

A Calculation of the Viscosity to Entropy Ratio of a Hadronic Gas

Shin MUROYA¹ and Nobuo SASAKI²

¹*Tokuyama University, Shunan 745-8566, Japan*

²*Research Center for Nanodevices and Systems,
Hiroshima University, Higashi-Hiroshima 739-8527, Japan*

(Received August 24, 2004)

We calculate thermodynamic quantities and transport coefficients of a hadronic gas using a Monte Carlo hadronic collision event generator URASiMA. The obtained shear viscosity to entropy density ratio of the hadronic gas is approximately 0.3 – 0.6 in the range $T = 100$ – 180 MeV and is quite insensitive to the baryon number density.

§1. Introduction

Hydrodynamical models are among the most thoroughly established models of multiple production phenomena.¹⁾ In particular for RHIC experiments, hydrodynamical models are accepted as the most successful models to reproduce v_2 data.²⁾ To this time, in almost all hydrodynamical models, viscosities and heat conductivity have been ignored for simplicity. Teaney³⁾ attempted to estimate the size of the effect that may appear in the present hydrodynamical models applied to RHIC experiment in the case that the shear viscosity is taken into account, and he concluded that the viscous term in the Navier-Stokes equation must be smaller than unity in order to retain good agreement between the blast wave model with viscous corrections and experimental results. A peculiarly small ratio of the viscosity to entropy has been identified as a possible special feature of the QGP liquid.⁴⁾ Here we evaluate the shear viscosity and entropy for a hadronic gas.

Calculating macroscopic material constants on the basis of a microscopic picture is one of the most important tasks in theoretical physics. This is, however, very difficult in general. Particularly in the case of strongly interacting hadrons, there exists no perturbation theory and no systematic method to calculate the relevant quantities.

In this paper, we evaluate the transport coefficients of dense, hot hadronic matter employing a hadro-molecular dynamic calculation that an event generator URASiMA (Ultra-Relativistic AA collision Simulator based on Multiple Scattering Algorithm).⁵⁾ The obtained heat conductivity and shear viscosity exhibit a temperature dependence of $\sim T^5$, which is stronger than the result obtained from naive dimensional analysis, $\sim T^3$. Muronga attempted a similar calculation for a pure meson gas using UrQMD, in which case, several interactions must be switched off by hand in order to realize the stationary state.⁶⁾ In the present paper, we investigate a finite baryon number system with all hadronic interactions, including those of baryons.

§2. Hadro-molecular dynamics

In order to construct a statistical ensemble state, we carry out a molecular dynamical calculation of hadrons in a box. The time evolution of the system is simulated with the Monte-Carlo collision event generator URASiMA. One of the present authors (N. S.) improved and tuned the parameters of URASiMA so as to realize an equilibrium state of hadrons for temperatures in the range 100 – 180 MeV for nuclear densities near twice the normal nuclear density.⁷⁾ In Ref. 7), equilibration of the system is discussed in detail and thermodynamical quantities are analyzed. Several kinds of diffusion constants have already investigated on the basis of the ensembles.^{8),9)}

The present version of the URASiMA contains the particles listed in the Table I. This version is the so-called “two-flavor” and “low-energy” version, which ignores anti-baryons and strangeness. As a hadronic gas model, this model covers the region characterized by temperature lower than Kaon’s mass and chemical potential lower than the baryon mass. The region on which we focus our consideration is below T_c (≈ 200 MeV) and up to twice the normal nuclear baryon number density. We believe that this region is well within the regime in which the present version of URASiMA is valid.

Thermodynamical quantities of the systems of interest are investigated in Ref. 7). Figure 1 displays the obtained equation of state (the pressure P as a function of the energy density ε). Throughout our simulation, the box size is fixed as 10^3 fm³. The black circles represent the normal nuclear density n_{b0} ($n_{b0} = 0.157$ (1/fm³)), the white boxes represent 1.5 times n_{b0} , and the triangles represent 2 times n_{b0} . We note from Fig. 1 that the bulk viscosity of the hadronic gas almost vanishes. Because

Table I. Baryons, mesons and their resonances included in the URASiMA.

nucleon	N_{938}	N_{1440}	N_{1520}	N_{1535}	N_{1650}	N_{1675}	N_{1680}	N_{1720}
Δ	Δ_{1232}	Δ_{1600}	Δ_{1620}	Δ_{1700}	Δ_{1905}	Δ_{1910}	Δ_{1950}	
meson	π	η_s	σ_{800}	ρ_{770}				

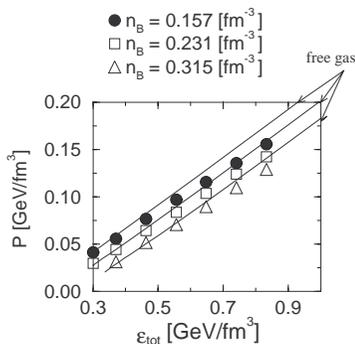


Fig. 1. Equation of state of a hadronic gas obtained using URASiMA.

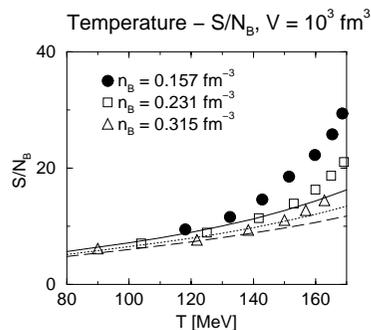


Fig. 2. Entropy of a hadronic gas obtained using URASiMA.

the current P' which corresponds to the bulk viscosity is given by¹⁰⁾

$$P' = P - \frac{\partial P}{\partial \varepsilon} \varepsilon. \quad (1)$$

P' vanishes if the pressure is proportional to energy density.

Figure 2 displays the entropy density calculated in Ref. 7) as $S = \sum \rho \ln \rho$, where ρ is the ensemble averaged density function and \sum represents the sum over phase space, respectively. Throughout our simulation, the baryon number is conserved. The entropy density grows more rapidly than that in the free hadron model (the curves in the figure) for the temperatures approximately equal to or larger than the pion mass.

§3. Relaxation of the currents

According to linear response theory, a transport coefficient is obtained from the correlation of the corresponding current.¹¹⁾ The shear viscosity and heat conductivity are obtained from the correlation of the heat current P_i and stress tensor π_{ij} as¹²⁾

$$\kappa = \lim_{\varepsilon \rightarrow +0} \frac{1}{T} \int d^3 \mathbf{x}' \int_{-\infty}^t dt' e^{-\varepsilon(t-t')} \langle P_x(t, \mathbf{x}), P_x(t', \mathbf{x}') \rangle, \quad (2)$$

$$\eta_s = \lim_{\varepsilon \rightarrow +0} \frac{2}{T} \int d^3 \mathbf{x}' \int_{-\infty}^t dt' e^{-\varepsilon(t-t')} \langle \pi_{xy}(t, \mathbf{x}), \pi_{xy}(t', \mathbf{x}') \rangle, \quad (3)$$

where P_i and π_{ij} are the spatial-temporal and spatial-spatial components of the energy-momentum tensor, respectively.

In the hadro-molecular dynamics described by URASiMA, the energy-momentum tensor is given by the sum of that of each particle:

$$T^{\mu\nu}(\mathbf{x}, t) = \sum_l T_{(l)}^{\mu\nu} \delta(\mathbf{x} - \mathbf{x}_{(l)}(t)) \quad (4)$$

$$= \sum_l \frac{p_{(l)}^\mu p_{(l)}^\nu}{p_{(l)}^0} \delta(\mathbf{x} - \mathbf{x}_{(l)}(t)). \quad (5)$$

Here, the subscript l indexes the particles. Average is taken as an ensemble average:

$$\langle \dots \rangle = \frac{1}{\text{number of ensembles}} \sum_{\text{ensemble}} \delta_{(l,l')}. \quad (6)$$

Correlation is assumed to exist only between the same particle in a state.^{*)}

Figure 3 displays the correlation function of the stress tensor π_{ij} , which represents the relaxation of the viscous shear tensor and appears in the integrand of Eq. (3). At temperatures near the pion mass, mesonic degrees of freedom gradually come to dominate transport phenomena. Detailed analyses of the separate contributions of the baryonic sector and mesonic sector will be reported elsewhere.¹⁶⁾

^{*)} A detailed explanation of the procedure will be given in Ref. 16).

In each case, for values of t greater than about $t = 20$ fm, the correlation function becomes smaller than $\sim 10^{-3}$ and can no longer be distinguished from the noise. Because we are interested in phenomena on the hadronic scale, in actual calculations, we cut off the infinite integration in Eqs. (2) and (3) at $t = 20$ fm by hand. It is known that the final results are not sensitive to this cutoff value.

§4. Transport coefficients

Figure 4 displays values of the calculated shear viscosity. Though the temperature range is not large, it is seen that the temperature dependence of η is $\sim T^5$, which is significantly stronger than that obtained from naive dimensional analysis, $\sim T^3$. This strong temperature dependence is caused by the liberation of the pion degrees of freedom, because the temperature is slightly above the pion mass. A similar temperature dependence also appears in the heat conductivity shown in Fig. 5. The somewhat peculiar behavior of the heat conductivity in the region of very low temperature can be attributed to the contribution of the baryons, whose mass is larger than the temperature and whose number must be conserved. A detailed investigation will be presented elsewhere.¹⁶⁾

The obtained shear viscosity is approximately $0.2 - 0.6$ [GeV/fm²] and is almost independent of the baryon number density. Though the system treated by the URASiMA is not a simple pion gas, the transport of stress is dominated by pions, because the temperature of the system is sufficiently smaller than the masses of

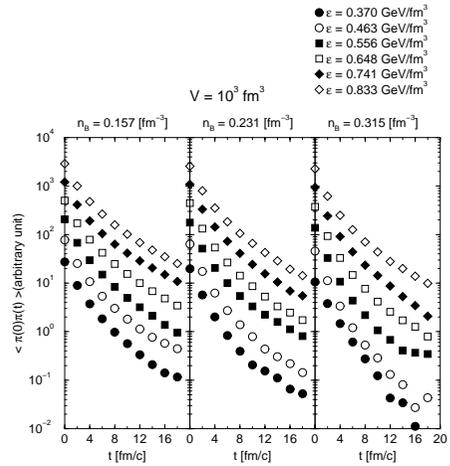


Fig. 3. Correlation function of the viscous shear tensor $\pi_{x,y}$.

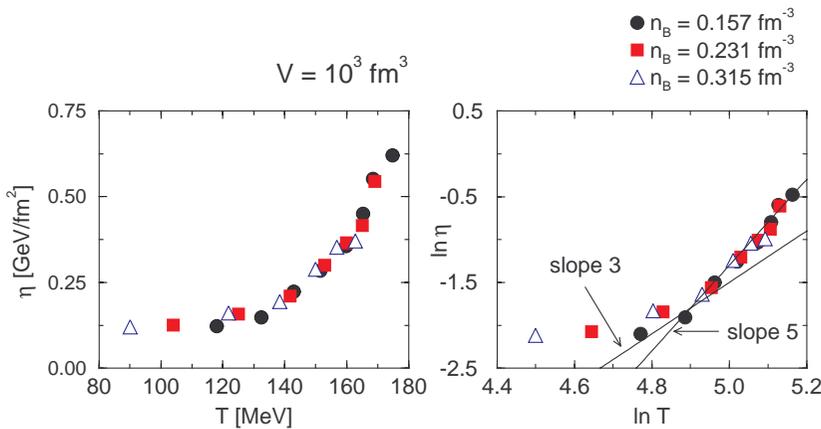


Fig. 4. Shear viscosity η_s as a function of temperature.

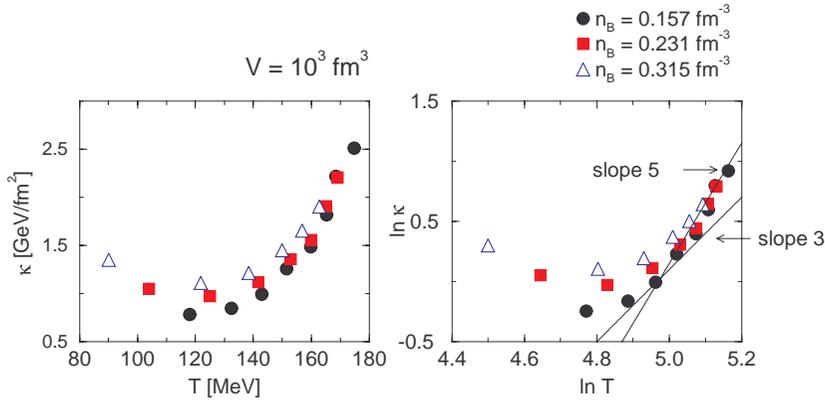


Fig. 5. Heat conductivity κ as a function of temperature.

hadrons other than pions.

About twenty years ago, Gavin calculated the shear viscosity of a pion gas¹³⁾ and obtained $0.05 - 0.15$ [GeV/fm^2], which is approximately one-fourth as large as our result. According to the recent calculation by Dobado and Llanes-Estrada,¹⁴⁾ the shear viscosity of a pion gas is approximately 0.4 [GeV^3] (~ 10 [GeV/fm^2]) if a constant scattering amplitude is assumed (see Fig. 2 of Ref. 13)) and 0.004 [GeV^3] (~ 0.1 [GeV/fm^2]) in the simple analytical phase shift model (see Fig. 3 of Ref. 13)). Our model gives a result between these values. For a pure meson gas, using UrQMD in which several interactions are modified to equilibrate the system, Muronga obtained a shear viscosity which is almost one-half of our result.⁶⁾

§5. η_s to entropy density ratio

Usually, the Reynolds number R is evaluated to estimate how large an effect is caused by viscous terms. If Bjorken's 1+1 dimensional Scaling solution is adopted as a particular solution, the inverse Reynolds number, R^{-1} , is given by¹⁵⁾

$$R^{-1} = \frac{\eta_s + \frac{2}{3}\eta_v}{\tau(\varepsilon + P)} \quad (7)$$

$$= \frac{\eta_s + \frac{2}{3}\eta_v}{s} \frac{1}{\tau T}, \quad (8)$$

where τ is the proper time.

Instead of the inverse Reynolds number, which depends on the solution of hydrodynamics, we can use the shear viscosity to entropy density ratio for simplicity.⁴⁾ The shear viscosity to entropy

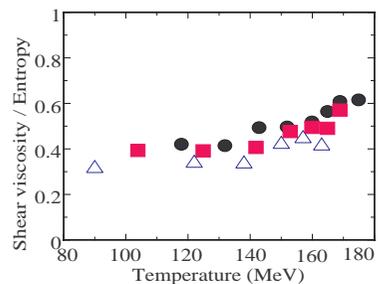


Fig. 6. Viscosity to entropy density ratio of a hadronic gas. The black circles represent the normal nuclear density n_{b0} , the boxes represent 1.5 times n_{b0} , and the triangles represent 2 times n_{b0} .

density ratio, η_s/s , obtained in our simulation is plotted in Fig. 6. It is seen that η_s/s is approximately 0.4 and quite insensitive to the baryon number density. The peculiarly small value η_s/s for QCD matter has been pointed out by several authors,⁴⁾ but even in the case of hadronic gas, the value of η_s/s obtained in our simulation is approximately order unity or smaller. The URASiMA consists of ordinary hadronic collisions only;⁷⁾ that is, there exists no special strong correlation, such as strings and potentials. The main difference between URASiMA and ordinary non-relativistic molecular dynamic simulations lies in the relativistic properties of the system. Therefore, we believe that the result $\eta_s/s \leq 1$ may be a common feature of a *relativistic gas*. More detailed analysis is now in progress.¹⁶⁾

Acknowledgements

We would like to thank Professor Teiji Kunihiro and Professor Atsushi Nakamura for their continuous encouragement. Collaboration with Chiho Nonaka in the early stage of the present work was very fruitful. Stimulating discussions at the workshop *Thermal Field Theory and Its Application* held at the Yukawa Institute for Theoretical Physics was extremely helpful. One of the authors (S. M.) thanks ERI of Tokuyama University for financial support. This work was started under the supervision of the late Professor Osamu Miyamura, Hiroshima University.

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