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Electromagnetic Structure Functions of Hadrons in Infinite-Multiplet Theory

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Remarkable properties of the electromagnetic structure functions of proton have been revealed through detailed experimental study of the deep inelastic electron-proton scattering.[1] Several authors[2] have attempted to uncover sub-nucleonic structure through theoretical analysis and speculation. The purpose of this note is to report our model calculation of structure functions of hadrons, which is based upon the infinite-component field theory of Nambu and Takabayasi.[3] Some characteristic features and shortcomings of the model are discussed.

As usual,[1] we introduce two structure functions $W_1^{(S)}$ and $W_2^{(S)}$ to describe the process $e+$hadron (mass M, spin S)$\rightarrow$ $e+$"anything" (we will consider two cases: $S=0$ and $S=1/2$). $W_i^{(S)}$ depends on the virtual photon's lab energy ν and invariant mass squared q^2. Since a treatment of many-particle states in the infinite-component field theory is far beyond our capability, we restrict ourselves here to work in a narrow-resonance-saturation approximation. That is, we assume that non-diffractive components of inelastic ep scattering consist of excitation of an infinite-tower of narrow resonances which lie on a family of Regge trajectories, the target hadron being the lowest (ground) state. Then we may write a total contribution from degnerate resonances with the same mass M_n to, for instance, $W_2^{(S)}$ in the form

$$W_2^{(S)}(\nu, q^2)=[G_n^{(S)}(q^2)]^2\delta(W^2-M_n^2), \tag{1}$$

near $W^2(\equiv M^2+2M\nu+q^2)=M_n^2$, where $G_n^{(S)}(q^2)$ is the excitation form factor of the "n-th resonance", i.e. a sum of transition form factors of all resonances degenerates with the mass M_n. Summing up, all contributions lead to an expression for $W_2^{(S)}$ which is a series of δ-functions weighted by form factors. To obtain smooth functions for W's we must go beyond the narrow-resonance approximation, which is not discussed in the present note.

The form factors and the mass spectrum are taken from a simple wave equation

$$(p^2-\kappa\Gamma\cdot p)\psi(p)=0, \qquad (\kappa>0)$$

providing an exactly soluble model. Here Γ_μ is a vector operator in the $SO(4,2)$ theory.[3] The mass spectrum is $M_n=\kappa n$, where n is the principal quantum number (positive integer for Bosons and half integer starting with 3/2 for Fermions). The electromagnetic field is assumed to couple with the conserved vector current $J_\mu(p',p)=\psi^\dagger(p')\{(p+p')_\mu-\kappa\Gamma_\mu\}\psi(p)$. We have calculated the vector transition form factors $\psi^\dagger(p')\Gamma_\mu\psi(p)$ in the W-spin representation introduced in a previous paper[4] in which the matrix elements $\psi^\dagger(p')\psi(p)$ have been given.

What we have done until now is summarized as follows:

$$\begin{aligned}W_1^{(S)}(\nu, q^2) &=\frac{8}{3}\kappa^2\left(\frac{M^2+2M\nu+q^2}{\nu^2-q^2}\right)^{S+2}\\ &\times\sum_{n=S+1}^{\infty}\theta(M+\nu)\delta(M_n^2-M^2-2M\nu-q^2)\\ &\times n_S(n+S)(n+S+1)(n-S-1)\\ &\times\left(\frac{\nu+M-M_n}{\nu+M+M_n}\right)^n(1+\delta_S R_n(\nu, q^2)),\end{aligned} \tag{2a}$$

$$W_2^{(S)}(\nu, q^2)$$
$$=8\frac{-q^2}{(\nu^2-q^2)^2}\left(\frac{M^2+2M\nu+q^2}{\nu^2-q^2}\right)^{S+1}$$
$$\times\sum_{n=S+1}^{\infty}\theta(M+\nu)\delta(M_n^2-M^2-2M\nu-q^2)$$
$$\times n_S(n+S)\left(\frac{\nu+M-M_n}{\nu+M+M_n}\right)^n$$
$$\times\Big[-q^2(M+\nu)^2$$
$$+\frac{1}{3}M_n^2(2M\nu+q^2)(1+\delta_S R_n(\nu, q^2))\Big], \quad (2b)$$

with $\delta_S=0\,(1)$ for $S=0\,(1/2)$ and

$$R_n(\nu, q^2)$$
$$=-\frac{3M^2\nu^2+2M\nu(q^2+M^2)+q^2M^2}{3M_n^2(2M\nu+q^2)}, \quad (3)$$

where the summation runs over integral (half-integral) n for $S=0\,(S=1/2)$, $n_0=1$, $n_{1/2}=n-1/2$, $\kappa=M$ for $S=0$ and $\kappa=2M/3$ for $S=1/2$.

Without appeal to any sort of unitarization, we may study generic behavior of the structure functions in the Bjorken limit $q^2\to-\infty$, $\nu\to+\infty$ (with $\omega=-2M\nu/q^2$ fixed).[5] This is because, in this limit, the level separation can be ignored so that $\sum_n$ in Eqs. (2) is safely replaced by $\int dn$. (This replacement, whose justification requires a careful consideration on the energy resolution of final electrons in such models as ours that contain narrow resonances alone, is valid only in this limit.) In this way we find

$$(\nu/M)\,W_2^{(S)}(\nu, q^2)\to F_2^{(S)}(\omega)$$
$$=2^3(2S+2)^{2S+2}\omega^{-2}(1-\omega^{-1})^{2S+1}$$
$$\times\exp[-\gamma_S(1-\omega^{-1})], \quad (4)$$

in the Bjorken limit, where $\gamma_0=4$ and $\gamma_{1/2}=6$. The structure functions $W_1^{(S)}$ turn out to vanish in this limit. We should emphasize that the non-trivial scaling law obtained here is a consequence of excitations of higher and higher states as $\nu\to\infty$ with reasonable behavior of form factors for $q^2\to-\infty$ as predicted in the $SO(4,2)$ theory.

Thanks to this scaling law, it can be shown on the basis of the finite-energy sum rules proposed by Bloom and Gilman,[6] that the asymptotic behavior of form factors (compare Eq. (1) with Eq. (2b))

$$(G_n^{(S)}(q^2))^2\to(1/-q^2)^{2S+2}\,(-q^2\to\text{large})$$

is related in the sense of duality to the threshold behavior of the scaling functions

$$F_2^{(S)}(\omega)\to(\omega-1)^{2S+1}\,(\omega\to1).$$

This, of course, does not mean that our model has duality. Any narrow-resonance model with scaling should enjoy this relation.

Finally, $F_2^{(S)}(\omega)\to0$ when $\omega\to\infty$ as expected from the non-diffractive nature of our model. From the experimental point of view[1] our scaling functions fall off drastically, however. In this and other respects our model in its present form is too simple to be capable of explaining the data.

1) See, E. D. Bloom et al., preprint SLAC-PUB-796 (1970).
2) See, for instance, F. J. Gilman, talk presented at *The 1969 International Symposium on Electron and Photon Interactions at High Energies, Liverpool* (1969) and references therein.
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3) Y. Nambu, Phys. Rev. **160** (1967), 1171.
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See, also, A. O. Barut, Rapporteur's talk given at *The XV-th International Conference on High Energy Physics, Kiev, September 1970*, and references therein.
4) K. Morita, Prog. Theor. Phys. **43** (1970), 1071; Soryusiron Kenkyu (mimeographed

circular in Japanese) **41** (1970), 363.
5) J. D. Bjorken, Phys. Rev. **179** (1969), 1547.
6) E. D. Bloom and F. J. Gilman, Phys. Rev. Letters **25** (1970), 1140.