

$U_A(1)$ Breaking Effects on the Light Scalar Meson Spectrum

Kenichi NAITO,^{1,*} Makoto OKA,^{2,**}
 Makoto TAKIZAWA^{3,***} and Toru UMEKAWA^{2,†}

¹*Meme Media Laboratory, Graduate School of Engineering, Hokkaido University, Sapporo 060-8628, Japan*

²*Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan*

³*Showa Pharmaceutical University, Machida 194-8543, Japan*

(Received December 9, 2002)

Effects of the $U_A(1)$ breaking interaction on the low-lying nonet scalar mesons are studied using the extended Nambu-Jona-Lasinio model. The strength of the $U_A(1)$ breaking interaction is determined by the electromagnetic decays of the η meson. We find that the $U_A(1)$ breaking interaction gives rise to about 150 MeV mass difference between the σ and a_0 mesons. We also find that the strangeness content in the σ meson is about 15%. The calculated mass of the $I = 1/2$ state is about 200 MeV heavier than that of the $I = 1$ state. The order of these masses is not likely to change within this model.

§1. Introduction

Understanding low-lying hadron spectrum is one of the most challenging problems in the quantum chromodynamics (QCD). The spectrum is highly nontrivial due to the nonperturbative complexity, such as spontaneous breaking of chiral symmetry and axial $U(1)$ anomaly. For instance, the pseudoscalar mesons are off-scale light, if we suppose that they are bound states of a quark and an antiquark, while their spin-flip partners, i.e., vector mesons, are normal with masses about $2/3$ of the baryon masses. This “anomaly” in the pseudoscalar mesons is attributed to their Nambu-Goldstone-boson nature associated with the spontaneous chiral symmetry breaking. This is strongly in contrast with the heavy meson spectroscopy, such as heavy quarkonia and heavy flavor mesons. There the spectrum is more like the hydrogen atom with slightly stronger fine and hyperfine splittings.

It should be noted that the low-lying hadrons are the key to explore the complicated QCD vacuum, as in QCD we are not able to “measure” the bulk properties of the ground state, which can be accessed directly in the case of condensed matter physics. Thus it is important to explore the properties of the low-lying hadrons from the viewpoints of QCD dynamics and symmetries.

Another nontrivial effect comes from $U_A(1)$ symmetry, which is expected to be broken by anomaly. Weinberg showed that the mass of η' should be less than $\sqrt{3}m_\pi$ if $U_A(1)$ symmetry were not explicitly broken.¹⁾ Thus the $U_A(1)$ symmetry must

*) E-mail: knaito@nucl.sci.hokudai.ac.jp

**) E-mail: oka@th.phys.titech.ac.jp

***) E-mail: takizawa@ac.shoyaku.ac.jp

†) E-mail: umekawa@th.phys.titech.ac.jp

be broken. In the following year, 't Hooft pointed out the relation between $U_A(1)$ anomaly and topological gluon configurations of QCD and showed that the interaction of light quarks and instantons breaks the $U_A(1)$ symmetry.²⁾ He also showed that such an interaction can be represented by a local $2N_f$ quark vertex, which is antisymmetric under flavor exchanges, in the dilute instanton gas approximation. The dynamics of instantons in the multi-instanton vacuum has been studied by many authors, either in the models or in the lattice QCD approach, and the widely accepted picture is that the QCD vacuum consists of small instantons of the size about $1/3$ fm with the density of 1 instanton (or anti-instanton) per fm^4 .³⁾

According to such an instanton vacuum picture, the hadron spectrum shows its signature. The $\eta - \eta'$ mass difference is the obvious one, which can be understood by flavor mixing in the $I = 0$ $(q\bar{q})_{NS} \equiv \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ and $s\bar{s}$. Without the flavor mixing, $(q\bar{q})_{NS}$ and $s\bar{s}$ would form mass eigenstates, and thus the ideal mixing is achieved. This is natural if the Okubo-Zweig-Iizuka (OZI) rule applies. However, the OZI rule is known to be significantly broken in the pseudoscalar mesons. For instance, according to our previous analyses in the Nambu-Jona-Lasinio model, the electromagnetic η decay processes indicate that the mixing of $(q\bar{q})_{NS}$ and $s\bar{s}$ is indeed strong so that the η meson is close to the pure octet state.⁴⁾

Recently, the scalar mesons, $J^\pi = 0^+$, attract a lot of attention by two reasons.⁵⁾ (1) Experimental evidence for σ ($I = 0$) scalar meson of mass around 500–800 MeV is overwhelming.⁶⁾ In particular the decays of heavy mesons show clear peaks in the $\pi\pi$ invariant mass spectrum. (2) The roles of the scalar mesons in chiral symmetry have been stressed in the context of high temperature and/or density hadronic matter.⁷⁾ It is believed that chiral symmetry will be restored in the QCD ground state at high temperature (and/or baryon density). Above the critical temperature of order 150 MeV the world is nearly chiral symmetric and we expect that hadrons belong to irreducible representations of chiral symmetry, if we neglect small mixing due to finite quark mass. The pion is not any more a Nambu-Goldstone boson, and has a finite mass and should be degenerate with a scalar meson, i.e., sigma.

In this paper, we study the masses and mixing angles of scalar mesons in the context of chiral symmetry and $U_A(1)$ breaking using the extended NJL model, in which the $SU(3)$ NJL model is supplemented with the Kobayashi-Maskawa-'t Hooft (KMT) determinant interaction.^{2), 8)} This is the simplest possible quark model with the correct symmetry structure for the present purposes. The chiral symmetry is broken both explicitly by a quark mass term and dynamically by quark loops, while $U_A(1)$ symmetry is broken by the KMT interaction term.

Why is the $U_A(1)$ expected to be important in the scalar mesons? It is because the KMT interaction selects out the scalar sector as well as the pseudoscalar mesons and therefore the OZI rule may be significantly broken also in the scalar mesons.

Recent experimental data suggest that the light scalar mesons (below 1GeV) show strange mass patterns, i.e., $\sigma(600) - \kappa(700 - 900) - f_0(980) - a_0(980)$.^{*)} This pattern cannot be explained by a $q\bar{q}$ nonet, because the $I = 1$ a_0 states are degenerate

*) The existence of a light and very broad κ meson is controversial. It is not our aim to claim the existence of the light κ meson.

with the second $I = 0$ state f_0 , while the first $I = 0$ σ is far below them. Furthermore the strange meson κ comes below a_0 .

Dmitrasinovic⁹⁾ has pointed out that the KMT interaction may change the spectrum of the scalar mesons significantly. He has shown that the mass splitting of the $I = 0$ and $I = 1$ nonstrange scalar mesons is generated by the KMT interaction. He, however, tried to assign the $I = 0$ state to $f_0(980)$ or higher, and the $I = 1$ to $a_0(1450)$. Thus, his results are not applied to the lower mass mesons phenomenologically.

We construct the model with the $U_A(1)$ anomaly which is strong enough to explain the η decay widths and apply it to the light scalar mesons. We will obtain significant $\sigma - a_0$ splitting and flavor mixing from the KMT interaction. The mixing of $s\bar{s}$ component in the σ meson is very interesting. As the extended NJL model has been used in the analyses of the pseudoscalar mesons, it has an advantage that the parameters have been all fixed in the pseudoscalar sector.

In §2, we present the extended NJL model. The formulation of solving the Bethe-Salpeter equation for the scalar channel is explained. In §3, we show our results and give discussions on the mass spectrum as well as the mixings. In §4, conclusion and future prospects are given.

§2. Formulation

We work with the following NJL model lagrangian density extended to three-flavor case:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_4 + \mathcal{L}_6, \tag{2.1}$$

$$\mathcal{L}_0 = \bar{\psi} (i\partial_\mu \gamma^\mu - \hat{m}) \psi, \tag{2.2}$$

$$\mathcal{L}_4 = \frac{G_S}{2} \sum_{a=0}^8 \left[(\bar{\psi} \lambda^a \psi)^2 + (\bar{\psi} \lambda^a i\gamma_5 \psi)^2 \right], \tag{2.3}$$

$$\mathcal{L}_6 = G_D \left\{ \det [\bar{\psi}_i (1 - \gamma_5) \psi_j] + \det [\bar{\psi}_i (1 + \gamma_5) \psi_j] \right\}. \tag{2.4}$$

Here the quark field ψ is a column vector in color, flavor and Dirac spaces and $\lambda^a (a = 0 \cdots 8)$ is the Gell-Mann matrices for the flavor $U(3)$. The free Dirac lagrangian \mathcal{L}_0 incorporates the current quark mass matrix $\hat{m} = \text{diag}(m_u, m_d, m_s)$ which breaks the chiral $U_L(3) \times U_R(3)$ invariance explicitly. \mathcal{L}_4 is a QCD motivated four-fermion interaction, which is chiral $U_L(3) \times U_R(3)$ invariant. The Kobayashi-Maskawa-'t Hooft determinant \mathcal{L}_6 represents the $U_A(1)$ anomaly. It is a 3×3 determinant with respect to flavor with $i, j = u, d, s$.

Quark condensates and constituent quark masses are self-consistently determined by the gap equations in the mean field approximation,

$$\begin{aligned} M_u &= m_u - 2G_S \langle \bar{u}u \rangle - 2G_D \langle \bar{d}d \rangle \langle \bar{s}s \rangle, \\ M_d &= m_d - 2G_S \langle \bar{d}d \rangle - 2G_D \langle \bar{s}s \rangle \langle \bar{u}u \rangle, \\ M_s &= m_s - 2G_S \langle \bar{s}s \rangle - 2G_D \langle \bar{u}u \rangle \langle \bar{d}d \rangle, \end{aligned} \tag{2.5}$$

with

$$\begin{aligned} \langle \bar{q}q \rangle &= -\text{Tr}^{(c,D)} [iS_F^q(x=0)] \\ &= -\int^A \frac{d^4p}{(2\pi)^4} \text{Tr}^{(c,D)} \left[\frac{i}{p_\mu \gamma^\mu - M_q + i\epsilon} \right]. \end{aligned} \tag{2.6}$$

Here the covariant cutoff Λ is introduced to regularize the divergent integral and $\text{Tr}^{(c,D)}$ represents trace in color and Dirac spaces.

The scalar channel quark-antiquark scattering amplitudes

$$\langle p_3, \bar{p}_4; \text{out} | p_1, \bar{p}_2; \text{in} \rangle = (2\pi)^4 \delta^4(p_3 + p_4 - p_1 - p_2) \mathcal{T}_{q\bar{q}} \tag{2.7}$$

are then calculated in the ladder approximation. We assume that $m_u = m_d$ so that the isospin is exact. In the σ and f_0 channel, the explicit expression is

$$\mathcal{T}_{q\bar{q}} = - \begin{pmatrix} \bar{u}(p_3)\lambda^8 v(p_4) \\ \bar{u}(p_3)\lambda^0 v(p_4) \end{pmatrix}^T \begin{pmatrix} A(q^2) & B(q^2) \\ B(q^2) & C(q^2) \end{pmatrix} \begin{pmatrix} \bar{v}(p_2)\lambda^8 u(p_1) \\ \bar{v}(p_2)\lambda^0 u(p_1) \end{pmatrix}, \tag{2.8}$$

with

$$A(q^2) = \frac{2}{\det \mathbf{D}(q^2)} \{2(G_0 G_8 - G_m G_m) I^0(q^2) - G_8\}, \tag{2.9}$$

$$B(q^2) = \frac{2}{\det \mathbf{D}(q^2)} \{-2(G_0 G_8 - G_m G_m) I^m(q^2) - G_m\}, \tag{2.10}$$

$$C(q^2) = \frac{2}{\det \mathbf{D}(q^2)} \{2(G_0 G_8 - G_m G_m) I^8(q^2) - G_0\}, \tag{2.11}$$

and

$$G_0 = \frac{1}{2} G_S - \frac{1}{3} (2\langle \bar{u}u \rangle + \langle \bar{s}s \rangle) G_D, \tag{2.12}$$

$$G_8 = \frac{1}{2} G_S - \frac{1}{6} (\langle \bar{s}s \rangle - 4\langle \bar{u}u \rangle) G_D, \tag{2.13}$$

$$G_m = -\frac{1}{3\sqrt{2}} (\langle \bar{s}s \rangle - \langle \bar{u}u \rangle) G_D. \tag{2.14}$$

The quark-antiquark bubble integrals are defined by

$$I^0(q^2) = i \int^A \frac{d^4p}{(2\pi)^4} \text{Tr}^{(c,f,D)} [S_F(p)\lambda^0 S_F(p+q)\lambda^0], \tag{2.15}$$

$$I^8(q^2) = i \int^A \frac{d^4p}{(2\pi)^4} \text{Tr}^{(c,f,D)} [S_F(p)\lambda^8 S_F(p+q)\lambda^8], \tag{2.16}$$

$$I^m(q^2) = i \int^A \frac{d^4p}{(2\pi)^4} \text{Tr}^{(c,f,D)} [S_F(p)\lambda^0 S_F(p+q)\lambda^8], \tag{2.17}$$

with $q = p_1 + p_2$. The 2×2 matrix \mathbf{D} is given by

$$\mathbf{D}(q^2) = \begin{pmatrix} D_{11}(q^2) & D_{12}(q^2) \\ D_{21}(q^2) & D_{22}(q^2) \end{pmatrix}, \tag{2.18}$$

with

$$D_{11}(q^2) = 2G_0I^0(q^2) + 2G_mI^m(q^2) - 1, \tag{2.19}$$

$$D_{12}(q^2) = 2G_0I^m(q^2) + 2G_mI^8(q^2), \tag{2.20}$$

$$D_{21}(q^2) = 2G_8I^m(q^2) + 2G_mI^0(q^2), \tag{2.21}$$

$$D_{22}(q^2) = 2G_8I^8(q^2) + 2G_mI^m(q^2) - 1. \tag{2.22}$$

From the pole positions of the scattering amplitude Eq. (2.8), the σ -meson mass m_σ and the f_0 -meson mass m_{f_0} are determined.

The scattering amplitude Eq. (2.8) can be diagonalized by rotation in the flavor space

$$\begin{aligned} \mathcal{T}_{q\bar{q}} = & - \begin{pmatrix} \bar{u}(p_3)\lambda^8v(p_4) \\ \bar{u}(p_3)\lambda^0v(p_4) \end{pmatrix}^T \mathbf{T}_\theta^{-1} \mathbf{T}_\theta \begin{pmatrix} A(q^2) & B(q^2) \\ B(q^2) & C(q^2) \end{pmatrix} \mathbf{T}_\theta^{-1} \\ & \times \mathbf{T}_\theta \begin{pmatrix} \bar{v}(p_2)\lambda^8u(p_1) \\ \bar{v}(p_2)\lambda^0u(p_1) \end{pmatrix}, \end{aligned} \tag{2.23}$$

$$\begin{aligned} = & - \begin{pmatrix} \bar{u}(p_3)\lambda^\sigma v(p_4) \\ \bar{u}(p_3)\lambda^{f_0}v(p_4) \end{pmatrix}^T \begin{pmatrix} D^\sigma(q^2) & 0 \\ 0 & D^{f_0}(q^2) \end{pmatrix} \\ & \times \begin{pmatrix} \bar{v}(p_2)\lambda^\sigma u(p_1) \\ \bar{v}(p_2)\lambda^{f_0}u(p_1) \end{pmatrix}, \end{aligned} \tag{2.24}$$

with $\lambda^\sigma \equiv \cos\theta\lambda^8 - \sin\theta\lambda^0$, $\lambda^{f_0} \equiv \sin\theta\lambda^8 + \cos\theta\lambda^0$ and

$$\mathbf{T}_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}. \tag{2.25}$$

The rotation angle θ is determined by

$$\tan 2\theta = \frac{2B(q^2)}{C(q^2) - A(q^2)}. \tag{2.26}$$

Note that θ therefore depends on q^2 . At $q^2 = m_\sigma^2$, θ represents the mixing angle of the λ^8 and λ^0 components in the σ -meson state.

The $U_A(1)$ breaking KMT 6-quark determinant interaction \mathcal{L}_6 contributes to the scalar $q\bar{q}$ channel only by the form of the effective 4-quark interaction, which is derived from \mathcal{L}_6 by contracting a quark-antiquark pair into the quark condensate. The explicit form of the effective KMT interaction is

$$\begin{aligned} \mathcal{L}_6^{\text{eff}} = & \left(\frac{-1}{2}\right) G_D \left\{ \left(\frac{-2}{3}\right) (2\langle\bar{u}u\rangle + \langle\bar{s}s\rangle) \left[(\bar{\psi}\lambda^0\psi)^2 - (\bar{\psi}\lambda^0i\gamma_5\psi)^2 \right] \right. \\ & + \langle\bar{s}s\rangle \sum_{i=1}^3 \left[(\bar{\psi}\lambda^i\psi)^2 - (\bar{\psi}\lambda^ii\gamma_5\psi)^2 \right] \\ & \left. + \langle\bar{u}u\rangle \sum_{i=4}^7 \left[(\bar{\psi}\lambda^i\psi)^2 - (\bar{\psi}\lambda^ii\gamma_5\psi)^2 \right] \right\} \end{aligned}$$

$$\begin{aligned}
& + \left(\frac{1}{3}\right) (4\langle\bar{u}u\rangle - \langle\bar{s}s\rangle) \left[(\bar{\psi}\lambda^8\psi)^2 - (\bar{\psi}\lambda^8i\gamma_5\psi)^2 \right] \\
& + \left(\frac{\sqrt{2}}{3}\right) (\langle\bar{u}u\rangle - \langle\bar{s}s\rangle) \left[(\bar{\psi}\lambda^0\psi) (\bar{\psi}\lambda^8\psi) + (\bar{\psi}\lambda^8\psi) (\bar{\psi}\lambda^0\psi) \right. \\
& \left. - (\bar{\psi}\lambda^0i\gamma_5\psi) (\bar{\psi}\lambda^8i\gamma_5\psi) + (\bar{\psi}\lambda^8i\gamma_5\psi) (\bar{\psi}\lambda^0i\gamma_5\psi) \right] \Big\}. \quad (2.27)
\end{aligned}$$

One can easily figure out from Eq. (2.27) that the $U_A(1)$ breaking KMT interaction gives the attractive force in the flavor singlet scalar $q\bar{q}$ channel. On the other hand, it gives the repulsive force in the isospin $I = 1$ (a_0) and $I = 1/2$ (K_0^*) channels. Because of the large strange quark mass, $|\langle\bar{s}s\rangle|$ is bigger than $|\langle\bar{u}u\rangle|$, and therefore, the repulsion in the $I = 1$ channel is stronger than that in the $I = 1/2$ channel.

§3. Results

We show our numerical results and give discussions on the mass spectrum as well as the mixings in this section. As the extended NJL model has been used in the analyses of the pseudoscalar mesons, here we have used the model parameters fixed in the study of the electromagnetic decays of the η meson. Since the η meson properties depend on the strength of the $U_A(1)$ breaking interaction rather sensitively, it is reasonable to determine the strength of the $U_A(1)$ breaking interaction from the η meson properties.

The parameters of the NJL model are the current quark masses $m_u = m_d$, m_s , the four-quark coupling constant G_S , the $U_A(1)$ breaking KMT six-quark determinant coupling constant G_D and the covariant cutoff Λ . We take G_D as a free parameter and study scalar meson properties as functions of G_D . We use the light current quark masses $m_u = m_d = 8.0$ MeV to reproduce $M_u = M_d \simeq 330$ MeV ($\simeq 1/3M_N$) which is the value commonly used in the constituent quark model. The other parameters, m_s , G_S and Λ , are determined so as to reproduce the isospin averaged observed masses, $m_\pi = 138.0$ MeV, $m_K = 495.7$ MeV and the pion decay constant $f_\pi = 92.4$ MeV. When we take the different value of G_D , we go through the fitting procedure each time.

We obtain $m_s = 193$ MeV, $\Lambda = 783$ MeV, $M_{u,d} = 325$ MeV, $M_s = 529$ MeV and $f_K = 97$ MeV, which are almost independent of G_D . The quark condensates are also independent of G_D and our results are $\langle\bar{u}u\rangle^{\frac{1}{3}} = -216$ MeV and $\langle\bar{s}s\rangle^{\frac{1}{3}} = -226$ MeV whenever we have fixed other model parameters from the observed values of m_π , m_K and f_π .

We define dimensionless parameters,

$$\begin{aligned}
G_D^{\text{eff}} &\equiv -G_D(\Lambda/2\pi)^4\Lambda N_c^2, \\
G_S^{\text{eff}} &\equiv G_S(\Lambda/2\pi)^2N_c.
\end{aligned} \quad (3.1)$$

As reported in Ref. 4), the experimental value of the $\eta \rightarrow \gamma\gamma$ decay amplitude is reproduced at about $G_D^{\text{eff}} = 0.7$. The calculated η -meson mass at $G_D^{\text{eff}} = 0.7$ is

$m_\eta = 510$ MeV which is 7% smaller than the observed mass. $G_D^{\text{eff}} = 0.7$ corresponds to $G_D \langle \bar{s}s \rangle / G_S = 0.44$, suggesting that the contribution from \mathcal{L}_6 to the dynamical mass of the up and down quarks is 44% of that from \mathcal{L}_4 . The calculated value of $\Gamma(\eta \rightarrow \pi^0 \gamma \gamma)$ is 0.92 eV at $G_D^{\text{eff}} = 0.7$, which is in good agreement with the experimental data: $\Gamma(\eta \rightarrow \pi^0 \gamma \gamma) = 0.93 \pm 0.19$ eV.

Before going to present the numerical results for the scalar mesons, let us summarize the properties of the scalar mesons in the NJL model. In the $SU_L(2) \times SU_R(2)$ version of the NJL with no explicit symmetry breaking term, the σ -meson mass can be calculated analytically in the mean field + ladder approximation, i.e., $m_\sigma = 2M_u$. The σ meson is therefore regarded as the lowest bosonic excitation, whose mass is twice of the gap energy, associated with chiral symmetry breaking. It should be noted that there is a cut above $q^2 = 4M_u^2$ in the complex q^2 -plane of the quark-antiquark scattering T -matrix, which corresponds to the unphysical decay: $\sigma \rightarrow \bar{q}q$. This is one of the known shortcomings of the NJL model. If one introduces a small symmetry breaking term, i.e., the current quark mass term, m_σ moves up and gets the imaginary part corresponding to the $\sigma \rightarrow \bar{q}q$ decay.¹⁰⁾ The pole position is in the second Riemann-sheet of the complex q^2 -plane, as is the case of ordinary resonances. It means that the Argand diagram for the T -matrix makes a circular resonance shape in the scalar $q\bar{q}$ channel.*)

It should be noted that the physical decay mode of σ , i.e., $\sigma \rightarrow \pi\pi$ is neither taken into account in the ladder approximation. As this decay makes the σ width significantly large, our result for σ mass is qualitative rather than quantitative. Nevertheless, the results shown below show that the scalar mesons in the NJL model are realized as the chiral partner of the Nambu-Goldstone bosons, and that they give systematic behavior for the orders of the masses and the splittings.

Let us now discuss our results of the scalar mesons. Here we call the lowest scalar meson states in the $I = 0, 1, 1/2$ channels and the second lowest one in the $I = 0$ channel σ , a_0 , K_0^* and f_0 , respectively. The identification of these states with the experimentally observed states will be given in the next section. The calculated results of the scalar-meson masses, $q\bar{q}$ decay widths and the mixing angle θ are shown in Figs. 1–3, respectively. The $q\bar{q}$ decay widths of the scalar mesons shown there are unphysical ones. We present them just for showing the pole positions in the complex q^2 -plane. When G_D^{eff} is zero, our lagrangian does not cause the flavor mixing and therefore the ideal mixing is achieved. The σ is purely $u\bar{u} + d\bar{d}$, which corresponds to $\theta = -54.7^\circ$, and is degenerate to the a_0 in this limit. When one increases the strength of the $U_A(1)$ breaking KMT interaction, the $q\bar{q}$ attraction in σ increases and σ state moves from the ideal mixing state toward the flavor singlet state. It means that the strange quark component of σ increases as G_D^{eff} becomes larger. Since the increase of the attractive force compensates with the increase of the strange quark component, m_σ is almost independent of the strength of the $U_A(1)$ breaking interaction and our result is $m_\sigma = 650$ MeV at $G_D^{\text{eff}} = 0.7$. The $\bar{q}q$ decay

*) The situation is quite different in the case of the vector meson. In the nonrelativistic limit, the scalar meson channel corresponds to the p -wave quark-antiquark state whereas the vector meson channel corresponds to the s -wave quark-antiquark state. See Ref. 11).

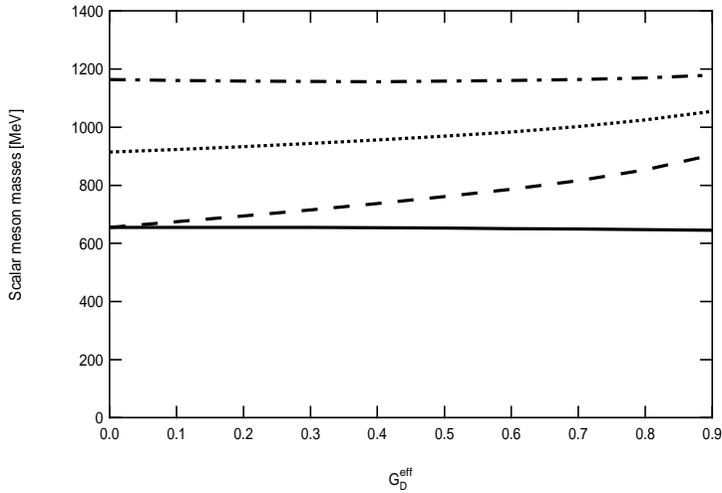


Fig. 1. The calculated scalar meson masses as functions of the effective coupling constant G_D^{eff} of the $U_A(1)$ breaking KMT interaction. The solid, dashed, dotted and dash-dotted lines represent m_σ , m_{a_0} , $m_{K_0^*}$ and m_{f_0} , respectively.

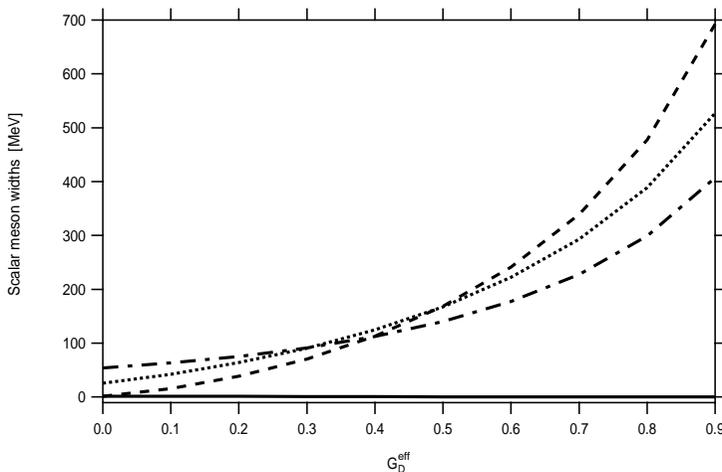


Fig. 2. The calculated $q\bar{q}$ decay widths of the scalar mesons as functions of the effective coupling constant G_D^{eff} of the $U_A(1)$ breaking KMT interaction. The solid, dashed, dotted and dash-dotted lines represent m_σ , m_{a_0} , $m_{K_0^*}$ and m_{f_0} , respectively.

width of the σ meson is very small, i.e., less than 2 MeV and therefore we neglect it in our calculation of the mixing angle. At $G_D^{\text{eff}} = 0.7$, the calculated mixing angle is $\theta = -77.3^\circ$, corresponding to about 15% mixing of the strangeness component in σ .

Hatsuda and Kunihiro have discussed the masses and mixing angle of the isoscalar nonstrange (σ_{NS}) and strange (σ_S) scalar mesons using the similar model.¹²⁾ They have reported a rather small mixing between σ_{NS} and σ_S . The reason of the difference between their result and our result is the strength of the $U_A(1)$ breaking

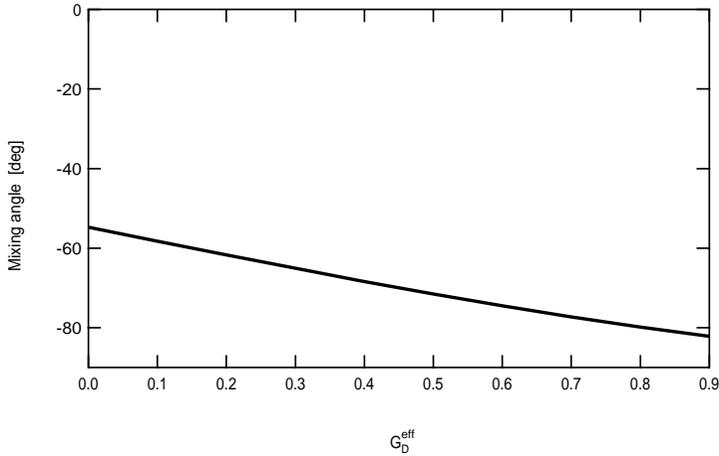


Fig. 3. The calculated mixing angle of the σ meson as a function of the effective coupling constant G_D^{eff} of the $U_A(1)$ breaking KMT interaction.

KMT interaction. The strength of the $U_A(1)$ breaking KMT interaction used in the present study is much stronger than that used in their study. They have determined the strength from the η' mass, while we have fixed it from the radiative decays of η . Strong $U_A(1)$ breaking interaction suggests that the instanton liquid picture of the QCD vacuum.¹³⁾ In Ref. 12), they have discussed the origin of the difference of the mixing properties of the scalar mesons and pseudoscalar mesons. We agree with their qualitative discussion, namely, the flavor mixing between σ and f_0 is weaker than that between η and η' . Shakin has also pointed out that the KMT interaction mixes the σ_{NS} and σ_S , while he assigned the lowest $I = 0$ $q\bar{q}$ state to $f_0(980)$.¹⁴⁾

Let us turn to the discussion of the a_0 and K_0^* mesons. As shown in Fig. 1, both m_{a_0} and $m_{K_0^*}$ increase as G_D^{eff} increases. The slope for m_{a_0} is steeper than that for $m_{K_0^*}$, which is consistent with the simple argument based on the form of the effective interaction Eq. (2.27). At $G_D^{\text{eff}} = 0.7$, the calculated masses are $m_{a_0} = 816$ MeV and $m_{K_0^*} = 1002$ MeV, therefore the $U_A(1)$ breaking interaction pushes up the a_0 and K_0^* masses about 161 MeV and 88 MeV, respectively. Although the effect of the $U_A(1)$ breaking interaction on the K_0^* meson is smaller than that on the a_0 meson, our numerical results show that it is not enough to support the existence of the light κ state.

As for the f_0 meson, we have shown our results in Figs. 1 and 2. At $G_D^{\text{eff}} = 0$, the f_0 state is expected to be pure $s\bar{s}$ state in our model. Because of the $q\bar{q}$ decay width, we cannot calculate the mixing angle for f_0 . The calculated mass of the f_0 meson at $G_D^{\text{eff}} = 0$ is $m_{f_0} = 1.163$ GeV which is above the $s\bar{s}$ threshold $2M_s = 1.113$ GeV. As shown in Ref. 9), the symmetry breaking effect by the current quark mass term pushes up the scalar meson mass above the $q\bar{q}$ threshold and the following relation is obtained by using the bosonization technique with the lowest order derivative

expansion in the NJL model.

$$\left(m_{\text{scalar meson}}^2 - (q\bar{q} \text{ threshold energy})^2\right) \propto m_{\text{current quark}}. \quad (3.2)$$

Our results at $G_D^{\text{eff}} = 0$ are $m_{a_0}^2 - 4M_u^2 = 0.008 \text{ GeV}^2$, $m_{K_0^*}^2 - (M_u + M_s)^2 = 0.060 \text{ GeV}^2$ and $m_{f_0}^2 - 4M_s^2 = 0.115 \text{ GeV}^2$, respectively. The above simple mass relation therefore holds in our case too. Figure 1 shows that the f_0 meson mass is almost independent of the strength of the KMT interaction. The situation is just opposite to the σ case, i.e., the increase of the repulsive force by the KMT interaction compensates with the decrease of the strange quark component of the f_0 meson when one increases the strength of the KMT interaction.

It should be noted here that in the $SU_L(3) \times SU_R(3)$ version of linear sigma model, not only the three-meson flavor determinant term but also the chiral invariant four-meson terms give rise to the $\sigma - a_0$ mass difference.^{15),16)} We note that the extended NJL model does not give such type of interaction.

§4. Conclusion

The aim of this study is to show the roles of two important symmetry structures of QCD, i.e., the $U_A(1)$ anomaly as well as the dynamical chiral symmetry breaking in the spectrum of the light scalar mesons. We suggest that the anomalous ordering of the $\sigma - a_0$ is understood by these features. To this end, we have studied the effects of the $U_A(1)$ breaking interaction on the low-lying nonet scalar mesons using the extended Nambu-Jona-Lasinio model. The strength of the $U_A(1)$ breaking interaction has been determined by the electromagnetic decays of the η meson and we have obtained rather strong $U_A(1)$ breaking interaction, which reminds us of the instanton liquid picture of the QCD vacuum.

We have found that the $U_A(1)$ breaking interaction gives rise to about 150 MeV mass difference between the σ and a_0 mesons. We obtain the low-lying $I = 0$ scalar meson as the chiral partner of the pion with mass about 650 MeV. We identify it with $\sigma(600)$. In our present scheme, it is the $q\bar{q}$ state. We have also found that the strangeness content in the σ meson is about 15%. The flavor contents of the σ meson may be observed in the analysis of the J/ψ decays. Furthermore, since the σ meson plays the central role in the intermediate range attraction of the nuclear force, the strange quark content in the σ meson may be important in the context of the hyperon-hyperon interactions.

As for the $I = 1$ channel, we find the $q\bar{q}$ resonance state at $m = 816 \text{ MeV}$. We identify it with $a_0(980)$ though the mass is still smaller than the observed value. In the $I = 1/2$ channel, we find the $q\bar{q}$ resonance state at $m = 1002 \text{ MeV}$, which is above the $I = 1$ state. A possible reason for the fact that the a_0 mass is not large enough is the large unphysical $q\bar{q}$ decay width shown in Fig. 2, which is an artifact of this model. On the other hand, the order of the $a_0 - K_0^*$ masses is not likely to change within this model. Because the unphysical $q\bar{q}$ widths are similar for a_0 and K_0^* , this conclusion may not be affected by this artifact. We therefore consider that the obtained K_0^* is not identified with recently reported $\kappa(700 - 900)$.

A possible candidate is $K_0^*(1430)$, although the mass difference between $K_0^*(1430)$ and $a_0(980)$ is rather large. It seems to be difficult to explain it from the symmetry breaking effect by the current strange quark mass. A possible scenario to explain this discrepancy is that the coupling of the $K\pi$ channel happens to be so large that its mixing leads to two scalar states $\kappa(700 - 900)$ and $K_0^*(1430)$. As for the second lowest state in the $I = 0$ channel, the pole appears at $m = 1164$ MeV, which is above the $s\bar{s}$ threshold and is about 350 MeV heavier than the mass of our $I = 1$ state. We therefore consider it may not be the $f_0(980)$ state. The possible candidates are $f_0(1370)$ and $f_0(1500)$. They again may be the results of mixings of two meson states, such as $K\bar{K}$, as well as glueball states.

We should note that the NJL model is a crude effective model of QCD with some shortcomings. In particular, as the model does not provide confinement of quarks, the scalar mesons are not free from $q\bar{q}$ decay channel. Therefore, the numerical results obtained in this paper should be taken at most qualitatively. Yet, we demonstrate that the significant $\sigma - a_0$ mass difference is induced by the $U_A(1)$ anomaly. Guided by this success, it is most desirable to confirm this mechanism directly in QCD. For instance, the lattice QCD calculation with controlled topological charge density and/or the QCD sum rule with direct instanton effects are future possibilities. Also a study of the nonet scalar mesons using the more realistic quark model approaches, such as the improved ladder model of QCD,¹⁷⁾ will give us further confidence on the mechanism and structure of the scalar meson spectrum. Such work is underway and is to be published elsewhere.¹⁸⁾

Acknowledgements

We are happy to thank Drs. Teiji Kunihiro and Muneyuki Ishida for helpful discussions. This work is supported in part by the Grant-in-aid for Scientific Research (C) (2) 11640261 of the Ministry of Education, Culture, Sports, Science and Technology of Japan.

References

- 1) S. Weinberg, Phys. Rev. D **11** (1975), 3583.
- 2) G. 't Hooft, Phys. Rev. Lett. **37** (1976), 8; Phys. Rev. D **14** (1976), 3432.
- 3) For a review, T. Schäfer and E. V. Shuryak, Rev. Mod. Phys. **70** (1998), 323.
- 4) M. Takizawa, Y. Nemoto and M. Oka, Phys. Rev. D **55** (1997), 4083.
- 5) See for recent developments, *Proceedings of the workshop "Possible existence of the σ -meson and its implications to hadron physics"*, June 2000, Yukawa Institute for Theoretical Physics, Kyoto, Japan, KEK-Proceedings 2000-4, Dec. 2000 and Soryushiron Kenkyu (Kyoto) **102** (2001), E1.
- 6) Particle Data Group, K. Hagiwara et al., Phys. Rev. D **66** (2002), 010001-450 and references therein.
- 7) T. Hatsuda and T. Kunihiro, Phys. Rev. Lett. **55** (1985), 158.
T. Hatsuda, T. Kunihiro and H. Shimizu, Phys. Rev. Lett. **82** (1999), 2840.
- 8) M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **44** (1970), 1422.
M. Kobayashi, H. Kondo and T. Maskawa, Prog. Theor. Phys. **45** (1971), 1955.
- 9) V. Dmitrasinovic, Phys. Rev. C **53** (1996), 1383; Nucl. Phys. A **686** (2001), 379.
- 10) M. Takizawa, K. Tsushima, Y. Kohyama and K. Kubodera, Nucl. Phys. A **507** (1990), 611.
- 11) M. Takizawa, K. Kubodera and F. Myhrer, Phys. Lett. B **261** (1991), 221.

- 12) T. Hatsuda and T. Kunihiro, *Phys. Rep.* **247** (1994), 221.
- 13) E. V. Shuryak, *Nucl. Phys. B* **203** (1982), 93; *ibid.* **203** (1982), 116.
- 14) C. M. Shakin, *Phys. Rev. D* **65** (2002), 114011.
- 15) L. H. Chan and R. W. Haymaker, *Phys. Rev. D* **10** (1974), 4143.
M. Ishida, *Prog. Theor. Phys.* **101** (1999), 661.
- 16) D. Black, A. H. Fariborz, S. Moussa, S. Nasri and J. Schechter, *Phys. Rev. D* **64** (2001), 014031, and references therein.
- 17) K. Naito, Y. Nemoto, M. Takizawa, K. Yoshida and M. Oka, *Phys. Rev. C* **61** (2000), 065201.
- 18) T. Umekawa, Ph. D. Thesis, hep-ph/0306040.