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Quantum Measurement Driven by Spontaneous Symmetry Breaking

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Quantum mechanics cannot be applied within a closed system. Inevitable measurement process in quantum mechanics is usually treated separately from the basic principles of the framework and requires outside observer of the system. In this paper, we propose that the quantum measurement process is actually a physical process associated with the ubiquitous mechanism of spontaneous symmetry breaking. Based on this proposal, we construct a quantum measurement model in which the mixed state evolves into a pure state as the dynamical pro-coherence process. Furthermore, the classically distinguishable pointer parameter emerges as the *c*-number order parameter in the formalism of closed time-path quantum field theory. We also discuss the precision of the measurement and the possible deduction of the Born probability postulate.

§1. Introduction

The standard formalism of quantum mechanics is so far complete and can be correctly applied to the most of phenomena in the world. However, this formalism cannot be directly applied within the inside of an isolated system. It always requires outside observer in the fundamental level and the measurement process*),1) can alter the state of the system. This *inside-outside duality* does characterize the quantum theory distinguished from the classical theory. The inside and outside of quantum theory are qualitatively quite different with each other. The inside is the unitary dynamical process described by the Schrödinger equation and the outside is the non-unitary in-deterministic process. The former process is deterministic and is uniquely described only by specifying the initial condition, while the latter process is essentially probabilistic and directly introduces the statistical nature into quantum mechanics.

This kind of dual-structure2) of quantum mechanics causes many technical disadvantages in its applications. This is most prominent in cosmology, in which no objective device for measurement can be found anywhere, anytime.**) Actually, this

*) This measurement process is often described by the von Neumann projection postulate in ordinary textbooks of quantum mechanics, though there is a fairly general approach using the positive operator-valued measure (POVM). This latter theory fully covers the whole measurement processes. Our goal is to construct a physical model which meets this general framework and to elucidate the fundamental mechanism of the measurement process.

**) An exception is ourselves, which appeared at the last moment of the long history of the universe. A theoretical formulation based strictly on the principle of quantum mechanics would be possible, in which the big wave function of the universe evolves according to the Schrödinger equation and is finally measured by us and materialized. However, this destroys the whole description of the

Table I. The basic quartet for the quantum measurement process. The standard construction of the measurement process is composed of the system S, the measurement apparatus A, and the implicit environment E, i.e. S+A+(E). In the table, each process is described for the simplest two-level state system. The total system initially is prepared as $|\psi\rangle \otimes |\phi\rangle$, where $|\psi\rangle = c_1 |\psi_1\rangle + c_2 |\psi_2\rangle$ is the system state, and $|\phi\rangle$ is the state of the measurement apparatus. Only the process I is described by a unitary evolution equation, while all the others, II, III, and IV, are non-unitary evolutions. In the literature, only the processes I and II have been relatively thoroughly studied. However, it is the processes III and IV that form the essential part of the von Neumann projection postulate.

PROCESSES	DESCRIPTION
I. Quantum Entanglement (=QE)	The states become entangled, keeping the initial condition of the system. $|\psi\rangle \otimes |\phi\rangle \to c_1 |\psi_1\rangle |\phi_1\rangle + c_2 |\psi_2\rangle |\phi_2\rangle$
II. Quantum Decoherence (=QD)	QD reduces the mixing term between the above two states. $\to \rho = |c_1|^2 |\psi_1\rangle |\phi_1\rangle \langle\psi_1| \langle\phi_1| + |c_2|^2 |\psi_2\rangle |\phi_2\rangle \langle\psi_2| \langle\phi_2|$
III. Classical Order parameter (=CO)	A finite c-number order parameter appears. $\to$ value φ_+ or value φ_-, but not both.
IV. Quantum Pro-coherence (=QP)	The system becomes a pure state $\to$ $|\psi_1\rangle$ or $|\psi_2\rangle$ with a strong correlation with the c-numbers in III.

fact makes the quantum origin of density fluctuations[3)] ambiguous and makes the wave function of the universe[4)] entirely meaningless. Therefore, it would be natural to explore the unification of the above two kinds of time evolutions, especially in the form that the latter projection process is described within the generalized form of the Schrödinger time evolution. This necessity for unification is not restricted to the field of cosmology, but actually many physicists in various fields have explored a similar issue.[5)]

Most attempts so far to clarify the quantum measurement process only deal with (I) the quantum entanglement of the system and the measuring apparatus, and (II) the quantum decoherence process of the full system, in which the off-diagonal interference term disappears. However the proper measurement process requires more.[*)] Actually, there are two further kinds of processes which have not been commonly discussed. One is (III) the dynamical appearance of c-number degrees of freedom, which distinguish the various quantum states of the system just after the measurement. Another one is (IV) the final transition to a pure state just after the measurement, reflecting the fact that the von Neumann postulate is a projection. The states realized in the process IV must have firm correlation with the c-number values in the process III for faithful measurement. These four processes I-VI form the *basic quartet* for the measurement process. This is summarized in Table I.

More precisely about the process III, the common description invokes the c-

standard scenario of cosmology, which is firmly based on the realism or the classical picture.

[*)] Here, we mainly treat the ideal (or first kind of) measurement, whose existence is the basic postulate of quantum theory. However, actual measurement processes necessarily are accompanied by finite errors and sometimes even lack projection processes. We will encounter this situation in later sections when we construct physical measurement processes.

number property of the pointer system in the introduction of the super-selection rule or many Hilbert spaces.[6] Although this is reasonable, since the completion of the quantum measurement requires the firm storage of information on the quantum state of the system, this static description is not enough. Because we attempt to describe the measuring dynamical process, we need a *dynamical realization of the c-number features.*

It is apparent from the beginning that an apparatus with a finite number of degrees of freedom is impossible to implement the measurement process according to von Neumann's uniqueness theorem of Hilbert space. We need an infinite number of degrees of freedom, which requires the use of quantum field theory. However, within ordinary quantum field theory, the most general description using the effective action only yields the c-number equation of motion for the transition amplitude, which is the operator $\hat{O}$ averaged with respect to the in-vacuum and the out-vacuum $\langle 0_{\rm in}|\hat{O}|0_{\rm out}\rangle$. Because the boundary conditions are set both on the initial in-state and on the final out-state, the evolution equation is not causal. This equation even includes an intractable imaginary part in the case of a general environment. We have to reformulate quantum field theory to incorporate an appropriate boundary condition. Such a formulation is generally possible[7] and has been applied in various fields of physics.[8] Moreover, for our purpose, further development of the formalism would be necessary to describe the evolution equation of the c-number order parameter with dissipation and fluctuations.[9] The present paper is based on this formalism.

More precisely about the process IV, the common argument on the quantum measurement ignores this process despite the fact that, after the decoherence process, the system obviously becomes a pure state especially in the ideal case. This common argument is partially correct, because the ordinary description of probability in quantum mechanics is for an ensemble of copies of the system, and in this case, the prediction based on the density matrix just after the decoherence and that based on the pure state with an appropriate probability weight are the same. However, these two methods, the density matrix and pure state, produce different predictions for multiple measurements and continuous measurements.[2] Thus this process IV is considered to be a basic ingredient of the quantum measurement process.

It is also apparent that this non-unitary evolution in the processes III and IV cannot be described only by a deterministic evolution equation, such as a simple Schrödinger equation. There must be a mechanism which selects, with appropriate probabilities, one among many possible equivalents. A physical evolution very similar to this is the process of the spontaneous symmetry breakdown (SSB) during phase transitions. The mechanism of SSB is ubiquitous in various fields of physics, such as solid state physics, super fluid, elementary particle physics, and cosmology. Therefore it would be natural to consider that such a universal process as SSB must constitute an important ingredient of quantum mechanics, just as the Schrödinger equation. Thus, we introduce the following hypothesis: The measurement process III and IV are always accompanied by an SSB.[*] In Table II, the measurement process

[*] Our original motivation to introduce SSB in quantum measurement process goes back to the necessity to describe inhomogeneous density fluctuations in the early Universe based on the sponta-

Table II. Comparison of the measurement processes III&IV and the SSB phase transition process. They have many common features.

NATURE	QUANTUM MEASUREMENT (III&IV)	SSB PHASE TRANSITION
Local	Individual measurement (Ψ)	A single domain (single phase)
Global	Measurement on an ensemble (ρ)	Many domains
Describe local process	POVM (positive operator-valued measure)[1]	Langevin equation
Describe Global process	Effective equation for density matrix	Fokker-Plank equation
Classical variables	Reading of a pointer variable	Order parameter (new degrees of freedom emerge)
Irreversibility	Yes. It is a generalized projection.	Yes. It is a 1st-order phase transition.
Meta- stability	Entangled states	Meta-stable states
Switching nature	POVM	Random force triggers the phase transition
Correlation length	Maximum size for the cluster property to hold	Determines the domain size
Formation of structures	Generation of classical fluctuations	Cosmological inflation, nucleation, spinodal decomposition, topological objects
Inter-phase dynamics	Fluctuations of pointer values	Goldstone mode promotes the recovery of symmetry

and the SSB process are compared with respect to various aspects.

As shown in detail in Table II, the quantum measurement process and the SSB process have many common features. Therefore, it is natural to develop the physical measurement process by applying the present general framework of SSB. It should be kept in mind, simultaneously, that the formalism of SSB at present is not complete, and this fact sets the limitation to describe the quantum measurement process in our approach. Our goal in this paper is not the complete resolution of the SSB dynamics but to elucidate the essence of quantum measurement in the ubiquitous SSB process.

In our approach, the triple role of the c-number random field, arising from the environment, is conspicuous. Firstly, it makes the system decohere in the measurement process II. This kills the quantum fluctuations and induces statistical fluctuations. Secondly, it triggers the SSB phase transition in the measurement process III. It is interesting, despite the existence of random fields, that the pure state quantum nature is finally recovered in the system through the phase transition process with a positive feedback mechanism, as we will explain in detail in subsequent sections. Thirdly, the appropriate strength of the c-number random field is essential to optimize the efficiency of the quantum measurement on an ensemble, as we will see in the latter part of this paper.

The construction of this paper is as follows. In §2, we derive the classical dynam-

neous breakdown of the spatial-translational symmetry.[10] Generalized effective action method was adopted there. Although recent algebraic method by Ojima[11] should be also promising, it seems difficult to extract an explicit time evolution of the measuring dynamics in a simple form. Therefore we adopt the generalized effective action method in this paper.

ics of SSB in the quantum field theory, in particular the emergence and evolution of the c-number degrees of freedom in the closed time-path formalism. This consists of the measurement process III. In §3, we analyze the quantum dynamics of a system in the environment, in particular the evolution from a mixed to a pure state is studied, as well as quantum decoherence. This consists of the measurement process IV. In §4, on the basis of the arguments of §§2 and 3, we introduce a simple model of a single quantum measurement. In §5, on the basis of this model, the precision of the measurement on an ensemble of the identical copies of the system is studied. In §6, we conclude our study and examine the possibility of the application to cosmology and that of the deduction of the Born probability postulate in quantum mechanics.

§2. Dynamics of SSB in quantum field theory: emergence and evolution of c-number degrees of freedom (process III)

Let us consider the dynamics of the SSB phase transition for clarifying the measurement process III. Here, the phase transition dynamics implies the existence of the c-number degrees of freedom whose development from zero to non-zero values describes the symmetry breaking dynamical process. In this context, the most appropriate tool is the effective action $\Gamma[\varphi]$, which is a quantum analog of the classical action functional of the c-number field, φ, where φ represents a general field. The effective action $\Gamma[\varphi]$ is defined, in ordinary quantum field theory, to be the Legendre transformation of the generating functional

$$Z\left[J\right]=\left\langle \Psi_f,t_f\left|T\exp\left[i\int d^4xJ\left(x\right)\phi\left(x\right)\right]\right|\Psi_i,t_i\right\rangle \tag{1}$$

as

$$\Gamma\left[\varphi\right]=-i\ln Z\left[J\right]-\int d^4xJ\left(x\right)\hat{\phi}\left(x\right) \tag{2}$$

and

$$\begin{aligned}\varphi\left(y\right)&=-i\left(\delta/\delta J\left(y\right)\right)\ln Z\\&=\left\langle \Psi_f,t_f\left|T\phi\left(y\right)\exp\left[i\textstyle\int d^4xJ\left(x\right)\phi\left(x\right)\right]\right|\Psi_i,t_i\right\rangle ,\end{aligned} \tag{3}$$

where $|\Psi_i,t_i,\rangle$ and $|\Psi_f,t_f,\rangle$ are the initial and final states. These definitions lead to the inverse relation

$$-J\left(y\right)=\delta\Gamma\left(\varphi\right)/\delta\varphi\left(y\right), \tag{4}$$

which can be regarded as the evolution equation for φ.[12] However, the boundary condition in Eq. (1) makes the equation of motion Eq. (4) non-causal and the quantity Eq. (3) more like a transition amplitude than an order parameter. Moreover, the quantity Eq. (3) becomes a complex field in general, even if we consider a real quantum field.

If the initial and final states $|\Psi_i,t_i,\rangle$ and $|\Psi_f,t_f,\rangle$ belong to the same Fock space in ordinary quantum field theory, we can make the generating functional state-independent

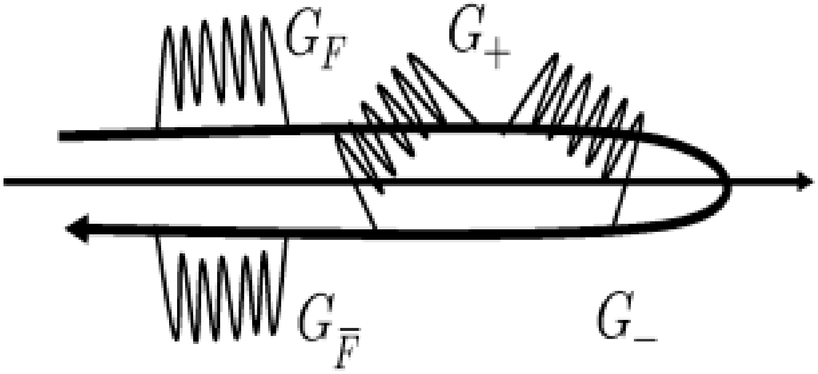

Fig. 1. The time integration contour in the CTP formalism is illustrated by the solid curve. The contour C is from $-\infty$ to $+\infty$, and then back to $-\infty$. The components of the two-point Green function Eq. (8) are also illustrated by wiggly curves.

$$Z\left[J\right]=\left\langle 0\left|T\exp\left[i\int d^4xJ\left(x\right)\hat{\phi}\left(x\right)\right]\right|0\right\rangle. \tag{5}$$

after the $-i\varepsilon$ prescription. However, we cannot expect this in the situation considered presently, because what we need now is the dynamical evolution from symmetric to non-symmetric states and $|\Psi_i,t_i,\rangle$ and $|\Psi_f,t_f,\rangle$ belong to different Fock spaces.

The static limit of the effective action $\Gamma[\varphi]$, multiplied by a minus sign, yields the effective potential $V_{\text{eff}}[\varphi]$, and its minima represent different kinds of vacua characterized by different values of φ. Each vacuum constructs an independent Fock space; these Fock spaces are in-equivalent with each other and cannot be connected by any unitary transformation. The effective action $\Gamma[\varphi]$ is the dynamical analog of $V_{\text{eff}}[\varphi]$, and its minima is expected to give some information regarding the non-unitary dynamics, inter-connecting the above inequivalent Fock spaces.

A minimal extension of the effective action formalism, free from the above mentioned difficulties of causality and complexity, is the closed-time-path (CTP) formalism,[7),8),9)] in which the boundary condition is altered. This method not only resolves the difficulties but it also properly describes effective dissipation and diffusion as well as quantum corrections in the ordinary quantum field theory, as we will see shortly.

The CTP formalism has an extra time-branch, and therefore all the time integrals are doubled as

$$\int_{-\infty}^{\infty}dt\rightarrow\int_C dt, \tag{6}$$

where the time contour C is depicted in Fig. 1: from $-\infty$ to $+\infty$, and then back to $-\infty$.

The generating functional in CTP formalism is defined as

$$\tilde{Z}[\tilde{J}]\equiv\mathrm{Tr}\left[\tilde{T}(\exp\left[i\int_C d^4x\tilde{J}\left(x\right)\tilde{\hat{\phi}}\left(x\right)\right]\rho)\right]\equiv\mathrm{Exp}[i\tilde{W}[\tilde{J}]], \tag{7}$$

where $\tilde{T}$ is the time-ordering operator along the extended time-contour C. The tildes here mean that the quantities are defined on this extended time contour C. The density matrix ρ is the initial state at $t = -\infty$. For example, the two point function is also doubled, and we have

$$\tilde{G}(x,y) = -i \begin{pmatrix} \mathrm{Tr}[T\phi(x)\phi(y)\rho] & \mathrm{Tr}[\phi(y)\phi(x)\rho] \\ \mathrm{Tr}[\phi(x)\phi(y)\rho] & \mathrm{Tr}[\bar{T}\phi(x)\phi(y)\rho] \end{pmatrix} = \begin{pmatrix} G_F(x,y) & G_+(x,y) \\ G_-(x,y) & G_{\bar{F}}(x,y) \end{pmatrix}, \tag{8}$$

where T and $\bar{T}$ are the time and anti-time ordering operators, and F and $\bar{F}$ are the ordinary Feynman and anti-Feynman two-point functions, respectively. The four components originate from the possible combinations of two branches of the contour C and two arguments, x and y. Among the four components, only three are independent. This becomes most apparent in the momentum representation:

$$G_F(k) = \frac{D(k) - iB(k)}{D(k)^2 + A(k)^2},\ G_{\bar{F}}(k) = \frac{-D(k) - iB(k)}{D(k)^2 + A(k)^2},\ G_{\pm}(k) = -i\frac{D(k) \mp iB(k)}{D(k)^2 + A(k)^2}. \tag{9}$$

The independent functions $D\,(k)$, $A\,(k)$ and $B\,(k)$ have the following interpretations.[9] The function $D\,(k)$ describes quantum corrections, such as the renormalization of the wave function, mass etc., the function $A\,(k)$ describes friction, which violates the time reversal symmetry, and the function $B\,(k)$ describes diffusion, which induce quantum decoherence effects.

The ordinary formulation of quantum field theory, given by Eqs. (1)–(4), is also possible in CTP formalism. Especially, the c-number field, defined by

$$\tilde{\varphi}(x) = \begin{pmatrix} \varphi_+(x) \\ \varphi_-(x) \end{pmatrix} \equiv \frac{\delta\tilde{W}}{\delta\tilde{J}} = \mathrm{Tr}\left[\tilde{\hat{\phi}}(x) \exp\left[i\int_C d^4x \tilde{J}\,(x)\,\tilde{\hat{\phi}}\,(x)\right]\rho\right] \tag{10}$$

has suitable properties as an order parameter. The suffices $\pm$ indicate that the arguments are located on the forward time contour ($x_0 \in [-\infty, \infty]$) or the backward time contour ($x_0 \in [\infty, -\infty]$), respectively. The effective action is again defined to be the Legendre transformation of $\tilde{W}[\tilde{J}]$.

In this construction, the effective action $\tilde{\Gamma}\,[\varphi]$ becomes complex in general, even if the classical action is real. This situation is the same as that of ordinary quantum field theory. However, in the present CTP formalism, the imaginary part of the effective action can be uniquely transformed into the kernel of statistical fluctuations. Actually,[9] we can rewrite $\tilde{\Gamma}$ as

$$e^{i\tilde{\Gamma}[\varphi]} = \int [d\xi]P[\xi] \exp\left[i\left(\mathrm{Re}\tilde{\Gamma} + \int d^4x \xi\,(x)\,\varphi_\Delta\,(x)\right)\right], \tag{11}$$

where we have introduce an auxiliary field $\xi\,(x)$. We have defined $\varphi_\Delta = \varphi_+ - \varphi_-$ and $\varphi_C = (\varphi_+ + \varphi_-)\,/2$. The exponential of the imaginary part of $\tilde{\Gamma}$,

$$e^{-\mathrm{Im}\tilde{\Gamma}} = \exp\left[\,-\frac{1}{2}\iint d^4x d^4y \varphi_\Delta(x) B(x-y) \varphi_\Delta(y) + O\left(\varphi_\Delta^3\right)\right], \tag{12}$$

is functionally Fourier transformed into the kernel of the statistical fluctuations for the random field $\xi\,(x)$,

$$P[\xi] = \exp\left[\, -\frac{1}{2}\iint d^4x d^4y \xi\,(x)\, B^{-1}\,(x-y)\,\xi\,(y) + O\left(\xi^3\right)\right], \tag{13}$$

which is the Gaussian statistical weight up to the second-order term in φ_Δ in Eq. (12). In the above, $B\,(x)$ is the Fourier transform of $B\,(k)$, and $B\,(x)^{-1}$ is that of $1/B\,(k)$. Thus, we can now identify the field $\xi\,(x)$ as a c-number statistical variable with the weight $P\,[\xi]$ given by Eq. (13). Actually, if we define the statistical weight as

$$\langle \cdots \rangle_{\rm st} \equiv \int [d\xi] \cdots P[\xi], \tag{14}$$

then the correlation of ξ is expressed by the kernel B as

$$\langle \xi(x)\xi(y)\rangle_{\rm st} = B(x-y). \tag{15}$$

Furthermore, this field $\xi\,(x)$ couples to the c-number order parameter φ_c through the real action of the exponent in Eq. (11). Because the field φ_c is already a c-number, the system is considered to possess some objective value at any time of further measurements.

We give here some brief comments on the appearance of the c-number order parameter φ_c in the CPT quantum field theory. (See Ref. 9) for further details.) We emphasize the validity of identifying the field $\xi\,(x)$ as a c-number statistical variable in the following three points. Firstly, our formalism also holds in a system of many harmonic oscillators,[13] and it is thought to be a quantum field theoretical generalization of Ref. 13). The diffusion term in Ref. 13), which represents statistical fluctuations, arises from the real part of the environment kernel, Re$[\alpha]$, which exactly corresponds to Im$\tilde{\Gamma}$ in our formalism. Therefore the function Im$\tilde{\Gamma}$ should contain all the information of the statistical fluctuations. Secondly, we point out the existence of the inverse of the functional Fourier transformation, from $\exp[-{\rm Im}\tilde{\Gamma}\,[\varphi_\Delta]]$ to $P\,[\xi]$, as in Eqs. (12) and (13); i.e. the map $\varphi_\Delta \mapsto \xi$ is one-to-one and onto. Therefore, the functional $P\,[\xi]$ inherits the entire statistical nature of Im$\tilde{\Gamma}$. Thirdly, use of the field ξ is one possible representation of the statistical fluctuations represented by Im$\tilde{\Gamma}$, and there may be other possible one-to-one, onto representation using other fields ξ'. However, they are statistically equivalent because the statistical properties are fundamentally governed by Im$\tilde{\Gamma}$.

As an example, in the $\lambda\phi^4$ model in an environment at temperature T, an imaginary part of the effective action of order $\lambda^2 T$ arises. This reflects the loss of information to the environment and the uncontrollable energy input from the environment.[14] Even in the vacuum, an imaginary part arises,[9]

$${\rm Im}\tilde{\Gamma}[\tilde{\varphi}] = \int \frac{\lambda^2}{128\pi}\theta\left(\left|m^2\right| - \frac{\lambda}{2}\varphi_C^2\right)\varphi_C^2(x)\varphi_\Delta^2(x), \tag{16}$$

reflecting the instability of the system for $\varphi_C^2 \le \frac{\lambda}{2}\left|m^2\right|$, and according to the above formalism, a c-number field $\varphi\,(x)$ arises, and it couples to the c-number random field

$\xi(x)$ with the correlation

$$\langle \xi(x)\,\xi(y)\rangle_{\rm st} = \frac{\lambda^2}{128\pi}\varphi_C^2(x)\,\delta^4(x-y)\,. \tag{17}$$

These random fields in general trigger the SSB phase transition.

The remaining real part $\mathrm{Re}\Gamma + \xi\varphi_\Delta$ of the effective action is considered to be the genuine dynamical action, including $D(k)$ and $A(k)$ corrections. Application of the least action principle for the variable $\varphi_\Delta(x)$ yields the Langevin equation for the order parameter $\varphi_C(x)$,

$$\partial_x\partial^x\varphi_C(x) + V'(\varphi_C(x)) + \int_{-\infty}^{t} dt' \int dx' A(x-x')\varphi_C(x') = \xi(x)\,, \tag{18}$$

where the second term, $V(\varphi)$, on the LHS includes the renormalized effects from the kernel $D(k)$, the third term includes the dissipative effects from the kernel $A(k)$, and the RHS reflects the diffusive effects from the kernel $B(k)$, through Eq. (15).

In order to elucidate the essence of the process in a technically tractable form, we introduce some approximations to make the system the simplest and non-trivial. First, we can apply the slow-rolling non-relativistic approximation, since the slowly evolving first-stage of the phase transition is relevant to determine the fate of the order parameter. Further, we consider the spatially uniform limit of the order parameter, since we are interested in the single domain which corresponds, according to II, to a single measurement of the system. Then, the above equation (18) reduces[*)] to

$$\frac{d\varphi(t)}{dt} = -\frac{1}{\kappa}\left(m^2\varphi(t) + \frac{\lambda}{3!}\varphi(t)^3\right) + \frac{1}{\kappa}\xi(t) \equiv \gamma\varphi(t) - g\varphi(t)^3 + \eta(t)\,. \tag{19}$$

In our case, we have $m^2 < 0$ and $\gamma > 0$, and the random field has a Gaussian white correlation,

$$\left\langle \eta(t)\,\eta(t')\right\rangle_{\rm st} = \varepsilon\delta(t-t')\,. \tag{20}$$

This equation cannot be solved exactly, even with these approximations. If we can ignore the non-linear term $g = 0$, the solution of the Fokker-Planck equation associated with this Langevin equation is given by[15)]

$$P(\varphi, t) = (2\pi\varepsilon(t))^{-1/2}\int_{-\infty}^{\infty} dy \exp\left[-\frac{\left(y - e^{-\gamma t}\varphi\right)^2}{2\varepsilon(t)}\right] P(y, 0)\,, \tag{21}$$

where

$$\varepsilon(t) = \frac{\varepsilon}{\gamma}\left(e^{2\gamma t} - 1\right), \tag{22}$$

[*)] The kernels A and B include the information regarding ρ, and so do the parameter coefficients γ, g and ε. Such reduction is possible only when the environment is stationary. Moreover, these parameters include information regarding the finite domain size formed after SSB.

and $P(y,0)$ is the initial distribution function at $t=0$. This form is sufficient for our purpose, because the first-stage of the SSB is the most relevant to determine the overall destination of the order parameter φ. Further, in the fully non-linear case, only the scaling solution is known.[15]

§3. Dynamics of a system in the environment: evolution from a mixed to a pure state (process IV)

We now turn our attention to the quantum evolution of the system in the environment. We are interested in how the quantum decoherence and the pro-coherence work. In this section, we study a spin of magnitude one half in the environment, which is a part of the complete measurement model we introduce in the next section.

The spin system in a static magnetic field $\vec{B}=(0,0,B)$ and the environment are described by the Hamiltonian

$$\hat{H}=\hat{H}_S+\hat{H}_I+\hat{H}_B, \tag{23}$$

where

$$\begin{aligned}\hat{H}_S&=\omega_0\hat{S}_3,\\ \hat{H}_I&=g\hat{\vec{S}}\cdot\vec{R},\end{aligned} \tag{24}$$

with $\omega_0=\mu B$. Because we are interested only in the dynamics of the spin, we coarse-grain the environmental degrees of freedom. The effective equation of motion is then given by[16]

$$\dot{\rho}(t)=-i\omega_0[\hat{S}_3,\rho(t)]+\left(a\left[\hat{S}_+\rho(t),\hat{S}_-\right]+b\left[\hat{S}_-\rho(t),\hat{S}_+\right]+c\left[\hat{S}_3\rho(t),\hat{S}_3\right]+\text{h.c.}\right), \tag{25}$$

where the coefficients a,b and c are the reservoir correlations

$$\begin{aligned}a&=\tfrac{g^2}{4}\int_0^\infty dte^{-i\omega_0 t}\left\langle\hat{R}_+(t)\,\hat{R}_-(0)\right\rangle,\\ b&=\tfrac{g^2}{4}\int_0^\infty dte^{i\omega_0 t}\left\langle\hat{R}_-(t)\,\hat{R}_+(0)\right\rangle,\\ c&=g^2\int_0^\infty dt\left\langle\hat{R}_3(t)\,\hat{R}_3(0)\right\rangle,\end{aligned} \tag{26}$$

and are set to be real and positive, discarding imaginary parts, which yield irrelevant phase factors.*) Note that Eq. (25) is a convolutionless local differential equation even after the coarse-graining, according to the beautiful formalism given in Ref. 16), which uses the pullback of the time evolution. A stationary environment guarantees the validity of the fluctuation-dissipation relation

$$a=\exp\left[\,-\hbar\omega_0/\,(kT)\right]b, \tag{27}$$

*) These parameters generally include complex dynamics of the reservoir. This approximation is valid only when the evolution of ρ is not too fast and the reservoir is stationary.

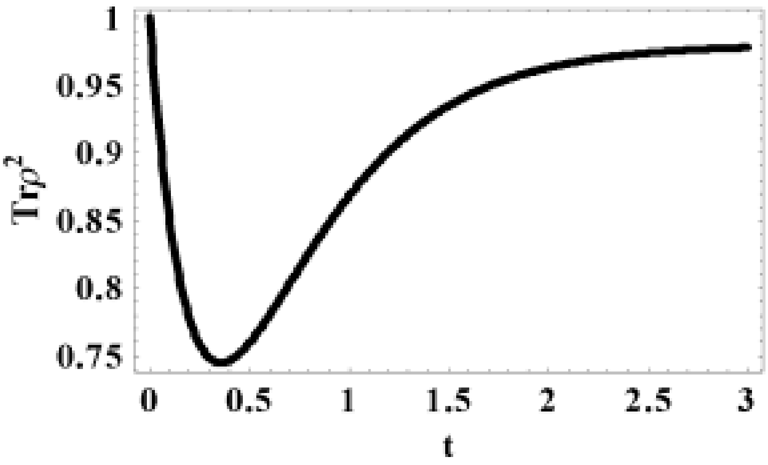

Fig. 2. The solid line represents the time evolution of $\mathrm{Tr}\rho^2$, a measure of the mixing of the spin state. The state starts from a pure state, $\mathrm{Tr}\rho^2 = 1$. The system then quantum de-coheres (QD) first and eventually quantum pro-coheres (QP) later. A quantum system in general can become pure from a mixed state after contact with a cold environment. The parameters in this demonstration are $c_1 = 0.5,\ c_2 = 0.5,\ c3 = 0.5,\ c4 = 0.5,\ a = 0.99,\ b = 0.01,\ c = 1,\ \omega_0 = 10$.

where T is the reservoir temperature. The above linear equation, Eq. (25), is readily solved as

$$\rho(t) = \begin{pmatrix} \frac{b(c_1+c_4)+e^{-2(a+b)t}(ac_1-bc_4)}{a+b} & c_2 e^{-(a+b+c+i\omega)t} \\ c_3 e^{-(a+b+c-i\omega)t} & \frac{a(c_1+c_4)+e^{-2(a+b)t}(-ac_1+bc_4)}{a+b} \end{pmatrix} \tag{28}$$

for the initial density matrix

$$\rho(t=0) = \begin{pmatrix} c_1 & c_2 \\ c_3 & c_4 \end{pmatrix}. \tag{29}$$

Here we have employed the matrix representation, so that S_3 is diagonalized.

Let us consider the high magnetic field (or low temperature) limit, $\omega_0\hbar \gg kT$. Then it is apparent from the solution given in Eq. (28) that the density matrix approaches

$$\rho(t \to \infty) = \begin{pmatrix} \frac{b}{a+b} & 0 \\ 0 & \frac{a}{a+b} \end{pmatrix}, \tag{30}$$

which represents either the spin-up pure state $|+\rangle = (1, 0)$ or the spin-down pure state $|-\rangle = (0, 1)$, but not both, depending on the signature of ω_0 (or the direction of the magnetic field), as in Eq. (27). Thus, the spin state after contact with the cold environment can become pure, contrary to the ordinary argument emphasizing the decoherence effect of the environment. This fact is demonstrated in Fig. 2. The present pro-coherence should be one of the key mechanisms that results in the realization of the von Neumann projection in physical systems.

The above pro-coherence is not restricted to the spin system. In the familiar Caldeira-Legget model[13)] for many coupled harmonic oscillators, with

$$\hat{H} = \hat{H}_A + \hat{H}_I + \hat{H}_B, \tag{31}$$

$$\begin{aligned} \hat{H}_A &= \frac{\hat{p}^2}{2m} + v\left(\hat{x}\right), \\ \hat{H}_I &= \hat{x}\sum_k C_k \hat{x}_k, \end{aligned} \tag{32}$$

the evolution equation for the reduced density matrix can be incorporated into the convolutionless formalism, similar to Eq. (25).[16)] The asymptotic expression of $\mathrm{Tr}\rho^2$ becomes[17)]

$$\mathrm{Tr}\rho^2 \underset{t\to\infty}{\longrightarrow} \left[\coth(\frac{\hbar\omega}{2kT})\right]^{-1} \approx \begin{cases} 1 & \text{for } \frac{\hbar\omega}{2kT} \gg 1 \\ \frac{\hbar\omega}{2kT} & \text{for } \frac{\hbar\omega}{2kT} \ll 1 \end{cases} . \tag{33}$$

Therefore, in the low temperature limit or in the high frequency limit, we have $\mathrm{Tr}\rho^2 \to 1$, and the system becomes a pure state.[*)] Thus in general, a cold environment can make the system pure.

§4. A simple model of spin measurement

We now introduce the simplest model which satisfies the basic properties of quantum measurement discussed in the introduction. The model is simply a combination of the systems introduced in §§2 and 3, and it is composed of a spin of magnitude one half (the system) and a scalar field with a $\lambda\varphi^4$ self-interaction (the apparatus), with a thermal bath. The Lagrangian is given by

$$\hat{L} = \frac{1}{2}\left(\partial_\mu\hat{\phi}\right)^2 - \frac{m^2}{2}\hat{\phi}^2 - \frac{\lambda}{4!}\hat{\phi}^4 + \mu\hat{\phi}\hat{\vec{S}}\cdot\vec{B} + (\text{bath}), \tag{34}$$

where $\mu > 0$, and the static magnetic field $\vec{B}$ is parallel to the z-direction. The "wrong" sign of the mass term, $m^2 < 0$, guarantees the occurrence of an SSB, and the c-number order parameter φ is described by the effective action $\mathit{\Gamma}\left[\varphi\right]$ in the CTP formalism. Although the variable field $\hat{\phi}$ does not have direct connection to any realistic measuring apparatus, this simple form of the Lagrangian is nevertheless useful to elucidate the relevance of the SSB phase transition for the quantum measurement process.

The evolution of the c-number order parameter can be derived through the variation of the generalized effective action in the CTP formalism. This reduces to the following form, similar to Eq. (19), after the slow-rolling and local approximations:

$$\dot{\varphi} = \gamma\varphi - g\varphi^3 + \mu\left\langle\hat{\vec{S}}\right\rangle\cdot\vec{B} + \eta. \tag{35}$$

Note that in the above, there appears an extra term $\mu\langle\hat{\vec{S}}\rangle\cdot\vec{B}$, which biases the SSB phase transition. On the other hand, the evolution of the system density matrix is

[*)] This is most apparent in the representation in which ρ is diagonal.

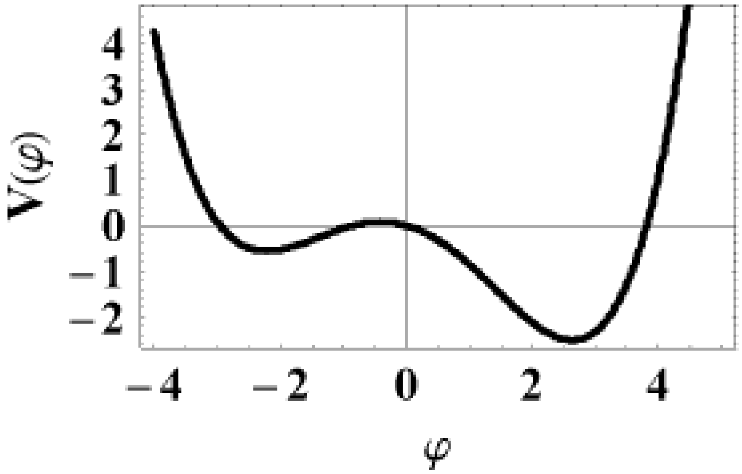

Fig. 3. A potential $V(\varphi)$ biased by the spin coupling $-\mu\varphi\langle\hat{\vec{S}}\rangle\cdot\vec{B}$. In the above case we have $\langle\hat{\vec{S}}\rangle\cdot\vec{B}>0$, and the slope of the potential at the origin is negative. This biases the c-number order parameter field φ to roll down, toward the right. If $\langle\hat{\vec{S}}\rangle\cdot\vec{B}<0$, the potential biases φ to roll down, toward the left.

given by Eq. (25), but now ω_0 is replaced by $\omega=\mu\varphi(t)\,B$ and is time dependent through $\varphi(t)$:

$$\dot{\rho}(t)=-i\omega[\hat{S}_3,\rho(t)]+\left(a\left[\hat{S}_+\rho(t),\hat{S}_-\right]+b\left[\hat{S}_-\rho(t),\hat{S}_+\right]+c\left[\hat{S}_3\rho(t),\hat{S}_3\right]+\text{h.c.}\right). \tag{36}$$

In the slow rolling regime, the standard fluctuation-dissipation relation Eq. (27) still holds. In the limit $\hbar\omega/(kT)\gg 1$, because the magnetic field B is constant, the asymptotic state becomes a pure spin up state or down state, depending on the signature of the order parameter φ.

Suppose initially the spin is prepared in a state for which $\langle\hat{\vec{S}}\rangle\cdot\vec{B}>0$. Then the *biased* potential $V(\varphi)$, which includes the term $-\mu\varphi\langle\hat{\vec{S}}\rangle\cdot\vec{B}$, has a negative gradient at the origin (Fig. 3). This causes the c-number order parameter φ to evolve from there toward the region $\varphi_+>0$. In this evolution, the potential energy for the spin $-\mu\varphi\hat{\vec{S}}\cdot\vec{B}$ favors the situation in which $\hat{\vec{S}}$ is parallel to $\vec{B}$. This is the *positive feedback* for the spin to be *locked* into the up-state with a strong correlation with the order parameter being φ_+. Biasing toward the other direction and positive feedback also hold for the initial state with $\langle\hat{\vec{S}}\rangle\cdot\vec{B}<0$; the spin is locked into the down-state, with a strong correlation with the order parameter being φ_-. Thus we obtain a strong correlation between the spin state and the order parameter after the phase transition, irrespective of the initial state of the system. This is the essence of quantum measurements and the von Neumann projection postulate. Due to the strong magnetic field ($\hbar\omega/(kT)\gg 1$), the spin evolves into a purely up-state or purely down-state in accordance with the c-number order parameter, φ_+ or φ_-, respectively.

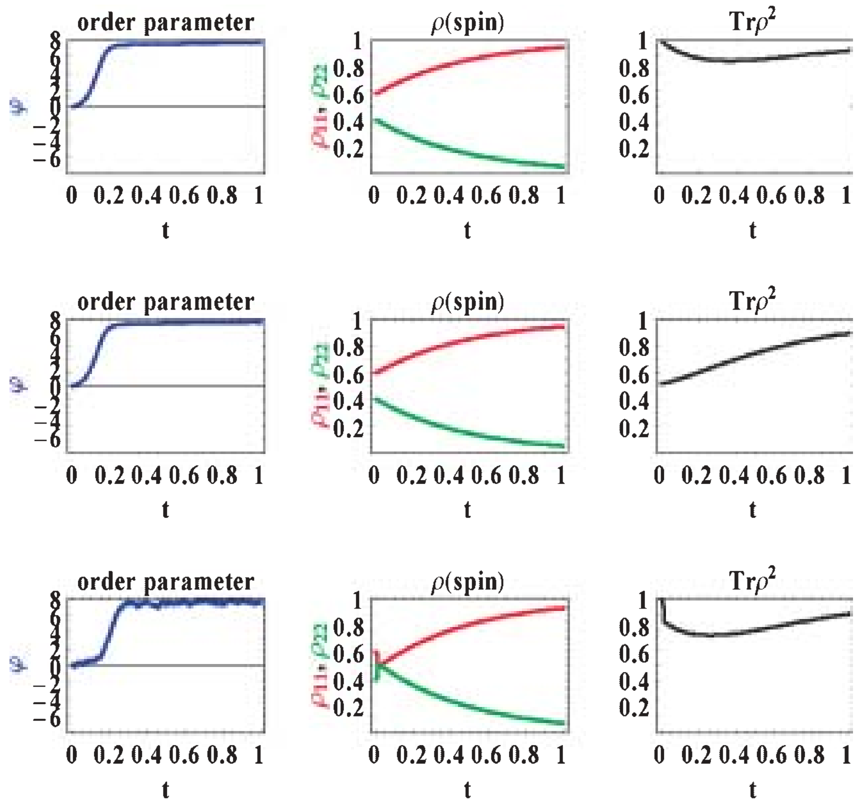

Fig. 4. Time evolution of the order parameter (the blue curve), spin probabilities (ρ_{11} for the red curve, and ρ_{22} for the green curve), and the measure of mixing (the black curve). The fourth-order Runge-Kutta method was used with a fixed interval, with which the coupling strength of the random field is properly normalized. The first and third rows start from pure states, while the second row starts from a mixed state. The optimization parameters, defined in Eq. (40), are $z = 10$, 2.0 and 0.08, respectively for the top to the bottom rows.

Numerical solutions for the coupled equations Eqs. (35) and (36) are depicted in Fig. 4. After the initial quantum entanglement (QE), the system generally decoheres (QD) first and then procoheres (QP), establishing a strong correlation with the fate of the order parameter φ (SSB).

The order parameter eventually rolls down the potential toward either φ_+ or φ_- positions, and simultaneously the spin state is locked in either the up-state or the down-state, with strong correlation with the order parameter. In this dynamics, the entropy of the spin system increases first, reflecting the QD process, and then decreases, reflecting the QP process. The initial spin state biases the direction of the rolling order parameter, though the bias is not always faithful, because the random force is applied in the SSB process. Though this random field seems to have no relevant role in the present one-event measurement, we will show that the random field plays a crucial role in the realistic ensemble measurement. This is the topic in the next section.

§5. Efficiency and precision of the measurement

After confirming that the above model works as a quantum measurement device, we would like to study to what extent the device actually performs faithful measurement. For this purpose, the balance between the initial biasing strength and the random force strength turns out to be essential.

The probability that the order parameter is located on the positive φ side at time t is obtained by integrating the solution Eq. (21),

$$P_{+} = \int_{0}^{\infty} P(\varphi, t)\, d\varphi = \frac{1 + \mathrm{erf}\left(\delta / \sqrt{2\varepsilon(t)}\right)}{2}, \tag{37}$$

where $\delta = (\mu/\gamma)\langle \hat{\vec{S}} \rangle \cdot \vec{B}$ is the initial bias. The probability that the order parameter is located on the negative φ side at time t is similarly given by

$$P_{-} = \int_{-\infty}^{0} P(\varphi, t)\, d\varphi = \frac{\mathrm{erfc}\left(\delta / \sqrt{2\varepsilon(t)}\right)}{2}. \tag{38}$$

In these expressions, the limit $t \to \infty$ would be useless, because the solution Eq. (21) is a linear approximation. We have to evaluate these quantities just after the completion of the SSB phase transition. The important time scale for this purpose is the onset time of order,

$$t_0 = \frac{1}{2\gamma} \ln \left[\frac{g}{\gamma} \left(\frac{\varepsilon}{\gamma} + \delta^2 \right) \right]^{-1}, \tag{39}$$

which plays an important role in the scaling solution.[15] This expression is the same as the deterministic rolling time duration of the order parameter from the initial position toward the inflection point of the potential; the initial position squared is defined to be the sum of the deviation squared δ^2 and the dispersion ε/γ caused by the random force during the characteristic time scale $1/\gamma$. For a faithful measurement, the relation $P_{\pm} \approx 1$ is necessary at $t = t_0$ for the initial pure state $\delta = \mu B/(2\gamma) \equiv \delta_{\max}$. This yields the approximate optimization condition for the measurement,*)

$$z \equiv \frac{\delta_{\max}}{\sqrt{2\varepsilon(t_0)}} \approx 2. \tag{40}$$

If this optimization condition is satisfied, then the initial expectation value of the system spin along the magnetic field $\langle \hat{\vec{S}}(t=0) \rangle \cdot \vec{B}$ has an approximate one-to-one, onto correspondence to $P_{+}(t_0)$. The probability $P_{+}(t_0)$ in this case is interpreted to

*) The value is somewhat arbitrary, since the error function exponentially approaches 1. If we choose values that are too large, the calibration curve in Fig. 5 becomes almost degenerate. Here we have chosen the value 2, in which case $P_{\pm} \approx 0.9953$. The following arguments are within this approximation.

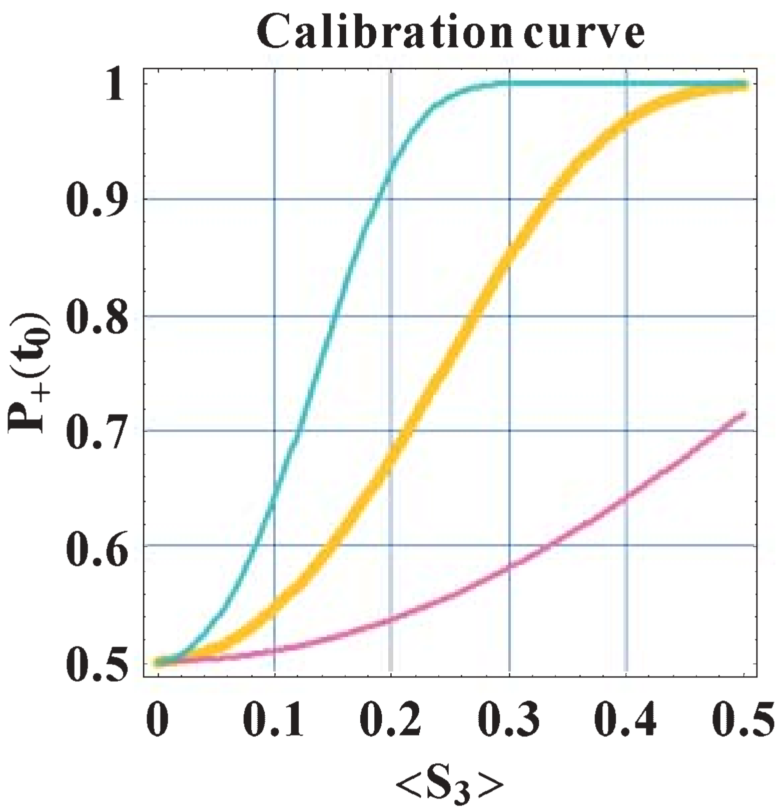

Fig. 5. The curves represent the probability function evaluated at $t_0 : P_+(t_0)$ as a function of the initial expectation value of the system spin $\hat{S}_3$. Regarding the spin variable $\langle\hat{S}_3\rangle$, only half of the full range $[-1/2, 1/2]$ is shown above, because the curves possess a rotational symmetry of degree π w.r.t. the point $(0, 0.5)$. The probability $P_+(t_0)$ is considered to be a relative frequency actually obtained after a large number of measurements of identically prepared systems. The thick gold curve represents the most optimized case, with $z = 2.0$. The magenta curve below is for $z \ll 2$. Even in this case, an appropriate calibration makes this measurement apparatus faithful. The cyan curve above is for $z \gg 2$. For this case, an extremely accurate calibration would be necessary, since the relative frequency is almost degenerate.

be the relative frequency actually obtained after a large number (ideally, an infinite number) of measurements of identically prepared systems. However, this correspondence is not linear in general, and therefore an appropriate calibration is necessary before a precise measurement. This situation is depicted in Fig. 5. If $z \gg 2$, then the initial deviation effect dominates the random force, and the correspondence becomes almost degenerate with respect to the relative frequency. In this case, an extremely accurate calibration is necessary. By contrast, if $z \ll 2$, then the random field dominates the initial deviation effect, and the correspondence is no longer an onto map; there appears a large range of relative frequencies which are never realized. Even in this case, an appropriate calibration still works.

Though the random field plays a crucial role in the first stage of SSB phase transitions, as we have seen above, it becomes obstructive in later stages of the SSB phase transition. Actually, it induces fluctuations of φ from the stationary values $\varphi_\pm$ and reduces the sharpness of the pointer readout, though this effect should be small. The random field promotes the transition only in the first stage of the SSB.

It is worth adding a comment on the actual calibration process mentioned in the above argument. A rough sketch of the process is as follows. We prepare the standard Stern-Gerlach (SG) spin filter, in which the original beam of atoms with spin one half is split into two beams by the spatial gradient of the magnetic field. One beam, which goes in the upper direction along the gradient (z' direction) has spin $+1/2$, and the other, which goes in the lower direction, has spin $-1/2$. This is simply a preparation of the state, not the measurement. Then, in the middle of the upper beam, we set our spin detector, whose z axis is tilted by an angle θ from the z' axis. The state injected into the detector is then a definite superposition of spin up $|+\rangle$ and down $|-\rangle$, measured along the z axis: $|\mathrm{in}\rangle = N\left(\cos\left(\theta/2\right)|+\rangle + \sin\left(\theta/2\right)|-\rangle\right)$. Although the overall normalization N of the state depends on the initial state before the SG splitter and is not known, the ratio of the coefficients is sufficient for the calibration. For each $\langle \hat{S}_z \rangle = \cos\theta/2$, we write the relative frequency that the order parameter indicates that the spin is up ($\varphi = \varphi_+$) as $P_+\left(t_0\right)$ and that it is down ($\varphi = \varphi_-$) as $P_-\left(t_0\right)$. This manipulation successfully determines the actual calibration curve. The details will be discussed in a separate report.

§6. Conclusions and some applications

Our original goal was to clarify the physical process of the measurement process, and resolve the original double-structure in the evolution rules in quantum mechanics. We first introduced the hypothesis that the crucial process in a quantum measurement is spontaneous symmetry breaking (SSB). The demonstration of this hypothesis requires that we study the description of the c-number order parameter in CTP quantum field theory and pro-coherence in the effective spin dynamics. Combining these basic considerations, we have introduced a measurement model with spin-field coupling. While the apparatus+spin system exhibits entanglement (QE) during their contact, the random fields acting from the outside environment provide the quantum decoherence (QD) of the spin system, as well as the trigger of the phase transition (SSB) of the apparatus. This measurement model with SSB guarantees the positive feedback of the evolution of the order parameter to the spin evolution, which promotes the quantum pro-coherence (QP) of the spin system. *These four processes, QE, QD, SSB, and QP, form the basic quartet of a quantum measurement.*

We have further considered the efficiency and the precision of the measurement on an ensemble of identically prepared systems. We have obtained the *calibration curve*, with which we can predict the initial spin state, and the *optimization condition* for the most efficient measurement. The necessity of this calibration and the characteristic time scale of the SSB phase transition, t_0, in Eq. (39) are characteristics of our approach, and also checkpoints for the validity of our scenario in comparison with actual quantum measurements.

We would like to conclude our paper by itemizing several considerations on the possible generalizations, applications, and further issues.

1. We have, so far, considered the simplest SSB of discrete Z_2 symmetry in the measurement model. Further application is, of course, possible to SSB models with continuous symmetry. For example, consider the measurement of the photon phase $\hat{\omega}$. We prepare the potential with the following coupling between the photon field a and the apparatus ϕ which is now a complex scalar filed:

$$V\left(\hat{\phi}\right) = -\mu^2\hat{\phi}^\dagger\hat{\phi} + \frac{\lambda}{2}\left(\hat{\phi}^\dagger\hat{\phi}\right)^2 + \mu\left(\hat{\phi}^\dagger\hat{a} + \hat{a}^\dagger\hat{\phi}\right) + O\left(\hbar\right). \tag{41}$$

The original $U\left(1\right)$ symmetry is broken after the evolution of the c-number order parameter from 0 toward finite values $\varphi = \left|\varphi\right| e^{i\theta}(\left|\varphi\right| : 0 \to \sqrt{2\mu^2/\lambda})$, with a spontaneously selected phase θ. During this process the photon phase is strongly locked into $\omega \approx \pi - \theta$ with positive feedback. Thus this system works as a proper quantum measurement apparatus. Also here the basic quartet is the essence of the quantum measurement. However, the appearance of the Goldstone mode may enhance the fluctuation effect by a random force, and therefore it may not be preferable for an exact measurement.

2. In the above models, we have studied almost projective (ideal) measurements. Similar modeling is possible also for absorptive measurements, in which the system is absorbed and the quantum information of the system is lost while the c-number order parameter newly appears. Suppose the field $\phi\left(t, \vec{x}\right)$, characteristic of the apparatus, has support in the spatial domain D in the symmetric phase. This means that the meta-stable spatial region D can accept the injection of a system. A local phase transition reaction at the some place in D completes the measurement. As a result, a special location in D is spontaneously selected, and it characterizes the injection of a system particle. In this case, the process QP is missing among the basic quartet.

3. There are many further applications such as the negative result measurements, quantum Zeno effects, etc., which will be reported in our future publications. It is apparent that, within our present spatially-local measurement model, a complete treatment of them is impossible. However, let us briefly mention an application of our method to the local version of negative result experiment, in which a definite eigenstate of spin is realized even nothing happens on the detector. For this purpose, we modify our previous detector so that it can only react to the up spin state. This is realized by modifying the potential for ϕ in Eq.(34) and set an infinitely tall wall for $\phi \leq -\varepsilon_0$ where ε_0 is a positive and infinitesimally-small constant. Then the order parameter either stays around meta-stable region $\phi \approx 0$ or roles-down toward the positive minimum φ_+. If the initial spin is in the up state, then the biasing and the positive feedback take place as before; the order parameter reads φ_+ and simultaneously the spin is locked-in at up state at the end of the interaction period t_0. On the other hand if the initial spin is in the down state, then the order parameter stays in the meta-stable state $\phi \approx 0$ and nothing happens. Moreover within a short time period t_0, the spin state almost stays within the initial down state leaving out irrelevant tiny quantum decoherence effect. This behavior is also confirmed by our numerical calculations. Thus, negative result measurement is accomplished. The detail in the case of general initial state and further extensions to non-local versions will be reported soon.

4. The application of our formalism to the Universe is very important. Originally, the resolution of the dual structure in quantum mechanics has been strongly desired in the field of cosmology. In the early universe, we especially have to clarify the quantum origin of density fluctuations after the complete leveling out by the inflationary mechanism.[3] The field φ in our model plays a very similar role to the inflaton field. In this cosmological context, we recognize that the relevant quantity is the c-number order parameter, which spontaneously breaks the translational invariance, but not the quantum state after some kind of quantum measurement. In this sense, the problem resembles that for an absorptive measurement because the QP process is lacking.
5. We would like to comment on the Born probability rule in quantum mechanics: After the measurement of the observable $\hat{A}$ in the state $\psi \in \mathcal{H}$, the obtained value is one of the eigenvalues of $\hat{A}$ with the probability $P(a)$ given by $P(a) = \|\wp_a |\psi\rangle\|^2$. In our argument given in §5, we did obtain some information on the probability as well as the von Neumann projection or the generalized POVM. If we adopt the most optimized case with an appropriate calibration, the probability is completely predicted by the thick gold curve in Fig. 5. In this sense, our measurement model claims more than a simple projection. Furthermore, if the parameters satisfy $\gamma \gg \sqrt{g\varepsilon}$ (strong friction) or $\varepsilon \gg \gamma\delta^2$ (strong random force), then we can have the linear relation for the spin-up probability $P_+ = \frac{1}{2} + \alpha\left(2\|\wp_+ |\psi\rangle\|^2 - 1\right)$, where α is a real constant. Moreover if $\alpha = 0.5$, then we have indeed obtain the result$P_+ = \|\wp_+ |\psi\rangle\|^2$, the exact form of the Born probability rule.
6. We would like to make some relation to the mathematical and general treatment of quantum measurement formalism by Ozawa.[1] According to this formalism, any measurement process is characterized by the quartet $\left\{K, \tilde{X}, \sigma, U\right\}$, where K is the Hilbert space of the apparatus, $\tilde{X}$ is an observable on K, σ is the initial state of the apparatus, and U is the unitary evolution operator for the whole system. Actually in our case, K is the Hilbert space of the apparatus field $\hat{\phi}$, $\tilde{X}$ is the order parameter φ, σ is the symmetric vacuum state which is supposed to be unstable or meta-stable, and the time evolution of U is given by the Lagrangian Eq.(34). Thus our present model meets the general framework of quantum measurement and moreover clarifies physical detail to elucidate the fundamental mechanism of the measurement process.
7. For more complete argument, we need at least to answer the following questions: Is the SSB mechanism always associated with the quantum measurement process? How is the exact derivation of the dynamics of an SSB phase transitions possible? Our present model of quantum measurement may also be the mechanism through which macroscopic irreversibility emerges. The problem is whether an SSB phase transition is always associated with the emergence of macroscopic irreversibility. By extending the algebraic method of Ojima[11] to include explicit dynamics, these problems should be addressed sometime in the future for a complete resolution of the problem of quantum measurement.

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