

## Supernova Cosmology and the Fine Structure Constant

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We study the dependence of the peak luminosity of Type Ia supernovae on the fine structure constant  $\alpha$ . We find that decreasing (increasing)  $\alpha$  enhances (reduces) the luminosity. Future experiments like SNAP could determine the variation of  $\alpha$  to a precision of  $10^{-2}$ .

### §1. Introduction

Recently, the possibility of the variation of fundamental “constants” (especially the fine structure constant  $\alpha$  and the gravitational constant  $G$ ) has attracted much attention.<sup>1)</sup> Recent evidence that the value of  $\alpha$  was smaller in the past from observations of a number of absorption systems in the spectra of distant quasars has stimulated interest in this topic.<sup>2),3)</sup> From an anthropic point of view, it is also intriguing to examine how changing physical constants would affect the structure of the universe we observe.

The magnitude of the variation of  $\alpha$  over time on a cosmological scale is constrained by several observations: those of the absorption/emission lines of distant objects, yielding information on the variation of fine-splitting ( $0.5 < z < 4$ ), those of spectrum of cosmic microwave background anisotropies, yielding information on the variation of the recombination process ( $z = 1000$ ), and those of Big Bang nucleosynthesis, yielding information on the variation of the masses of the proton and neutron ( $z = 10^{10}$ ). In this paper, we consider another method to constrain the cosmological time variation of  $\alpha$ , observations of Type Ia supernovae. A Type Ia supernova is considered to be a good standard candle, because its peak luminosity correlates with the rate of decline of the magnitude. Observations of Type Ia supernovae have been used to constrain cosmological parameters.<sup>4),5)</sup> The homogeneity of the peak luminosity is essentially due to the homogeneity of the progenitor mass, and this is primarily determined by the Chandrasekhar mass, which is proportional to  $G^{-3/2}$ . The peak luminosity also depends on the diffusion time of photons, which depends on  $\alpha$  through the opacity. A decrease in opacity reduces the diffusion time, allowing trapped radiation to escape more rapidly, leading in turn to an increase in the luminosity.

## §2. Type Ia supernovae and $\alpha$

Now we consider the effect of changing  $\alpha$  on the bolometric absolute magnitude of the peak luminosity of Type Ia supernovae. We limit our study to the peak luminosity, because the physics behind the relation between the peak luminosity and the rate of its decline is not fully understood (see Ref. 6) for a recent attempt). We write the energy deposition rate from the  $^{56}\text{Ni} \rightarrow ^{56}\text{Co} \rightarrow ^{56}\text{Fe}$  decay chain inside the photosphere of Type Ia supernovae as

$$F(t) = M_{56} q(t), \quad (2.1)$$

where  $M_{56}$  is the mass of  $^{56}\text{Ni}$  in grams ( $\sim 0.6M_{\odot} \sim 1.2 \times 10^{33}$  g),<sup>9),10)</sup> and  $q(t)$  is given by<sup>11)</sup>

$$q(t) = \left[ S_{\text{Ni}}^{\gamma} e^{-t/\tau_{\text{Ni}}} + S_{\text{Co}}^{\gamma} \left( e^{-t/\tau_{\text{Co}}} - e^{-t/\tau_{\text{Ni}}} \right) \right] f_{\text{dep}}^{\gamma}(t) + S_{\text{Co}}^{\beta} \left( e^{-t/\tau_{\text{Co}}} - e^{-t/\tau_{\text{Ni}}} \right), \quad (2.2)$$

with

$$\begin{aligned} S_{\text{Ni}}^{\gamma} &= 4.03 \times 10^{10} \text{ erg s}^{-1} (\tau_{\text{Ni}}/8.51 \text{ days})^{-1}, \\ S_{\text{Co}}^{\gamma} &= 6.78 \times 10^9 \text{ erg s}^{-1} (\tau_{\text{Co}}/111.5 \text{ days})^{-1}, \\ S_{\text{Co}}^{\beta} &= 0.232 \times 10^9 \text{ erg s}^{-1} (\tau_{\text{Co}}/111.5 \text{ days})^{-1}. \end{aligned}$$

In Ref. 12), the most recent value of the mean lifetime of  $^{56}\text{Ni}$  ( $^{56}\text{Co}$ ), which is determined by the weak interaction, is stated to be  $\tau_{\text{Ni}} = 8.51$  days ( $\tau_{\text{Co}} = 111.5$  days). For simplicity, in this paper we assume that the differences in energy between the excited states and the ground states of  $^{56}\text{Ni}$ ,  $^{56}\text{Co}$  and  $^{56}\text{Fe}$  are mainly determined by nuclear forces; the Coulomb part in excitation energy is assumed to be small. In general, this assumption is almost valid. The  $\gamma$ -ray deposition function,  $f_{\text{dep}}^{\gamma}(t)$ , i.e., the fraction of  $\gamma$ -ray energy deposited in supernova matter, is fitted by<sup>13)</sup>

$$f_{\text{dep}}^{\gamma}(t) = G(\tau) (1 + 2G(\tau) [1 - G(\tau)] [1 - 0.75G(\tau)]), \quad (2.3)$$

with

$$G(\tau) = \tau / (\tau + 1.6), \quad (2.4)$$

where  $\tau = \tau(t)$  is the optical depth.

The peak luminosity of the optical light curve is essentially proportional to the value of  $F(t)$  at a time  $t_p$ , when the expansion timescale is equal to the diffusion timescale:  $L_{\text{peak}} \sim F(t = t_p)$  with  $t_p \sim t_{\text{exp}} \sim t_{\text{diff}}$ . Here the diffusion timescale  $t_{\text{diff}}$  is given by

$$t_{\text{diff}} = \kappa \rho R^2 / c, \quad (2.5)$$

where  $\kappa$  is the mean opacity ( $\sim 0.1 \text{ cm}^2 \text{ g}^{-1}$  in Ref. 14)),  $\rho$  is the matter density, and  $R$  is the radius. On the other hand, the expansion timescale  $t_{\text{exp}}$  is given approximately by

$$t_{\text{exp}} = R/v, \quad (2.6)$$

where  $v$  is the expansion velocity of the matter. The total mass of the progenitor is determined by the Chandrasekhar mass and is given by  $M = 4R^3\rho/3\pi \simeq 3\mu_e^{-2}G^{-3/2}m_p^{-2} \simeq 1.4M_\odot$ , where  $\mu_e$  is the mean molecular weight of electrons and  $m_p$  is the proton mass.<sup>\*)</sup> The total energy of the explosion is  $E = Mv^2/2 \sim 10^{51}$  erg, which is due to the difference between the binding energies of  $^{56}\text{Ni}$  and C ( $\sim 1$  MeV times the number of nucleons). Through the relation

$$t_{\text{diff}} = \frac{3\kappa M}{4\pi cR} = \frac{3\kappa M}{4\pi cVt_{\text{exp}}} = \frac{3\kappa M^{3/2}}{4\sqrt{2}\pi cE^{1/2}t_{\text{exp}}}, \quad (2.7)$$

we obtain

$$t_p \sim \left( \frac{3}{4\sqrt{2}\pi} (\kappa/c) \right)^{1/2} \left( \frac{M^3}{E} \right)^{1/4}, \quad (2.8)$$

$$\sim 19 \text{ days} \left( \frac{\kappa}{0.1 \text{ cm}^2 \text{ g}^{-1}} \right)^{1/2} \left( \frac{M}{1.4M_\odot} \right)^{3/4} \left( \frac{E}{10^{51} \text{ erg}} \right)^{-1/4}. \quad (2.9)$$

In this case we find that  $L_{\text{peak}} \sim 0.97 \times 10^{43} \text{ erg s}^{-1}$ , and the velocity is given by  $v \simeq 8.5 \times 10^8 \text{ cm s}^{-1} (E/10^{51} \text{ erg})^{1/2} (M/1.4M_\odot)^{-1/2}$ .

The mean opacity  $\kappa$  should be proportional to  $\alpha^n$  because emitted photons scatter off ions through the electromagnetic interaction ( $n = 2$  for Thomson scattering). Because the density in the supernova shell is very low ( $\sim 10^{-13} \text{ g cm}^{-3}$ ) after 10 days, the opacity may be almost entirely due to electron scattering ( $n = 2$ ).<sup>15)</sup> However, for generality we include the  $n$  dependence.<sup>\*\*)</sup> Then, the uncertainty in  $t_p$  is related with the change of  $\alpha$  as

$$\frac{\Delta t_p}{t_p} = \frac{1}{2} \frac{\Delta \kappa}{\kappa} = \frac{n}{2} \frac{\Delta \alpha}{\alpha}. \quad (2.10)$$

From Eq. (2.2), we see that the uncertainty in  $q(t_p)$  caused by the variation of  $t_p$  or  $\alpha$  can be expressed by

$$\begin{aligned} & \Delta q(t_p) \\ &= -\frac{\Delta t_p}{\tau_{\text{Ni}}} \left[ S_{\text{Ni}}^\gamma f_{\text{dep}}^\gamma(t_p) e^{-t_p/\tau_{\text{Ni}}} + \left( S_{\text{Co}}^\gamma f_{\text{dep}}^\gamma(t_p) + S_{\text{Co}}^\beta \right) \left( \frac{\tau_{\text{Ni}}}{\tau_{\text{Co}}} e^{-t_p/\tau_{\text{Co}}} - e^{-t_p/\tau_{\text{Ni}}} \right) \right] \\ &+ \Delta f_{\text{dep}}^\gamma(t_p) \left[ S_{\text{Ni}}^\gamma e^{-t_p/\tau_{\text{Ni}}} + S_{\text{Co}}^\gamma \left( \frac{\tau_{\text{Ni}}}{\tau_{\text{Co}}} e^{-t_p/\tau_{\text{Co}}} - e^{-t_p/\tau_{\text{Ni}}} \right) \right], \end{aligned} \quad (2.11)$$

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<sup>\*)</sup> Changing  $\alpha$  causes a change in the nucleon mass through electromagnetic radiative corrections,<sup>7),8)</sup> and hence a change in the Chandrasekhar mass. However, the resulting change in the luminosity is found to be smaller by four orders of magnitude than the value given in Eq. (2.15).

<sup>\*\*)</sup> A photon in the expanding shell suffers a continuous Doppler shift of frequency with respect to the rest frame of the material. Those photons which are redshifted to the frequency of a sufficiently strong line will be absorbed by the corresponding bound-bound transition. This effect would effectively increase the opacity.<sup>16),17)</sup> Such an enhancement factor has a complicated form as a function of the bound-bound transition opacity and is non-linear in  $\alpha$ .<sup>16)</sup> Near the peak luminosity, however, the effective value of  $n$  modified by the enhancement factor is of the same order of magnitude, and therefore this effect should not affect the following analysis significantly.

where

$$\Delta f_{\text{dep}}^{\gamma}(t_p) = \Delta G(\tau_p) \left[ 1 + 4G(\tau_p) - 10.5G(\tau_p)^2 + 6.0G(\tau_p)^3 \right], \quad (2.12)$$

with  $\tau_p \equiv \tau(t = t_p)$ , and

$$\Delta G(\tau_p) = \frac{1.6}{(1.6 + \tau_p)^2} \Delta\tau_p. \quad (2.13)$$

In our simple treatment, the optical depth ( $\tau = \kappa\rho R$ ) is assumed to be proportional to  $\kappa t^{-2} v^{-2} M$ . If we adopt the value of  $\tau$  in Ref. 13) ( $\tau \simeq 1.0$  at  $t = 20$  days for  $v = 1.5 \times 10^9$  cm s<sup>-1</sup>), we obtain

$$\tau_p \sim 3.6 \left( \frac{E}{10^{51} \text{ erg}} \right)^{-1/2} \left( \frac{M}{1.4M_{\odot}} \right)^{1/2}. \quad (2.14)$$

From Eq. (2.14), we see that  $\tau_p$  does not depend on  $\kappa$ , and the second term in Eq. (2.11) disappears. We note that in this case  $f_{\text{dep}}^{\gamma}(t_p) \sim 0.83$ .

Using Eqs. (2.1), (2.10) and (2.11), we obtain the relation

$$\frac{\Delta L_{\text{peak}}}{L_{\text{peak}}} = \frac{\Delta q(t_p)}{q(t_p)} = -a \frac{n}{2} \frac{\Delta\alpha}{\alpha}, \quad (2.15)$$

where  $a \sim 0.94$ . Thus, we obtain the following relationship between  $\Delta\alpha$  and the change in the absolute magnitude  $\Delta\mathcal{M}$  at the peak luminosity:

$$\frac{\Delta\alpha}{\alpha} = \frac{4 \ln 10}{5an} \Delta\mathcal{M} = 0.98 \left( \frac{0.94}{a} \right) \left( \frac{2}{n} \right) \Delta\mathcal{M}. \quad (2.16)$$

This is the main result of this paper. It can be understood as follows. Decreasing  $\alpha$  causes the opacity to decrease, which allows photons to escape more rapidly, thereby leading to an increase in the luminosity. Thus a smaller (larger) value of  $\alpha$  would make supernovae brighter (fainter).

Let us now estimate the severity of the constraint derived from observations of Type Ia supernovae. Future experiments like SNAP (SuperNova/Acceleration Probe)<sup>\*)</sup> are planned to observe thousands of supernovae and should be able to reduce systematic errors to a magnitude of 0.02 mag, which corresponds to  $\Delta\alpha/\alpha < 2 \times 10^{-2}$ . This is larger than the current limit for  $0.16 < z < 0.80$ ,  $\Delta\alpha/\alpha = (-2 \pm 1.2) \times 10^{-4}$ ,<sup>18)</sup> although the limit obtained from supernovae could apply to higher redshifts and depends on the nature of the time evolution of  $\alpha$ .

The sensitivity of this ‘‘supernova method’’ to the variation of  $\alpha$  might be on the same order as that of CMB.<sup>19)</sup> (For the case of varying  $G$  see Refs. 20) and 21).) In any case, it is important to determine the possible limits of the variation of fundamental constants by various means. As one such effort, we have proposed a method to determine the limit of the time variation of  $\alpha$  through examination of the variation of the peak luminosity of supernovae. Detailed analysis will be published elsewhere.

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<sup>\*)</sup> <http://snap.lbl.gov>

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