous light-cone sum rules using the continuum factor that we have considered in this work.

In summary, a proper method of subtracting the continuum contribution in light-cone QCD sum rules has been demonstrated in the work. In particular, by closely looking into the double dispersion relation and QCD duality, we have isolated the spurious contributions in LCQSR. They are spurious because (1) they belong to subtraction terms in the dispersion relation, and (2) in the limit of vanishing external momentum, they are precisely the terms that do not match with that obtained using the single dispersion relation. We then proposed the proper continuum factors to appear in the sum rules for a given OPE. We found that the continuum factor in this approach is slightly different from the usual one appearing in LCQSR, but the effect of this difference is found to be enormous. It has been demonstrated that the conclusions are altered greatly by this modification.

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