On the Solution of the N/D Method

Toshimi ADACHI*
and Tsuneyuki KOTANI**

* *The Tokyo Metropolitan Technical College, Shinagawa, Tokyo*

** *Department of Physics Tokyo Metropolitan University Setagaya, Tokyo*

May 15, 1964

Recently, Shaw[1] proposed a useful approximate solution to the N/D equations.

His proposed solution is independent of a subtraction point and is symmetric in the multi-channel problem. It, however, seems that his procedure contains an obscure approximation to derive an expression for the partial wave amplitude. In this note we derive an expression for the amplitude in the N/D method. The meaning of Shaw's approximate solution becomes clearer from this alternate expression.

We consider the usual N/D equations for the case where there are n-coupled two body channels; n by n matrices are denoted hereafter by the boldface. According to Shaw's notation,[1] the partial wave amplitude $\boldsymbol{A}(s)$ is defined by $\boldsymbol{A}=\boldsymbol{N}\boldsymbol{D}^{-1}$. In terms of the "generalized potential" $\boldsymbol{B}(s)$ which is regular in the physical region, the N and D equations are[2]

$$\boldsymbol{D}(s, s_0)=\boldsymbol{M}(s, s_0)-i\boldsymbol{\theta}\boldsymbol{\rho}(s)\boldsymbol{N}(s, s_0), \quad (1)$$

$$\boldsymbol{N}(s, s_0)=\boldsymbol{B}(s)\boldsymbol{M}(s, s_0)+\frac{\mathrm{P}}{\pi}\int_0^\infty \boldsymbol{B}(s')\boldsymbol{\theta}\boldsymbol{\rho}(s')\boldsymbol{N}(s', s_0)\frac{ds'}{s'-s}, \quad (2)$$

where

$$\boldsymbol{M}(s, s_0)=\boldsymbol{1}-\frac{(s-s_0)}{\pi}\mathrm{P}\int_0^\infty \boldsymbol{\theta}\boldsymbol{\rho}(s')\boldsymbol{N}(s', s_0)\times\frac{ds'}{(s'-s)(s'-s_0)}, \quad (3)$$

and $\boldsymbol{\theta}$ ensures that the right-hand cuts start at the appropriate thresholds, s_0 is a subtraction point and $\boldsymbol{\rho}$ is the diagonal matrix depending on kinematics. Eliminating $\boldsymbol{M}(s, s_0)$, we obtain the other expression for the $\boldsymbol{D}$-matrix as follows:

$$\boldsymbol{D}(s, s_0)=\boldsymbol{B}(s)^{-1}\boldsymbol{N}(s, s_0)-\boldsymbol{B}(s)^{-1}\frac{\mathrm{P}}{\pi}\int_0^\infty \boldsymbol{B}(s')\boldsymbol{\theta}\boldsymbol{\rho}(s')\boldsymbol{N}(s', s_0)\frac{ds'}{s'-s}-i\boldsymbol{\theta}\boldsymbol{\rho}(s)\boldsymbol{N}(s, s_0). \quad (4)$$

Thus we get

$$\boldsymbol{A}(s)=\Big[\boldsymbol{B}(s)^{-1}-\boldsymbol{B}(s)^{-1}\frac{\mathrm{P}}{\pi}\int_0^\infty \boldsymbol{B}(s')\boldsymbol{\theta}\boldsymbol{\rho}(s')\boldsymbol{N}(s', s_0)\times\frac{ds'}{s'-s}\boldsymbol{N}(s, s_0)^{-1}-i\boldsymbol{\theta}\boldsymbol{\rho}(s)\Big]^{-1}. \quad (5)$$

It is worthwhile to note that the above expression Eq. (5) is independent of s_0, because $\boldsymbol{N}(s, s_0)$ can be expressed in the from

$$\boldsymbol{N}(s, s_0)=\boldsymbol{n}(s)\boldsymbol{m}(s_0)^{-1}. \quad (6)$$

This factorization can be proved from the fact that, for example, the following relations for $\boldsymbol{N}(s, s_0)$ and $M(s, s_0)$ are derived from Eqs. (2) and (3):[3]

$$\boldsymbol{N}(s, t)=\boldsymbol{N}(s, u)\boldsymbol{M}(u, t), \quad (7)$$

$$\boldsymbol{M}(s, t)=\boldsymbol{M}(s, u)\boldsymbol{M}(u, t). \quad (8)$$

Then, in order to satisfy these relations, the necessary and sufficient condition is that $\boldsymbol{N}(s, t)=\boldsymbol{n}(s)\boldsymbol{m}(t)^{-1}$ and $\boldsymbol{M}(s, t)=\boldsymbol{m}(s)\boldsymbol{m}(t)^{-1}$ where $\boldsymbol{n}$ and $\boldsymbol{m}$ are arbitrary matrices.[4] The proof by Bjorken and Nauenberg may guarantee the symmetrical property of $\boldsymbol{A}(s)$, Eq. (5).[5]

Clearly we see that Shaw's solution is no other than the approximation $\boldsymbol{N}(s, s_0)=\boldsymbol{B}(s)$ in the general expression Eq. (5). In previous paper,[6] we discussed the s_0 dependence for the single channel problem. In that paper we solved the integral equation for N, Eq. (2), by approximating $B(s)$ by a set of poles and showed that the result was a form like Eq. (6). So we believe that our solution is the better approximation than Shaw's one, so far as the single-channel problems are concerned. The generalization of our procedure to the multi-channel problems will be discussed elsewhere.

The authors wish to thank Mrs. M. Imoto and Dr. M. Yamazaki for the valuable discussions. They also wish to express thanks to Prof. T. Asano and Prof. M. Sasaki for hospitality.

1) G. L. Shaw, Phys. Rev. Letters **12** (1964), 345.

2) J. L. Uretsky, Phys. Rev. **123** (1961), 1459.
3) H. Suura, private communication.
4) T. Adachi and T. Kotani, Soryusiron Kenkyu (Mimeographed Circular in Japanese) **29** (1964), No. 4.
5) J. D. Bjorken and M. Nauenberg, Phys. Rev. **121** (1961), 1250.
6) T. Adachi, R. Kawamura and T. Kotani, Prog. Theor. Phys. **32** (1964), 126.