

Λ^* -Hypernuclei in Phenomenological Nuclear ForcesAkiyuki ARAI, Makoto OKA^{*)} and Shigehiro YASUI^{**)}*Department of Physics, Tokyo Institute of Technology,
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The Λ^* -hypernuclei, which are bound states of $\Lambda(1405)$ and nuclei, are studied as a possible interpretation of the \bar{K} -nuclei. The Bonn and Nijmegen potentials are extended and used as a phenomenological potential between Λ^* and N . The K -exchange potential is also considered in the interaction of Λ^* and N . The two-body (Λ^*N) and three-body (Λ^*NN) systems are investigated using a variational method. It is shown that the spin and isospin of the ground states are given by $\Lambda^*N(S = 1, I = 1/2)$ and $\Lambda^*NN(S = 3/2, I = 0)$, respectively. The binding energies of the Λ^* -hypernuclei are compared with experiment.

§1. Introduction

The possibility of kaon-nuclear bound states is of great interest in hadron and nuclear physics. A deeply bound kaonic nuclear state with a relatively small decay width was recently predicted and further studied in the literature.^{1)–4)} Possible high density matter caused by the strong kaon attractive force is also a subject of much study.^{1)–4)} In order to find such exotic states, several experimental searches have been carried out.^{5),6)} The FINUDA collaboration reported the observation of bound ppK^- state at a binding energy of 115 MeV.⁶⁾ The interpretation of the peak found in this experiment is still under discussion, while some advanced calculations, such as a 3-body calculation employing the Faddeev equation with $\bar{K}N - \pi\Sigma$ channel coupling, have been performed.^{7),8)} Other approaches include an interpretation as a nine-quark state studied in the MIT bag model,⁹⁾ and a kaon absorption process between two nucleons.¹⁰⁾ While the J-PARC facility will certainly answer to the question whether such deeply bound kaonic nuclear states exist, we need to study such a system in a more general context.

The purpose of this paper is to give a new interpretation of the “kaon-nucleus” states. We consider Λ^* -hypernuclear states. Λ^* is the lowest negative parity baryon, with a mass around 1405 MeV and strangeness -1 . This baryon is in many senses unique. For instance, it is below any other non-strange baryons with negative parity and is isolated, not forming an octet in $SU(3)$. If it is assumed to be a p -wave baryon with spin $1/2$, then it requires a large spin-orbit splitting. Its uniqueness has attracted a lot of attention, and various exotic views of Λ^* have been proposed.

Here, we do not consider the specific composition of Λ^* , but simply assume that it belongs to a flavor singlet representation, and is thus isolated. Our claim is that the so-called kaon-nuclear bound states can be interpreted as a bound state of Λ^*

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in a nucleus or a Λ^* -hypernucleus. Then, the FINUDA observation is regarded as a two-body bound state of Λ^* and N , whose binding energy is about 88 MeV. Larger systems, such as strange tribaryons could be a Λ^*NN system, and so on.

In this paper, we construct a model of the Λ^*N interaction according to the one-boson exchange model and consider the two-body (Λ^*N) and three-body (Λ^*NN) systems. As the NN interaction, we choose the same one-boson exchange model, the Bonn potential¹¹⁾ and the Nijmegen soft-core potential models.^{12),13)} We extend these models to the Λ^*N systems, and the K -exchange is also included in the same manner between Λ^* and N . We then solve the two-body and three-body Schrödinger equations using a variational method.

The content of the paper is as follows. In §2, we construct the potential model for the Λ^*N system. The features of the model in the context of possible quantum numbers of the Λ^* -nuclear states are also given. In §3, the numerical results for the binding energies of two-body and three-body bound states are presented. In §4, some discussion of the present results is given. A conclusion is given in §5.

§2. Model

The phenomenological nuclear force has been quite successful in the study of nuclear systems. In order to formulate the model for the Λ^* -hypernuclei, we investigate the phenomenological Λ^*N interaction. The nuclear forces among hyperons are not yet established, although many studies have been carried out both experimentally and theoretically. In general, it is considered that the interaction between baryons can be described by the exchange of mesons supplemented by a phenomenological short-range repulsion. In the present study, based on the one-boson exchange picture, we extend the phenomenological nuclear forces between the NN pair to the Λ^*N pair interaction. In the Bonn¹¹⁾ and Nijmegen potentials,^{12),13)} scalar (σ , a_0), pseudoscalar (π , η) and vector (ω , ρ) bosons are exchanged by the NN pair. The explicit equations and the parameter sets for the Bonn potential are given in the appendix. By considering the $SU(3)$ symmetry, we assume that these potentials are also applied to the Λ^*N pair. Here the isovector mesons (a_0 , π , ρ) are irrelevant to the Λ^*N interaction, since the Λ^* is an isosinglet. Instead, the K (and \bar{K}) meson appears in the exchange process $\Lambda^*N \rightarrow N\Lambda^*$. Then, the Λ^*N potential is given by the sum

$$V_{\Lambda^*N} = V_\sigma + V_\eta + V_\omega + V_K. \quad (2.1)$$

The σ -exchange potential, V_σ , is the most attractive force, and the ω -exchange potential, V_ω , plays an essential role in determining the spin, as we see below. In the following, we investigate several possibilities for the Λ^*N potential.

In the extended phenomenological nuclear forces to the Λ^*N pair, there are some unknown parameters: the coupling constants g_{Λ^*NM} (for the M meson) and the momentum cutoffs, which appear in the form factors. Among them, the $\Lambda^*N\bar{K}$ vertex constant, $g_{\Lambda^*N\bar{K}}$, is determined by the $SU(3)$ relation from the observed decay width of the $\Lambda^* \rightarrow \Sigma\pi$ channel. The $\Lambda^*\Sigma\pi$ coupling $g_{\Lambda^*\Sigma\pi}^2/4\pi = 0.064$ is obtained from the decay width $\Gamma(\Lambda^* \rightarrow \Sigma\pi) = 50.0 \pm 2.0$ MeV. Assuming that Λ^* is a purely

$SU(3)$ singlet state, the $SU(3)$ isoscalar factor

$$(\Lambda^*) = (N\bar{K} \ \Sigma\pi \ \Lambda\eta \ \Xi K) = \frac{1}{\sqrt{8}}(2 \ 3 \ -1 \ -2)^{1/2} \quad (2.2)$$

leads to the prediction $g_{\Lambda^*NK}^2/4\pi = g_{\Lambda^*\Sigma\pi}^2/4\pi = 0.064$ for the coupling constant. It should be noted that this coupling constant is much smaller than the $NN\sigma$ coupling constant, $g_{NN\sigma}^2/4\pi = 7.78$. Therefore, the K meson plays only a minor role in the Λ^*N bound state, as seen in the results of the numerical calculation, given below.

Here, the K -exchange has to be treated carefully. Because Λ^* lies close to the $\bar{K}N$ threshold, the kaon may have a four momentum q_μ near the on-mass-shell values. Then the kaon propagator can be strongly enhanced. In order to include this effect, we replace the kaon mass by an effective mass. Assuming that the baryons are static, we obtain a finite energy transfer, $q_0 \simeq M_{\Lambda^*} - M_N$, and the effective mass

$$\tilde{m}_K \equiv \sqrt{m_k^2 - q_0^2} \simeq \sqrt{m_K^2 - (M_{\Lambda^*} - M_N)^2} = 171 \text{ MeV}/c^2. \quad (2.3)$$

This replacement indeed enhances the Λ^*N potential more strongly than in the case of the ΛN or ΣN potential. This is because the $\Lambda^*N\bar{K}$ coupling is a scalar coupling, and the effect of \tilde{m}_K appears only in the range of the potential. By contrast, a p -wave coupling gives rise to an extra \tilde{m}_K^2 factor, which significantly suppresses its effect on the ΛN or ΣN potential. As a result, the K -exchange potential for the Λ^*N pair is given by the coupling constant $g_{\Lambda^*NK}^2/4\pi = 0.064$ and the effective mass \tilde{m}_K given in Eq. (A.1) of the Appendix.

The other parameters in the Λ^*N interaction are not yet fixed, due to a lack of experimental information. In the present study, we treat $g_{\Lambda^*\Lambda^*\sigma}$ as a free parameter, because the results happen to be most sensitive to the σ -exchange. We further assume that the other parameters are the same as those in the NN interaction, for simplicity.

Here, it is important for subsequent discussion to elucidate the properties of the Λ^*N interaction. Let us first consider the spin dependence of the Λ^*N interaction. The spin-spin interaction is induced by the η , K and ω mesons. We consider the ω -exchange force, because the ω meson coupling is much stronger than the η and K meson couplings. The static ω -exchange potential is written in the momentum space as

$$\begin{aligned} V_\omega(\vec{k}^2) &= -C \frac{(\vec{\sigma}_1 \cdot \vec{\sigma}_2) \vec{k}^2}{\vec{k}^2 + m_\omega^2} \\ &= -C \left(1 - \frac{m_\omega^2}{\vec{k}^2 + m_\omega^2} \right) (\vec{\sigma}_1 \cdot \vec{\sigma}_2), \end{aligned} \quad (2.4)$$

with a positive constant C , and the ω meson mass m_ω . In real space, the first term is a delta-type function, while the second term is the standard Yukawa potential. With a form factor introduced, the delta-type potential becomes a finite range potential. For the Yukawa-type potential, the spin singlet is more attractive than the spin triplet. Contrastingly, for the delta-type potential, the spin triplet is more attractive than the spin singlet. Usually, the delta-type potential at short distance

plays a minor role, due to the strong repulsive core of the NN force, and hence the Yukawa potential is much more important. However, this is not the case for the Λ^*N potential. There, at short distances, the delta function overwhelms the Yukawa-type potential. Consequently, in the Λ^*N potential, the spin triplet potential is more attractive at short distances than the spin singlet potential.

Now we consider the possible quantum states of the Λ^* -hypernuclei, which are obtained in the spin and isospin combinations. It is assumed that all the particles occupy the lowest energy s -orbit. Because Λ^* has spin $S = 1/2$ and isospin $I = 0$, the Λ^*N system has two states of spin and isospin, $(S = 0, I = 1/2)$ and $(S = 1, I = 1/2)$, whereas the Λ^*NN system has the isosinglet and isotriplet states. The isosinglet state has spin $S = 1/2$ and $S = 3/2$ states, where the NN pair is an isosinglet and spin triplet. The isotriplet state has spin $S = 1/2$, where the NN pair is isotriplet and spin singlet. Therefore, the possible quantum numbers of the s -wave Λ^*NN system are $(S = 1/2, I = 0)$, $(S = 3/2, I = 0)$ and $(S = 1/2, I = 1)$.

Here, we elucidate the quantum numbers of the ground states of the Λ^* -hypernuclei, using the spin dependence of the ω -exchange at short distance. It is expected that the ground state of the two-body system (Λ^*N) is a spin triplet. In order to understand the ground state of the three-body system (Λ^*NN), the three possible states, $(S = 1/2, I = 0)$, $(S = 3/2, I = 0)$ and $(S = 1/2, I = 1)$, are expanded in terms of the Λ^*N pair as

$$|\Lambda^*(NN)_{S=1}; S = 1/2, I = 0\rangle = \frac{1}{2}|(\Lambda^*N)_{S=1}N\rangle + \frac{\sqrt{3}}{2}|(\Lambda^*N)_{S=0}N\rangle, \quad (2.5)$$

$$|\Lambda^*(NN)_{S=1}; S = 3/2, I = 0\rangle = |(\Lambda^*N)_{S=1}N\rangle \quad (2.6)$$

for the isosinglet states and as

$$|\Lambda^*(NN)_{S=0}; S = 1/2, I = 1\rangle = \frac{\sqrt{3}}{2}|(\Lambda^*N)_{S=1}N\rangle - \frac{1}{2}|(\Lambda^*N)_{S=0}N\rangle \quad (2.7)$$

for the isotriplet state. Among these states, the spin triplet interaction is contained most strongly in the $(S = 3/2, I = 0)$ state, because in this case, the values of the coefficient of the $(\Lambda^*N)_{S=1}$ component are largest. Therefore, it is expected that the $(S = 3/2, I = 0)$ is the ground state for the Λ^*NN state. In the next section, it is shown that the above analysis is consistent with the numerical results.

The above conclusion concerning the spin of the two-body and three-body states is in strong contrast with obtained using the kaon bound state approach. In that case, the spin dependence is induced by the isospin dependence of the $\bar{K}N$ interaction. For instance, in the $\bar{K}NN$ system, the $(\bar{K}N)_{I=0}$ interaction is the driving force for the bound state. It is easy to see that the $\bar{K}(NN)_{I=1, S=0}$ state contains a larger $(\bar{K}N)_{I=0}$ component than the $\bar{K}(NN)_{I=0, S=1}$ state does. Thus, the ground state of $\bar{K}NN$ is expected to have $S = 0$.

§3. Numerical results

In this section, we study the numerical results for the two-body (Λ^*N) and three-body (Λ^*NN) systems by solving the Schrödinger equation with the Bonn¹¹⁾ and

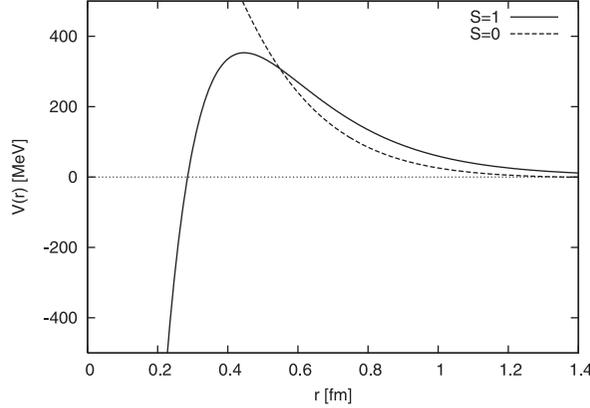


Fig. 1. The Bonn potentials applied to the Λ^*N pair in the $S = 0$ (dashed curve) and $S = 1$ (solid curve) channels for the coupling constants $g_{\Lambda^*\Lambda^*\sigma}/g_{NN\sigma}=0.39$.

Nijmegen (SC89¹²) and ESC04¹³) potentials for the Λ^*N and NN interactions. For simplicity, derivative terms are not considered. We mainly present the results for the Bonn potential. The results do not differ qualitatively for the Nijmegen potentials with SC89 and ESC04.

3.1. Two-body system

The two-body system (Λ^*N) has spin singlet ($S = 0, I = 1/2$) and triplet ($S = 1, I = 1/2$) states. In Fig. 1, the Bonn potentials are plotted as functions of the relative distance r between Λ^* and N in the $S = 0$ and $S = 1$ channels for the coupling constant $g_{\Lambda^*\Lambda^*\sigma}/g_{NN\sigma} = 0.39$. The $S = 1$ potential is strongly attractive at short distance ($r \lesssim 0.4$ fm), while the $S = 0$ potential is repulsive. This difference is caused by the delta-type interaction in the ω potential (2.4), which is attractive for $S = 1$ and repulsive for $S = 0$. It should be noted that the delta function is smeared and becomes a finite range potential due to the form factor in phenomenological nuclear forces.

In order to see the contributions from individual mesons in the $S = 1$ Λ^*N potential, the components from the σ , ω and η -exchange potentials and the additional K -exchange potentials are plotted in Fig. 2. The σ -exchange potential is attractive at medium range ($r \gtrsim 0.4$ fm), while the ω potential is repulsive. However, the ω potential is strongly attractive at shorter distances ($r \lesssim 0.4$ fm), due to the delta function in (2.4). Hence, their sum is a deeply attractive potential at short distance, which is one of the characteristic properties of the Λ^*N potential. The contributions from the η and K mesons are relatively weak.

Here, we investigate the on-shell effect of the K -exchange. As we have already discussed, the K meson propagating between the Λ^* and N may have a large energy, and thus can be close to the on-mass-shell kinematics. Such a K exchange might be largely enhanced. We treat this effect as the effective mass \tilde{m}_K defined in Eq. (2.3). In Fig. 3, the K -exchange potential is plotted in the case of the bare mass $m_K = 495$ MeV (solid curve) and the reduced mass $\tilde{m}_K = 171$ MeV (dashed curve). As the

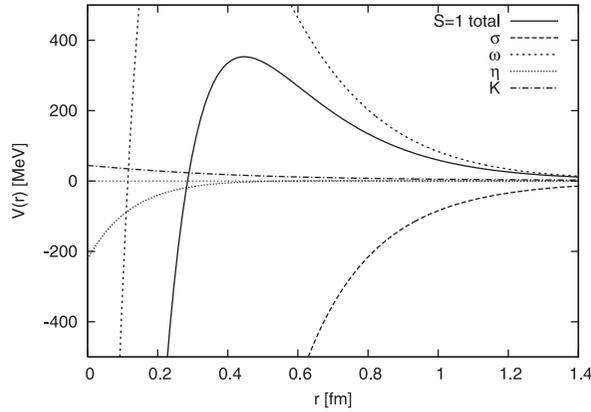


Fig. 2. The components from the σ , ω , η and K mesons in the Bonn potential between the Λ^*N pair for $S = 1$ for $g_{\Lambda^*\Lambda^*\sigma}/g_{NN\sigma}=0.39$. The Bonn potential is indicated by the solid curve.

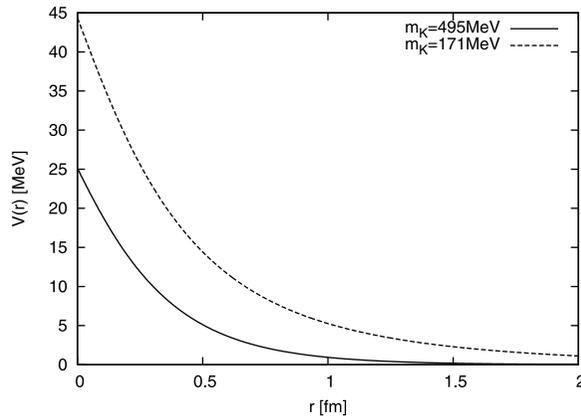


Fig. 3. The K -exchange component in the Λ^*N potential. The solid curve represents the bare mass, $m_K = 495$ MeV, and the dashed curve represents for the reduced mass, $\hat{m}_K = 171$ MeV, see the text.

effective K meson mass decreases, both the potential strength and the range increase. However, this enhancement has little effect on the binding energy of Λ^*N , since the K -exchange potential is still weaker than the σ and ω -exchange potentials.

Solving the Schrödinger equation for Λ^*N with the Bonn potential, we obtain the binding energy and the wave function of the Λ^*N bound states. As a set of trial wave functions, we use a linear combination of Gaussian functions:

$$\psi(r) = \sum_{i=1}^n a_i e^{-\alpha_i r^2}. \quad (3.1)$$

Here the quantities a_i are the variational parameters, α_i the width parameters, and r the distance between Λ^* and N . As a result, the binding energy depends strongly on the coupling constant $g_{\Lambda^*\Lambda^*\sigma}/g_{NN\sigma}$, as shown in Fig. 4. The σ potential plays an essential role in the formation of the bound state. Indeed, without the $\Lambda^*\Lambda^*\sigma$ coupling, the bound state does not exist. For $S = 1$, Λ^*N is bound in the case

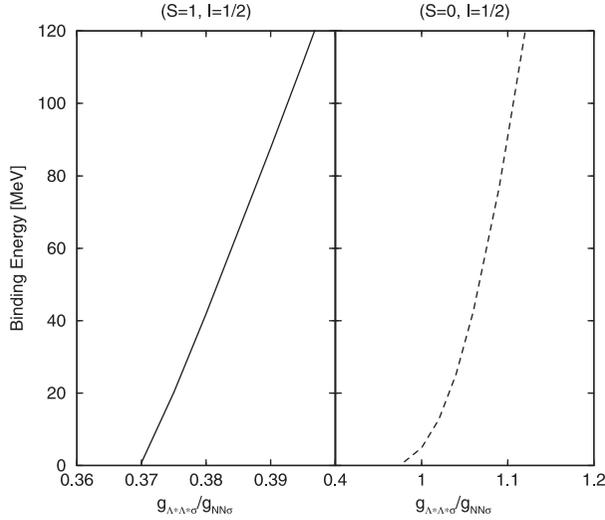


Fig. 4. The binding energy of Λ^*N as functions of the coupling constant $g_{\Lambda^*\Lambda^*\sigma}/g_{NN\sigma}$ for the Bonn potential. The solid curve is for $S = 1$ and the dashed curve for $S = 0$.

$g_{\Lambda^*\Lambda^*\sigma}/g_{NN\sigma} \gtrsim 0.37$.

At $g_{\Lambda^*\Lambda^*\sigma}/g_{NN\sigma} = 0.39$, the binding energy reaches 88 MeV. The observed binding energy, 115 MeV, of the ppK^- state reported by the FINUDA collaboration⁶⁾ is interpreted as 88 MeV for the binding energy of the Λ^*N state. For $S = 0$, a stronger coupling of $g_{\Lambda^*\Lambda^*\sigma}/g_{NN\sigma} \gtrsim 0.98$ is required for the bound state. A binding energy of 88 MeV is obtained at $g_{\Lambda^*\Lambda^*\sigma}/g_{NN\sigma} = 1.01$. With the extent of the present experiment results, the quantum number of the bound state is not yet known. Our result suggests an $S = 1$ bound state rather than $S = 0$. The probability density of Λ^*N with $S = 1$ is plotted in Fig. 5. The wave function is very compact in comparison with the nucleon size, suggesting that the obtained bound state is produced mainly by the short-range ω attraction, with a smaller contribution of the medium-range σ -exchange, which cancels the medium range ω repulsion.

To this point, we have discussed the results obtained using the Bonn potential. In order to check the model dependence, we consider different types of nuclear forces. In particular, as the short-range part of the interaction is important, the mechanisms of short-range NN repulsion must be investigated. The Bonn potential acquires a short-range repulsion mainly from the ω -exchange potential, while the other models, such as the Nijmegen potential, introduce a new component for the repulsion.

The pomeron exchange is represented in the Nijmegen potential by a Gaussian potential with short range. In the case of the NN interaction, it is strong enough to expel the wave functions away from the center. When we apply the same repulsion to the Λ^*N system, we find that the short-range attraction from ω -exchange may still create a bound state if the σ -exchange attraction at medium range is sufficiently strong. However, compared with the Bonn potential, the extra pomeron repulsion for Λ^*N makes the system less bound. This implies that the required $g_{\Lambda^*\Lambda^*\sigma}$ coupling constant becomes larger.

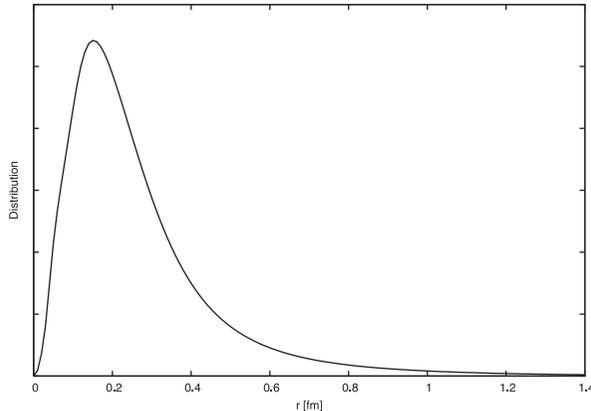


Fig. 5. The probability density $|\psi(r)|^2 r^2$ of the bound $\Lambda^* N$ system with $S = 1$ and $g_{\Lambda^* \Lambda^* \sigma} / g_{NN\sigma} = 0.39$. The binding energy is 88 MeV. The horizontal axis represents the distance between Λ^* and N .

Table I. Ratios of the coupling constants $g_{\Lambda^* \Lambda^* \sigma} / g_{NN\sigma}$ required to obtain the binding energy 88 MeV for the $\Lambda^* N$ two-body system.

$\Lambda^* N$	Bonn	Nijmegen (SC89)	Nijmegen (ESC04)	B.E. [MeV]
$S = 1$	0.39	0.832	1.118	88
$S = 0$	1.1	1.128	1.398	88

We employ the versions SC89 and ESC04 of the Nijmegen potential. It is found that SC89 requires the minimal coupling constants $g_{\Lambda^* \Lambda^* \sigma} / g_{NN\sigma} = 0.77$ and $g_{\Lambda^* \Lambda^* \sigma} / g_{NN\sigma} = 0.98$ to form a $\Lambda^* N$ bound state for $S = 1$ and $S = 0$, respectively. The binding energy (88 MeV) reported by the FINUDA group is obtained by setting $g_{\Lambda^* \Lambda^* \sigma} / g_{NN\sigma} = 0.832$ for $S = 1$ and $g_{\Lambda^* \Lambda^* \sigma} / g_{NN\sigma} = 1.128$ for $S = 0$. The probability density is pushed away from the center in comparison with the Bonn potential. This is because the range of the $\Lambda^* N$ potential is longer than that of the Bonn potential.

The result obtained using ESC04 is qualitatively the same as that obtained using SC89. The difference is the absolute values of the σ , ω and pomeron exchange potentials in ESC04 are smaller than those in SC89. For this reason, the minimum coupling constants of $\Lambda^* \Lambda^* \sigma$ become larger, and the experimental binding energy is reproduced at $g_{\Lambda^* \Lambda^* \sigma} / g_{NN\sigma} = 1.118$ for $S = 1$ and $g_{\Lambda^* \Lambda^* \sigma} / g_{NN\sigma} = 1.398$ for $S = 0$.

The results discussed above are listed in Table I. The ratios of the coupling constants $g_{\Lambda^* \Lambda^* \sigma} / g_{NN\sigma}$ that correspond to the experimental observation in the FINUDA collaboration are listed. A smaller value indicates that the system can form a bound state more easily. We find that in all cases that the coupling constant in the $S = 1$ bound state is smaller than that in the $S = 0$ one in the Bonn and Nijmegen potentials. Therefore, we conjecture that the lowest-energy bound state is the $S = 1$ state.

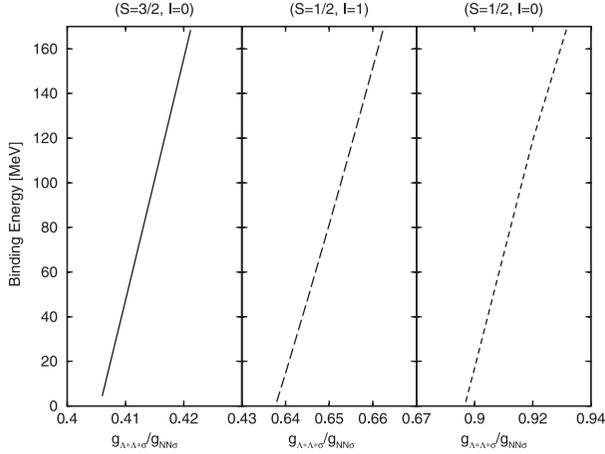


Fig. 6. The binding energy of Λ^*NN as functions of the coupling constant $g_{\Lambda^*\Lambda^*\sigma}/g_{NN\sigma}$ for the Bonn potential. The solid, long-dashed and short-dashed curves represent the $(S = 3/2, I = 0)$, $(S = 1/2, I = 1)$ and $(S = 1/2, I = 0)$ states, respectively.

3.2. Three-body system

Now we discuss the three-body system (Λ^*NN). Here, the binding energies and wave functions are obtained variationally by solving the Schrödinger equation, using the phenomenological NN and Λ^*N potentials. The trial wave function is given by

$$\psi(r, r') = \sum_{i,j} a_{ij} e^{-\alpha_i r^2} e^{-\beta_j r'^2}, \quad (3.2)$$

where a_{ij} are the variational parameters, α_i and β_j the width parameters, and r and r' the Jacobi coordinates for Λ^*NN . Then, we obtain the binding energies as functions of the coupling constant $g_{\Lambda^*\Lambda^*\sigma}/g_{NN\sigma}$ for the $(S = 3/2, I = 0)$, $(S = 1/2, I = 1)$ and $(S = 1/2, I = 0)$ states plotted by the solid, long-dashed, short-dashed curves, respectively, in Fig. 6. We find that a bound $(S = 3/2, I = 0)$ state can be formed only for $g_{\Lambda^*\Lambda^*\sigma}/g_{NN\sigma} \gtrsim 0.406$, while the $(S = 1/2, I = 1)$ and $(S = 1/2, I = 0)$ bound states require a stronger coupling constant. Therefore, our analysis shows that the $(S = 3/2, I = 0)$ state is the ground state of the Λ^*NN system. Note that for the Λ^*N pair, the $S = 1$ channel is more attractive than the $S = 0$ channel. From the Λ^*NN states written as combinations of $(\Lambda^*N)_{S=1}N$ and $(\Lambda^*N)_{S=0}N$ as in Eq. (2.6) and (2.7), it is seen that the $(S = 3/2, I = 0)$ state contains a larger $(\Lambda^*N)_{S=1}$ component.

From the discussion given above, it is clear that the short-distance behavior of the nuclear potential plays an important role. We recall that at short distances, the smeared delta interaction in the ω -exchange of the Λ^*N interaction overwhelms the Yukawa-type potential, and it induces a strong attractive potential for $S = 1$. This contrasts with the long-range behavior of the ω -exchange potential, which is repulsive for $S = 1$ and attractive for $S = 0$. A strong attractive potential at short distances is also seen in the three-body system. This is directly observed from the probability densities of Λ^*N and Λ^*NN , which are concentrated at short distances.

Table II. The ratios of the coupling constants $g_{\Lambda^* \Lambda^* \sigma} / g_{NN\sigma}$ required to obtain the binding energy 167 MeV for the $\Lambda^* NN$ three-body system.

$\Lambda^* NN$	Bonn	Nijmegen (SC89)	Nijmegen (ESC04)	B.E. [MeV]
$(S = 3/2, I = 0)$	0.424	0.898	1.217	167
$(S = 1/2, I = 1)$	0.64	0.987	1.325	167

§4. Discussion

Here, we compare our results with those of some other studies of the same system. In the original Akaishi-Yamazaki picture,^{1)–3)} the binding energy of $ppn\bar{K}^-$ has been shown to be larger than $pp\bar{K}^-$. Furthermore, it has been found that \bar{K} -nuclei can be stable for larger baryon numbers. However, this is not the case for the Λ^* -hypernuclei. Let us now compare the minimum coupling constants for the two-body (Λ^*N) and three-body (Λ^*NN) bound systems. For example, for the Bonn potential, the coupling $g_{\Lambda^* \Lambda^* \sigma} / g_{NN\sigma} = 0.37$ gives a bound state of Λ^*N with $(S = 1, I = 1/2)$, while $g_{\Lambda^* \Lambda^* \sigma} / g_{NN\sigma} = 0.406$ is required for Λ^*NN with $(S = 3/2, I = 0)$. Therefore, the present framework leads to the conclusion that the Λ^*N two-body state is bound more easily than the Λ^*NN three-body state. This is qualitatively the same for the Nijmegen potentials with SC89 and ESC04. Furthermore, it may be expected that the Λ^* -hypernuclei become unbound as the baryon number increases. This picture contradicts sharply with that for the \bar{K} -nucleus bound states.^{1)–3)}

A recent proposal¹⁵⁾ for interpreting the binding of \bar{K} in terms of the “migration” of \bar{K} is in fact quite similar to the K -exchange part of our picture. The difference between the analysis in Ref. 15) and ours mainly regards the coupling strengths. We determine the coupling constant so as to reproduce the relatively narrow width of Λ^* , while Ref. 15) employs a coupling to reproduce the binding energy of Λ^* as a $\bar{K}N$ bound state. It should be noted that the K exchange is attractive in the S -wave Λ^*N ($S = 0$) state, while it is repulsive for the $S = 1$ state. When the K exchange is supposed to be the driving force for the bound state, the K migration picture prefers the $S = 0$ $\bar{K}NN$ states. Further analysis of the comparison is needed.

We next give a comment concerning the dependence of the result on the coupling constant in the phenomenological nuclear potentials for Λ^*N . The coupling constant $g_{\Lambda^* \Lambda^* \sigma}$ is taken as a free parameter, and the couplings for the other mesons are fixed to the same values as in the NN interaction. The σ potential is the most attractive force among them, and the ω potential is the second strongest force. Therefore, as a further step, it would be interesting to alter the coupling strength of $\Lambda^* \Lambda^* \omega$. However, concerning the spin of the bound state, the result will not be changed as long as the ω -exchange potential is attractive at short range. This is because the spin of the Λ^*N pair is almost completely determined by the ω -exchange potential, not by the other mesons.

Lastly, we note that in the present framework, only the meson exchange potential has been considered as the interaction between Λ^* and N . However, the resulting bound states are compact where the substructure of the baryons should be important. In the present analysis, we have introduced form factors representing the baryon structure. It may be important to consider the quark structure of baryons

explicitly so that the short range baryon-baryon interactions are correctly taken into account.¹⁴⁾ This subject is left for a future work.

§5. Conclusion

The possibility of a Λ^* -hypernuclei was investigated by considering Λ^* as a compound state in kaonic nuclei. The two-body (Λ^*N) and three-body (Λ^*NN) systems were investigated by using phenomenological nuclear forces. Based on the one-boson exchange picture, the Bonn and Nijmegen (SC89 and ESC04) potentials were extended to the Λ^*N interaction. The K exchange was also included in the extended nuclear forces. The $g_{\Lambda^*N\bar{K}}$ coupling was determined from the experimental value of the decay width of the Λ^* . Only the $\Lambda^*\Lambda^*\sigma$ coupling constant is left as a free parameter, with the other parameters fixed.

We found that for the two-body system (Λ^*N), an appropriate $\Lambda^*\Lambda^*\sigma$ coupling constant reproduces the binding energy of the ppK^- which is comparable to the value reported by the FINUDA collaboration. The most stable state in the Λ^*N bound state is the $S = 1$ state. For the three-body system (Λ^*NN), the ($S = 3/2, I = 0$) state is the most stable state. These results can be understood by considering the fact that the ω -exchange potential is strongly attractive at short distances for $S = 1$ channel rather than for $S = 0$. There, the K exchange plays only a minor role. It was obtained that the minimum coupling constant $g_{\Lambda^*\Lambda^*\sigma}$ in Λ^*N was smaller than that in Λ^*NN . Therefore, the Λ^*N state is more easily produced than the Λ^*NN state. Our conclusion is qualitatively the same for the Bonn and Nijmegen (SC89 and ESC04) potentials.

In the present study, Λ^* is regarded as a stable particle. In reality, however, multi-channel decays are open for Λ^* . The decay to $\Sigma\pi$ is the main source of Λ^* free decay, while the in-medium decays $\Lambda^*N \rightarrow \Lambda N, \Sigma N$ are new and interesting. In our study, the bound Λ^*N and Λ^*NN states are very compact objects. Therefore, the conversion width to ΛN or ΣN may be modified. These subjects are closely related to experimental studies of DAΦNE and J-PARC. Further investigation is left to future works.

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Appendix

Here, the Bonn potential used in the present paper is explicitly presented in a coordinate formalism.¹¹⁾ In the following, m is the mass of the meson, and g_{ij} and f_{ij} are the coupling constants in the reaction process $1+2 \rightarrow 3+4$, with the baryon mass M_1 and M_2 . The parameter values are listed in Table III. The pseudoscalar-type

potential is given by

$$V_{\text{ps}}(m, r) = \frac{g_{13}g_{24}}{4\pi} \frac{m^2}{4M_1M_2} m \left[\frac{1}{3} \vec{\sigma}_1 \cdot \vec{\sigma}_2 \phi(mr) + S_{12} \chi(mr) \right]. \quad (\text{A}\cdot 1)$$

The scalar-type potential is given by

$$V_{\text{s}}(m, r) = -\frac{g_{13}g_{24}}{4\pi} \left(1 - \frac{m^2}{8M_1M_2} \right) \phi(mr). \quad (\text{A}\cdot 2)$$

The vector-type potential is given by

$$\begin{aligned} & V_{\text{v}}(m, r) \\ &= \frac{m}{4\pi} \left[\left\{ g_{13}g_{24} \left(1 + \frac{m^2}{8M_1M_2} \right) + g_{13}f_{24} \frac{m^2}{4MM_1} \right. \right. \\ &\quad \left. \left. + f_{13}g_{24} \frac{m^2}{4MM_2} + f_{13}f_{24} \frac{m^4}{16M^2M_1M_2} \right\} \phi(mr) \right. \\ &\quad \left. + \frac{m^2}{4M_1M_2} \left\{ g_{13}g_{24} + g_{13}f_{24} \frac{M_2}{M} + f_{13}g_{24} \frac{M_1}{M} + f_{13}f_{24} \frac{M_1M_2}{M^2} \left(1 + \frac{m^2}{8M_1M_2} \right) \right\} \right. \\ &\quad \left. \left(\frac{2}{3} (\vec{\sigma}_1 \cdot \vec{\sigma}_2) \phi(mr) - S_{12} \chi(mr) \right) \right]. \end{aligned} \quad (\text{A}\cdot 3)$$

In the above equations, we define

$$\phi(x) = \frac{e^{-x}}{x}, \quad (\text{A}\cdot 4)$$

$$\chi(x) = \left(\frac{m}{M} \right)^2 \left(1 + \frac{3}{x} + \frac{3}{x^2} \right) \phi(x), \quad (\text{A}\cdot 5)$$

$$S_{12} = 3 \frac{(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{r^2} - \vec{\sigma}_1 \cdot \vec{\sigma}_2, \quad (\text{A}\cdot 6)$$

with the propagating meson mass m . In the case that the exchanged meson possesses isospin, the quantify $\tau_1 \cdot \tau_2$ is multiplied. The form factor is introduced for each meson by making the replacement

$$V_{\alpha}(m, r) \rightarrow V_{\alpha}(m, r) - \frac{\Lambda_2^2 - m^2}{\Lambda_2^2 - \Lambda_1^2} V_{\alpha}(\Lambda_1, r) + \frac{\Lambda_1^2 - m^2}{\Lambda_2^2 - \Lambda_1^2} V_{\alpha}(\Lambda_2, r), \quad (\text{A}\cdot 7)$$

where

$$\Lambda_1 = \Lambda + \epsilon, \quad \Lambda_2 = \Lambda - \epsilon, \quad (\text{A}\cdot 8)$$

with $\epsilon/\Lambda \ll 1$, for example, $\epsilon \approx 10$ MeV.

Table III. The parameter values for the Bonn potential.¹¹⁾ (The superscripts *a* and *b* indicate the *NN* pairs with isospin *T* = 1 and *T* = 0, respectively.)

Meson	Mass (MeV)	$g^2/4\pi$	$f^2/4\pi$	Λ (GeV)
π	138.03		14.9	1.3
η	548.8	3		1.5
ρ	769	0.95	5.8	1.3
ω	782.6	20		1.5
a_0	983	2.6713		2.0
σ	550 ^a	7.7823 ^a		2.0
	715 ^b	16.2061 ^b		2.0

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