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The *p-n* Mass Difference Based on the Reggeized Tadpole Model

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Recently Srivastava[1] has calculated the *p-n* mass difference, assuming the Regge asymptotic behavior, and using the generalized superconvergence relation(G.S.C.R.) and has shown that the result is in good agreement with experiment especially on its sign, i.e. $\Delta m(p-n)<0$. But he overlooks minus sign in Eq. (7) of reference 1) and this gives $\Delta m(p-n)>0$. In this letter we investigate the *p-n* mass difference, following Srivastava's idea, but using some assumptions different from his. Our assumptions are as follows:

(i) The contribution to $\Delta m(p-n)$ comes from ordinary low energy diagrams and high energy ones, as shown by Harari.[2]

(ii) The high energy contribution to $\Delta m(p-n)$ may be calculated by the Regge pole theory as the Reggeized tadpole model suggested by Okubo.[3] We will consider two trajectories

$A_2(1300)$ and $\pi_V(1016)$ as Regge poles.

(iii) Two generalized superconvergence relations are used as shown in Eqs. (5a) and (5b).

As shown by Cottingham et al.[4] we can express the p-n mass difference to first order in α by the forward amplitude for Compton scattering of a virtual photon of mass q^2 and energy $q^0\equiv\nu$ by a proton or neutron as follows,

$$\begin{aligned}\Delta m &\equiv m_p - m_n \\ &= -\frac{1}{2\pi}\int_0^\infty \frac{dq^2}{q^2}\int_0^q d\nu\sqrt{q^2-\nu^2} \\ &\quad\times\{3q^2 t_1(q^2, i\nu) - (2\nu^2+q^2) t_2(q^2, i\nu)\},\end{aligned} \tag{1}$$

where the t_i's are gauge invariant and even functions of ν. Low energy contribution to Δm is assumed to come mainly from nucleon pole and can be estimated by using the dipole-like form factor of proton and neutron.[5] Thus we obtain $\Delta m^L(p-n)=0.62\,\text{MeV}$ as low energy contribution. Following the discussion by Harari, the high energy contribution to Δm is assumed to come from the high energy part of $t_1(q^2, \nu)$, not from $t_2(q^2, \nu)$, and is shown as

$$\begin{aligned}\Delta m^H(p-n) &= -\frac{3}{2\pi}\int_0^\infty dq^2 \\ &\quad\times\int_0^q d\nu\sqrt{q^2-\nu^2}\, t_1{}^R(q^2, i\nu)\,\tilde{\theta}((i\nu)^2-\nu_1{}^2),\end{aligned} \tag{2}$$

where ν_1 is shown in Eqs. (5a) and (5b) and $\tilde{\theta}(\nu-\nu_1)$ is an analytic function of ν which plays the same role as a step function. From our assumption (ii),

$$\begin{aligned}&t_1{}^R(q^2, \nu) \\ &= \beta_1(q^2)\frac{1+e^{-i\pi\alpha_1}}{\sin\pi\alpha_1}\nu^{\alpha_1} + \beta_2(q^2)\frac{1+e^{-i\pi\alpha_2}}{\sin\pi\alpha_2}\nu^{\alpha_2},\end{aligned} \tag{3}$$

which comes from A_2- and π_V-trajectory respectively. The existence of $\tilde{\theta}(\nu^2-\nu_1{}^2)$ avoids the double counting as Srivastava does, and we can rewrite Eq. (2) by using a dispersion-like form of Eq. (3) (with $\alpha_1=\frac{1}{2}$ and $\alpha_2=-1$) as

$$\begin{aligned}&\Delta m^H(A_2) \\ &= -\frac{3}{2\pi}\int_0^\infty dq^2\int_0^q d\nu\sqrt{q^2-\nu^2}\frac{\nu^2}{\pi}\beta_1(q^2) \\ &\quad\times\int_{\nu_1{}^2}^\infty \frac{d\nu'^2(\nu'^2)^{1/4}}{\nu'^2(\nu'^2+\nu^2)},\end{aligned} \tag{4a}$$

$$\begin{aligned}&\Delta m^H(\pi_V) \\ &= \frac{3}{2\pi}\int_0^\infty dq^2\int_0^q d\nu\sqrt{q^2-\nu^2}\frac{\beta_2(q^2)}{\pi} \\ &\quad\times\int_{\nu_1{}^2}^\infty \frac{d\nu'^2(\nu'^2)^{-1/2}}{\nu'^2+\nu^2}.\end{aligned} \tag{4b}$$

In order to determine the form of $\beta_i(q^2)$ we assume two G. S. C. R. s:

$$\int_0^{\nu_1{}^2} \text{Im}\, t_1{}^R(q^2, \nu)\, d\nu^2 \cong \int_0^{\nu_1{}^2} \text{Im}\, t_1{}^P(q^2, \nu)\, d\nu^2, \tag{5a}$$

$$\int_0^{\nu_1{}^2} \nu^2\, \text{Im}\, t_1{}^R(q^2, \nu)\, d\nu^2 \cong \int_0^{\nu_1{}^2} \nu^2\, \text{Im}\, t_1{}^P(q^2, \nu)\, d\nu^2, \tag{5b}$$

where ν_1 is a finite energy. We put $\nu_1=(q^2+x_1)/2m$ with a positive free parameter x_1. Srivastava takes $x_1=0$ which corresponds to nucleon pole but generally x_1 is not zero and is rather large. We estimate the high energy contribution to Δm by using $m=940\,\text{MeV}$, $x_0=(850\,\text{MeV})^2$ and $x_1\equiv\gamma x_0$ and obtain $\Delta m^H(A_2)+\Delta m^H(\pi_V)\geq -0.17\,\text{MeV}$ which corresponds to $\gamma=1$, i.e. ν_1 corresponds to N^*_{33} resonance. Therefore we obtain $\Delta m(p-n)\gtrsim 0.45\,\text{MeV}$ which gives wrong sign.

In conclusion, we would like to make some comments on our unsuccessful result.

(i) If a few resonances and a background term contribute remarkably to the Compton scattering amplitude in low energy region, we will have larger β_i than before.

(ii) Some poles except for A_2 and π_V, for example, a conspirator of pion or fixed pole at $J=0$, will make a fairly large contribution to Δm.

(iii) It is doubtful to use G. S. C. R. s for electromagnetic mass difference.

(iv) If we assume the Regge asymptotic behavior for the amplitude $t=t_1 - [(2\nu^2+q^2)/3q^2]t_2$ instead of t_1 we may expect that the results will be improved.

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