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# Gauge Invariant Quantum Electrodynamics with the Mass-Changing Minimal Current

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A gauge invariant local electrodynamics with the mass-changing minimal current giving rise to excitation processes of baryon or lepton is exactly formulated by means of introducing two ghost gauge fields. It is shown that the whole $S$-matrix in this formalism is equivalent to that in the conventional formalism with the Feynman gauge and without the ghost fields. On the bases of these arguments, a possible form of phenomenological mass-changing current vertex function of nucleon is suggested.

## § 1. Introduction

Recent experiments on $ep \to ep$, $ep \to eN^*$ and $N^* \to N\gamma$ give us the knowledge of elastic and inelastic nucleon form factors for a wide range of momentum transfer. Many authors have tried to explain the experimental behaviour of the form factors in terms of the internal structure of nucleon and its isobars based on some speculative particle models. The essential point of their works, especially for $ep \to eN^*$ and $N^* \to N\gamma$, is how to construct a mass-changing electromagnetic current vertex function corresponding to the transition $N^* \rightleftarrows N +$ *one real* or *virtual photon.*[1] Besides these processes, similar mass-changing currents would also appear in possible electromagnetic excitation processes of the electron (or generally lepton) as recently suggested by a few papers.[2] We may, therefore, point out the important role of the mass-changing electromagnetic current in future elementary particle physics.

As is well known, however, we do not necessarily have a satisfactory theory of the mass-changing current, because the mass-changing minimal electromagnetic current is not subject to the conservation law or gauge invariance. Fujimura, Kobayashi and Namiki[1] avoided this serious problem, in their quark model-theoretical arguments, by introducing the *ad hoc* replacement of the non-conserved current $j_\mu'$ by $(\delta_{\mu\nu} - q_\mu q_\nu/q^2)j_\nu'$, $q_\mu$ being the momentum transfer due to photon. However, we do not know why this replacement is justified.

In this paper we shall discuss gauge invariance and the conservation law of current in a quantum electrodynamics modified by introducing the mass-changing minimal current. The whole theory is formulated only within the framework of

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local field theory, because troubles of the mass-changing current in the quark model theory are essentially the same as those of the mass-changing current in the local field theory. We start with introducing ghost gauge fields which serve to keep the conservation law of current and gauge invariance (§ 2). After free field quantization (§ 3), it is shown that the whole $S$-matrix in the present formalism is equivalent to that in the conventional formalism with the Feynman gauge and without the ghost fields, if the gauge parameter is appropriately chosen (§ 4). Finally, we suggest a possible form of phenomenological mass-changing current vertex function of nucleon for the processes $ep\rightarrow eN^*$ and $N^*\rightarrow N\gamma$ on the basis of the above arguments in the local electrodynamics (§ 5).

## § 2. Gauge invariant formalism

In what follows, we deal with a system in which mutual transitions between two kinds of Dirac particles with masses different from each other take place through emission or absorption of a photon as well as the usual electromagnetic process. The vertex leading to such transitions for hadron or lepton is considered to come from the strong or other interactions different from the pure electromagnetic interaction. In this paper, however, we confine ourselves within the electrodynamics alone and introduce the corresponding vertex as an effective local electromagnetic interaction with coupling constant $eg$. The constant $g$ is introduced as a substitute for the above strong or other interactions. We shall not enter into discussion of the origin of $g$.

In order to obtain a gauge invariant formalism for the above system, we can see that, in addition to two kinds of Dirac fields $\psi(x)$ with mass $m$ and $\psi'(x)$ with mass $m'$, a massless scalar dipole field $B(x)$ and its pair field $B_0(x)$ are further required as gauge fields. As will be shown later, we can eliminate these ghost fields from the $S$-matrix by making proper choice for the dipole character. In terms of the field operators $\psi(x)$, $\psi'(x)$, $A_\mu(x)$, $B(x)$ and $B_0(x)$, the Lagrangian density desired is given in the form

$$L=-\bar{\psi}(\gamma\partial+m)\psi-\bar{\psi}'(\gamma\partial+m')\psi'-\frac{1}{4}F_{\mu\nu}F_{\mu\nu}-\frac{1}{2}(\partial_\mu A_\mu)^2$$

$$-\varepsilon\partial_\mu B_0\Big(\partial_\mu B+\frac{1}{a}A_\mu\Big)-\frac{b}{2}B_0^2+j_\mu A_\mu+j_\mu'(A_\mu+a\partial_\mu B), \qquad (2\cdot1)$$

where $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ and

$$j_\mu=ie(\bar{\psi}\gamma_\mu\psi+\bar{\psi}'\gamma_\mu\psi'), \qquad (2\cdot2)$$

$$j_\mu'=ieg(\bar{\psi}'\gamma_\mu\psi+\bar{\psi}\gamma_\mu\psi'). \qquad (2\cdot3)$$

Note that $m\neq m'$ here. The constants $a, b$ and $g$ are all real, and $\varepsilon$ denotes a sign factor ($\varepsilon^2=1$) which determines the metric of the space of ghost states. The current $j_\mu(x)$ is the usual one, while $j_\mu'(x)$ represents the mass-changing current giving rise to mutual transitions between two kinds of Dirac particles.*) Although the constant $g$ in (2·3) is a substitute reflecting the effect of the strong or other interactions, its value is completely arbitrary within the present framework.

From (2·1), (2·2) and (2·3), we have the equations of motion for the Dirac fields $\psi(x)$ and $\psi'(x)$ in the form

$$\begin{aligned}(\gamma\partial+m)\psi&=ie\gamma_\mu A_\mu\psi+ieg\gamma_\mu(A_\mu+a\partial_\mu B)\psi',\\(\gamma\partial+m')\psi'&=ie\gamma_\mu A_\mu\psi'+ieg\gamma_\mu(A_\mu+a\partial_\mu B)\psi\,.\end{aligned}\tag{2·4}$$

It then follows that

$$\partial_\mu j_\mu=0\,,\tag{2·5}$$

$$\partial_\mu j_\mu'=ieg(m-m')(\bar\psi\psi'-\bar\psi'\psi).\tag{2·6}$$

Equation (2·6) shows the non-conserved property of $j_\mu'(x)$. The equations of motion for other fields become

$$\left.\begin{aligned}&\Box A_\mu=-J_\mu\,,\\&J_\mu=j_\mu+j_\mu'-\frac{\varepsilon}{a}\partial_\mu B_0\,,\end{aligned}\right\}\tag{2·7}$$

$$\Box B=\varepsilon bB_0-\frac{1}{a}\partial_\mu A_\mu\,,\tag{2·8}$$

$$\Box B_0=\varepsilon a\partial_\mu j_\mu'.\tag{2·9}$$

Equations (2·5), (2·7) and (2·9) yield

$$\partial_\mu J_\mu=0\,.\tag{2·10}$$

We note that the total current $J_\mu(x)$ is conserved, in spite of $\partial_\mu j_\mu'\neq 0$.

The Lagrangian density and all the equations of motion are invariant under a $c$-number gauge transformation defined by

$$\left.\begin{aligned}&A_\mu\to A_\mu+\partial_\mu\Lambda\,, &&\Box\Lambda=0\,,\\&B\to B-\frac{1}{a}\Lambda\,, &&B_0\to B_0\,,\\&\psi\to e^{ie\Lambda}\psi\,, &&\psi'\to e^{ie\Lambda}\psi'.\end{aligned}\right\}\tag{2·11}$$

The transformation (2·11) represents a straightfoward extension of the usual gauge transformation, and therefore the gauge invariant character of the whole theory is guaranteed.

If the current $j_\mu'(x)$ is absent, that is, $g=0$, the last term in $J_\mu(x)$ becomes redundant. Therefore, we may understand that $a^{-1}$ is of the same order of mag-

*) In this paper we shall simply call $j_\mu'(x)$ proportional to $\gamma_\mu$ the *minimal* mass-changing current, even though the name of minimal current is to be ascribed to those which are introduced through the additional term $j_\mu A_\mu$ obtained by replacing $\partial_\mu\to\partial_\mu-ieA_\mu$ in a free Lagrangian.

nitude as $g$. It will, however, be shown in the next section that the presence of this term is not essential if $\Box B_0=0$.

## § 3. Free field equations and quantization

If the electromagnetic interaction is absent ($e=0$), our equations of motion become

$$\Box A_\mu=\frac{\varepsilon}{a}\partial_\mu B_0\,,$$

$$\Box B=\varepsilon b B_0-\frac{1}{a}\partial_\mu A_\mu\,, \tag{3·1}$$

$$\Box B_0=0\,,$$

which yield

$$\Box\partial_\mu A_\mu=\Box^2 A_\mu=\Box^2 B=0\,. \tag{3·2}$$

Equation (3·1) seems to show that some unknown background interactions like the strong or other interactions still remain in the field equations even if $e\to 0$, unless $a^{-1}$ is of the order of $e$. In this case, however, we can eliminate the contribution of $B_0(x)$ and $\partial_\mu A_\mu(x)$ to the equations of motion in the sense of expectation values of field operators. Since both $B_0(x)$ and $\partial_\mu A_\mu(x)$ satisfy the massless Klein-Gordon equation, we can always put the supplementary conditions

$$B_0^{(+)}(x)|\Phi\rangle=0\,, \tag{3·3}$$

$$\partial_\mu A_\mu^{(+)}(x)|\Phi\rangle=0 \tag{3·4}$$

on the physical state $|\Phi\rangle$, and can have the Maxwell equations in the form[3]

$$\langle\Box A_\mu(x)\rangle=\langle\partial_\mu A_\mu(x)\rangle=0\,.$$

Therefore, we can take (3·1) as our free field equations, so that the interaction Lagrangian becomes

$$L_{\rm int}=j_\mu A_\mu+j_\mu'(A_\mu+a\partial_\mu B)\,. \tag{3·5}$$

In the case of the interacting fields, we have no guarantee for (3·3), though the Lorentz condition (3·4) still holds due to (2·10). As will be shown in the next section, however, the contribution of $B(x)$ and $B_0(x)$ can be eliminated in the $S$-matrix and we can always put the condition (3·3) for the asymptotic incoming and outgoing fields.

Quantization of field operators in (3·1) can be done by means of the following commutation relations:

$$\left.\begin{aligned}
&[A_\mu(x), A_\nu(x')] = i\delta_{\mu\nu}D(x-x'),\\
&[A_\mu(x), B(x')] = i\frac{1}{a}\partial_\mu\tilde{D}(x-x'),\\
&[B(x), B(x')] = i\left(b-\frac{1}{a^2}\right)\tilde{D}(x-x'),\\
&[B(x), B_0(x')] = i\varepsilon D(x-x'),\\
&[A_\mu(x), B_0(x')] = [B_0(x), B_0(x')] = 0,
\end{aligned}\right\} \tag{3·6}$$

where the distribution $\tilde{D}(x)$ is defined by

$$\tilde{D}(x) = \frac{-i}{(2\pi)^3}\int d^4k\varepsilon(k)\delta'(k^2)e^{ikx}, \tag{3·7}$$

and it satisfies

$$\Box\tilde{D}(x) = D(x), \qquad \tilde{D}(-x) = -\tilde{D}(x), \tag{3·8}$$

$$\tilde{D}(x) = \partial_\mu\tilde{D}(x) = \partial_\mu\partial_\nu\tilde{D}(x) = 0, \quad \text{at} \quad x_0 = 0. \tag{3·9}$$

It is shown that the commutation relations (3·6) are consistent with the free field equations (3·1). If we require consistency between commutation relations and field equations alone, we have a possibility of taking more general forms for the commutation relations. For example, we can quantize $A_\mu(x)$ by

$$[A_\mu(x), A_\nu(x')] = i(\delta_{\mu\nu} + \alpha\partial_\mu\partial_\nu)D(x-x') + i\beta\partial_\mu\partial_\nu\tilde{D}(x-x')$$

with arbitrary constants $\alpha$ and $\beta$. We, however, further require the canonical quantization which compels us to take $\alpha = \beta = 0$ in the above relation. The detailed derivation of (3·6) is given in the Appendix.

It is also possible to take our free field equations and interaction Lagrangian in the form

$$\left.\begin{aligned}
&\Box A_\mu = 0,\\
&\Box B = \varepsilon b B_0, \qquad \Box B_0 = 0,
\end{aligned}\right. \tag{3·10}$$

and

$$L'_{\text{int}} = -\frac{\varepsilon}{a}\partial_\mu B_0 A_\mu + j_\mu A_\mu + j_\mu'(A_\mu + a\partial_\mu B), \tag{3·11}$$

respectively. In this case, we have the following canonical quantization:

$$\left.\begin{aligned}
&[A_\mu(x), A_\nu(x')] = i\delta_{\mu\nu}D(x-x'),\\
&[B(x), B(x')] = ib\tilde{D}(x-x'),\\
&[B(x), B_0(x')] = i\varepsilon D(x-x'),\\
&[A_\mu(x), B(x')] = [A_\mu(x), B_0(x')] = [B_0(x), B_0(x')] = 0,
\end{aligned}\right\} \tag{3·12}$$

which are equivalent to (3·6) in the canonical sense [see (3·9)]. It is easily shown that the contents of the $S$-matrix do not alter in either case of taking (3·5) or (3·11) as the interaction Lagrangian, apart from treatment with respect to external lines.

We have so far applied only the Feynman gauge formalism to $A_\mu(x)$. We can also apply any covariant gauge formalism to $A_\mu(x)$. For example, if we want to use the Nakanishi-Lautrup formalism[4)] in this case, it is sufficient to replace the Lagrangian density (2·1) with

$$L \to L - G\partial_\mu A_\mu + \frac{\lambda}{2}G^2 + \frac{1}{2}(\partial_\mu A_\mu)^2$$

and set up another supplementary condition to restrict $|\Phi\rangle$ with $G(x)$ as $G^{(+)}(x)|\Phi\rangle = 0$, where $G(x)$ is a newly introduced scalar field and $\lambda$ represents a gauge parameter. We can develop the theory in a way parallel to that in the Feynman gauge. In the next section, however, we show that a covariant gauge to $A_\mu(x)$ can be obtained in the interaction picture by a unitary transformation to state vectors without changing the $S$-matrix. Therefore, there is no necessity in starting with covariant gauge formalism, as far as we stay within the present framework.

## § 4. The $S$-matrix and elimination of ghosts

We here take (3·5) as the interaction Lagrangian. The free field equations and commutation relations are given by (3·1) and (3·6), respectively. The $S$-matrix in this case becomes

$$\left.\begin{aligned} S &= T^* \exp\left[i\int d^4x L_{\text{int}}(x)\right], \\ L_{\text{int}}(x) &= j_\mu(x)A_\mu(x) + j_\mu'(x)\{A_\mu(x) + a\partial_\mu B(x)\}. \end{aligned}\right\} \tag{4·1}$$

Equations (2·5) and (2·6) remain as they are. Boson propagators appearing in $S$ can be obtained from (3·6) as follows:

$$\begin{aligned} \langle 0|TA_\mu(x)A_\nu(x')|0\rangle &= \delta_{\mu\nu}D_F(x-x'), \\ \langle 0|TA_\mu(x)B(x')|0\rangle &= -\langle 0|TB(x)A_\mu(x')|0\rangle \\ &= \frac{1}{a}\partial_\mu\tilde{D}_F(x-x'), \\ \langle 0|TB(x)B(x')|0\rangle &= \left(b-\frac{1}{a^2}\right)\tilde{D}_F(x-x'), \end{aligned} \tag{4·2}$$

where

$$\begin{aligned} D_F(x) &= \frac{1}{(2\pi)^4}\int d^4k D_F(k)e^{ikx}, \qquad iD_F(k) = \frac{1}{k^2-i\varepsilon}, \\ \tilde{D}_F(x) &= \frac{1}{(2\pi)^4}\int d^4k \tilde{D}_F(k)e^{ikx}, \qquad -i\tilde{D}_F(k) = \frac{1}{(k^2-i\varepsilon)^2}. \end{aligned} \tag{4·3}$$

We note that, on account of (3·9), the distinction between $T$ and $T^*$ operations disappears in this case.

In every diagram of $S$, there are only three types of propagation as far as Boson lines are concerned. Two points $x$ and $x'$ are joined with $\delta_{\mu\nu}D_F(x-x')$ if they are specified with the $j_\mu$- and $j_\nu$-vertices (Fig. 1). If they are specified with the $j_\mu$- and $j_\nu'$-vertices, the propagator $D_{\mu\nu}(x-x')$ defined by

$$\left.\begin{aligned} D_{\mu\nu}(x-x') & \\ &=\langle 0|TA_\mu(x)\{A_\nu(x')+a\partial_\nu B(x')\}|0\rangle \\ &=\langle 0|T\{A_\mu(x)+a\partial_\mu B(x)\}A_\nu(x')|0\rangle, \\ D_{\mu\nu}(x)=\delta_{\mu\nu}D_F(x)-\partial_\mu\partial_\nu\tilde{D}_F(x) & \end{aligned}\right\} \qquad (4\cdot4)$$

joins them (Fig. 2). If the two points are specified with the $j_\mu'$- and $j_\nu'$-vertices, they are joined with the propagator $D'_{\mu\nu}(x-x')$ defined by

$$\left.\begin{aligned} &D'_{\mu\nu}(x-x')=\langle 0|T\{A_\mu(x)+a\partial_\mu B(x)\}\{A_\nu(x')+a\partial_\nu B(x')\}|0\rangle, \\ &D'_{\mu\nu}(x)=\delta_{\mu\nu}D_F(x)-(1+a^2b)\partial_\mu\partial_\nu\tilde{D}_F(x) \end{aligned}\right\} \qquad (4\cdot5)$$

(Fig. 3). In the momentum space, the Fourier transforms of $D_{\mu\nu}(x)$ and $D'_{\mu\nu}(x)$ are

$$\begin{aligned} &D_{\mu\nu}(k)=\delta_{\mu\nu}D_F(k)+k_\mu k_\nu\tilde{D}_F(k), \\ &D'_{\mu\nu}(k)=\delta_{\mu\nu}D_F(k)+(1+a^2b)k_\mu k_\nu\tilde{D}_F(k), \end{aligned} \qquad (4\cdot6)$$

respectively. We note that $D_{\mu\nu}(k)$ is nothing other than the propagator in the Landau gauge. Such a representation has already been used by Fujimura, Kobayashi and Namiki[3] in their *ad hoc* replacement of the non-conserved current $j_\mu'(k)$ with $(\delta_{\mu\nu}-k_\mu k_\nu/k^2)j_\nu'(k)$.

The propagators $D_{\mu\nu}(k)$ and $D'_{\mu\nu}(k)$ have the double pole character due to $\tilde{D}_F(k)$ (see (4·3)). From the fact that $D_{\mu\nu}(k)$ is always incident with the $j_\mu$-vertex (the conserved vertex), we can eliminate its double pole contribution, as will be shown later. For $D'_{\mu\nu}(k)$, however, we have no such favourable circumstance, and therefore we should put

$$b=-\frac{1}{a^2} \qquad (4\cdot7)$$

in order to eliminate its double pole contribution. The above relation corresponds

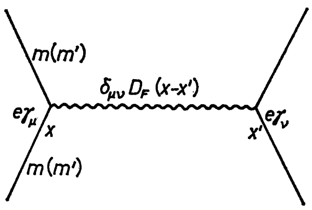


Fig. 1.

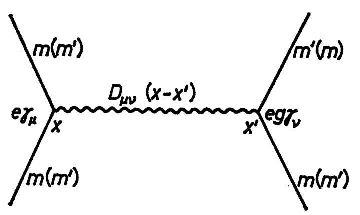


Fig. 2.

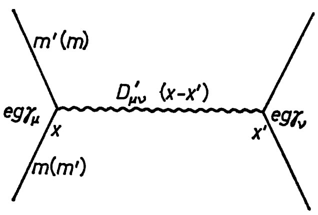


Fig. 3.

to a ghost-killing condition which is inevitable to guarantee the unitarity of the $S$-matrix. In this way, the dipole structure of the fields $B(x)$ and $B_0(x)$ is determined by the parameter $a$.

We are now in a position to show that the double pole in $D_{\mu\nu}(k)$ can really be eliminated. The reason why such an elimination is possible is simple. The theory is invariant under the gauge transformation (2·11) from the beginning, but the interaction Lagrangian $L_{\rm int}(x)$ suffers a superficial change as $j_\mu(x)\partial_\mu \Lambda(x)$. Therefore, it is rather natural that the term proportional to $k_\mu k_\nu \tilde{D}_F(k)$ can be eliminated if it is incident with the $j_\mu$-vertex, in spite of the presence of the non-conserved $j_\mu'$-vertex.

To show the circumstances strictly, let us treat the Schrödinger equation in the interaction picture. Equivalence of $T$ and $T^*$ operations in this theory tells us that the interaction Hamiltonian is simply given by

$$H_{\rm int}(x) = -L_{\rm int}(x). \tag{4·8}$$

We can show that (4·8) is really a valid Hamiltonian in this interaction picture. To prove this, it is to be noted that the commutation relations (3·6) play essential roles. The Schrödinger equation of the state $|\Phi(t)\rangle$ is

$$\left.\begin{aligned} &|\Phi(t)\rangle = S(t)|\Phi(-\infty)\rangle, \\ &i\dot{S}(t) = H_{\rm int}(t)S(t), \qquad H_{\rm int}(t) = -\int_t d^3x L_{\rm int}(x). \end{aligned}\right\} \tag{4·9}$$

From (3·4), we have the Lorentz condition in the usual form

$$\left.\begin{aligned} &\Omega(x;t)|\Phi(t)\rangle = 0, \\ &\Omega(x;t) = \partial_\mu A_\mu^{(+)}(x) - \int_{x_0'=t} d^3x' D^{(+)}(x-x')j_0(x'), \end{aligned}\right\} \tag{4·10}$$

where $x_0$ is not necessarily equal to $t$. This form is derived from the fact

$$[A_\mu(x) + a\partial_\mu B(x), \partial_\nu A_\nu(x')] = 0, \tag{4·11}$$

which is a direct consequence of (3·6) and (3·8). To show that the condition (4·10) is compatible with (3·1), (3·6) and (4·9), it is sufficient to see that the following relations hold similarly to the usual theory:

$$\left.\begin{aligned} &\Box_x \Omega(x;t) = 0, \qquad [\Omega(x;t), \Omega(x';t)] = 0, \\ &i\frac{\partial}{\partial t}\Omega(x;t) + [\Omega(x;t), H_{\rm int}(t)] = 0, \end{aligned}\right\} \tag{4·12}$$

the last of which is derived from (4·11) and

$$[j_\mu(x), j_0(x')] = [j_\mu'(x), j_0(x')] = 0 \quad \text{for} \quad x_0 = x_0'. \tag{4·13}^{*)}$$

Here we have neglected possible Gotō-Imamura-Schwinger terms[5] which play only

*) It also holds that

$$[j_\mu(x), j_0'(x')] = [j_\mu'(x), j_0'(x')] = 0 \quad \text{for} \quad x_0 = x_0'.$$

pathological roles.

As the next step we transform $|\Phi(t)\rangle$ into $|\Phi'(t)\rangle$ by

$$|\Phi'(t)\rangle = U(t)|\Phi(t)\rangle,\ S'(t) = U(t)S(t), \tag{4·14}$$

where $U(t)$ is a unitary operator given by

$$U(t) = e^{iG(t)},\ G(t) = \omega\int_t d^3x j_0(x)B(x) \tag{4·15}$$

with a real parameter $\omega$. The procedure is analogous to that of Ref. 6). It follows from (3·6) and (3·9) that the field operators $A_\mu(x)$ and $\partial_\mu B(x)$ are invariant under this transformation. The currents $j_\mu(x)$ and $j_\mu'(x)$ also remain invariant on account of (4·13). The Schrödinger equation (4·9) suffers from its change by (4·14). A simple calculation leads to

$$\left.\begin{aligned} i\dot{S}'(t) &= H'_{\rm int}(t)S'(t),\\ H'_{\rm int}(t) &= H_{\rm int}(t) - \dot{G}(t), \end{aligned}\right\} \tag{4·16}$$

giving

$$H'_{\rm int}(t) = -\int d^3x\{j_\mu(A_\mu + \omega\partial_\mu B) + j_\mu'(A_\mu + a\partial_\mu B)\}. \tag{4·17}$$

The last step is due to (2·5).

In this new representation (4·14), the propagators

$$\begin{aligned} &\delta_{\mu\nu}D_F(k) + \omega\left(\omega b - \frac{\omega}{a^2} + \frac{2}{a}\right)k_\mu k_\nu\tilde{D}_F(k),\\ &\delta_{\mu\nu}D_F(k) + (1 + \omega ab)k_\mu k_\nu\tilde{D}_F(k) \end{aligned} \tag{4·18}$$

appear in place of the old $\delta_{\mu\nu}D_F(k)$ and $D_{\mu\nu}(k)$, respectively. In this way, we have obtained a general covariant gauge representation with an arbitrary $a$, $b$ and $\omega$.*) The Lorentz condition (4·10) is transformed by (4·14) into

$$\begin{aligned} &\Omega'(x;t)|\Phi'(t)\rangle = 0,\\ &\Omega'(x;t) = \partial_\mu A_\mu^{(+)}(x) - \left(1 - \frac{\omega}{a}\right)\int_{x_0'=t} d^3x' D^{(+)}(x-x')j_0(x'), \end{aligned} \tag{4·19}$$

where we have used (3·6), (3·8) and (4·13). The compatibility relations of the type (4·12) are also proved after (4·14).

If the usual adiabatic switching of interaction is carried out, that is, $j_\mu(x)\to 0$ as $t\to\pm\infty$, then we have $S'(\infty) = S(\infty)$ from (4·14) and (4·15). Therefore, we can calculate our $S$-matrix with any covariant gauge under the relation (4·7). Elimination of the double poles in the propagators (4·18) is now a trivial problem.

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*) If we substitute (4·7) into the first of (4·18), the coefficient of $k_\mu k_\nu\tilde{D}_F(k)$ becomes $2(1-\omega/a)\omega/a \equiv \lambda$, showing $1/2 \geq \lambda > -\infty$. If we want to get the range $(\infty, 1/2)$ for $\lambda$, we may replace $B(x)$ in (4·15) with a new field $B'(x)$ which is completely independent of other operators and satisfies $[B'(x), B'(x')] = i\tilde{D}(x-x')$.[6]

Choosing $\omega=a$, we see at once that both of the propagators (4·18) become identical with $D'_{\mu\nu}(k)$ in (4·6). Under the relation (4·7), the propagator $D'_{\mu\nu}(k)$ is equal to $\delta_{\mu\nu}D_F(k)$. Thus, it is proved that all double poles of the propagators can be eliminated by the relation (4·7) even if we work with an arbitrary gauge.

In the case $\omega=a$ with (4·7), the $S$-matrix is equivalent to that of the Feynman gauge formalism with an interaction Lagrangian given simply by

$$(j_\mu+j_\mu')A_\mu .$$

This fact tells us that whether the current $j_\mu'(x)$ is conserved or not brings no change to the whole structure of the $S$-matrix if (2·5) and (4·13) hold. Although we have been able to eliminate the dynamical contribution of the ghost fields $B(x)$ and $B_0(x)$, it is to be noted that their existence is necessary in the gauge invariant formalism.

## § 5. Discussion on phenomenological current vertex

In previous sections we have shown that the gauge invariance of the whole theory is still kept in spite of the presence of the mass-changing minimal current.

It is true that the conclusion is obtained within the framework of local electrodynamics, but the essential connection of gauge invariance to the mass-changing current is common both in the point particle model and in the extended particle model. We can understand this situation from the fact that the non-conserved property of the mass-changing current in the extended particle model[1] is quite similar to (2·6) in the present point particle model. Therefore, we can suggest that the gauge invariance of the phenomenological $S$-matrix with the mass-changing current in the extended particle model is kept.

Fujimura, Kobayashi and Namiki's *ad hoc* replacement of the non-conserved current vertex $\langle P_F|J_\mu'|P_I\rangle$ for $ep\rightarrow eN^*$ or $N^*\rightarrow N\gamma$ with $\langle P_F|\hat{J}_\mu|P_I\rangle=(\delta_{\mu\nu}-k_\mu k_\nu/k^2)\langle P_F|J_\nu'|P_I\rangle$, in which $k=P_F-P_I$, may be justified by the above arguments. Here the second term ($\propto k^{-2}$) must have no effect because of the conserved property of the electron current or of the Lorentz condition of the real photon fields. The conclusion of this paper is that such a term ($\propto k^{-2}$) can be completely eliminated from the whole $S$-matrix. Thus, we may safely use the current vertex function

$$\langle P_F|J_\mu'|P_I\rangle=ie\bar{u}(P_F)\left\{\gamma_\mu F_1(k^2)+\frac{\kappa}{2M}\sigma_{\mu\nu}k_\nu F_2(k^2)\right\}u(P_I),$$

where we have followed the usual notation.


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## Appendix

*Derivation of the commutation relations (3·6)*

The commutation relations among the field operators should be consistent with the field equations (3·1). Since $A_\mu(x)$ and $B(x)$ satisfy (3·2) and $\Box B_0(x)=0$, we generally have the following form:

$$[A_\mu(x), A_\nu(x')]=i(\delta_{\mu\nu}+\alpha\partial_\mu\partial_\nu)D(x-x')+i(\alpha'\delta_{\mu\nu}+\alpha''\partial_\mu\partial_\nu)\tilde{D}(x-x'), \tag{A·1}$$

$$[A_\mu(x), B(x')]=i\beta\partial_\mu D(x-x')+i\beta'\partial_\mu\tilde{D}(x-x'), \tag{A·2}$$

$$[A_\mu(x), B_0(x')]=i\gamma\partial_\mu D(x-x'), \tag{A·3}$$

$$[B(x), B(x')]=i\delta D(x-x')+i\delta'\tilde{D}(x-x'), \tag{A·4}$$

$$[B(x), B_0(x')]=i\rho D(x-x'), \tag{A·5}$$

$$[B_0(x), B_0(x')]=0, \tag{A·6}$$

where all the coefficients are real constants. The last relation (A·6) is a direct consequence of (3·1), (A·3) and (A·5).

Operating $\Box$ on both sides of (A·1) with respect to $x'$, we find

$$\frac{\varepsilon}{a}[A_\mu(x), \partial_\nu B_0(x')]=i(\alpha'\delta_{\mu\nu}+\alpha''\partial_\mu\partial_\nu)D(x-x'),$$

the left-hand side of which should also be

$$-i\frac{\varepsilon}{a}\gamma\partial_\mu\partial_\nu D(x-x')$$

from (A·3), giving

$$\alpha'=0, \qquad \gamma=-\varepsilon a\alpha''. \tag{A·7}$$

Operation of $\Box$ on both sides of (A·2) with respect to $x$ leads to

$$\frac{\varepsilon}{a}[\partial_\mu B_0(x), B(x')]=i\beta'\partial_\mu D(x-x').$$

The left-hand side should also become

$$i\frac{\varepsilon}{a}\rho\partial_\mu D(x-x')$$

from (A·5). Therefore, we have

$$\rho=\varepsilon a\beta'. \tag{A·8}$$

On the other hand, if the operation is carried out with respect to $x'$, it follows from (3·1), (A·1) and (A·3) that

$$\varepsilon b[A_\mu(x), B_0(x')] - \frac{1}{a}[A_\mu(x), \partial_\nu A_\nu(x')] = i\beta'\partial_\mu D(x-x')$$
$$= i\varepsilon b\gamma\partial_\mu D(x-x') + \frac{i}{a}(1+\alpha'')\partial_\mu D(x-x').$$

Utilization of (A·7) and (A·8) leads to

$$(1-\varepsilon\rho)a = \varepsilon(1-a^2b)\gamma\,. \tag{A·9}$$

Finally, operating $\Box$ on both sides of (A·4) and using (3·1), (A·2) and (A·5), we find

$$\varepsilon b[B_0(x), B(x')] - \frac{1}{a}[\partial_\mu A_\mu(x), B(x')] = i\delta' D(x-x')$$
$$= i\varepsilon b\rho D(x-x') - i\frac{1}{a}\beta' D(x-x'),$$

giving

$$\delta' = \varepsilon\left(b - \frac{1}{a^2}\right)\rho \tag{A·10}$$

due to (A·8).

The field equations (3·1) give only the above five relations in (A·1)$\sim$(A·6), and therefore the most general form of the commutators becomes

$$\begin{aligned}
&[A_\mu(x), A_\nu(x')] = i(\delta_{\mu\nu} + \alpha\partial_\mu\partial_\nu)D(x-x') - i\frac{1-\varepsilon\rho}{1-a^2b}\partial_\mu\partial_\nu\tilde{D}(x-x'),\\
&[A_\mu(x), B(x')] = i\beta\partial_\mu D(x-x') + i\frac{\varepsilon}{a}\rho\partial_\mu\tilde{D}(x-x'),\\
&[A_\mu(x), B_0(x')] = i\frac{\varepsilon(1-\varepsilon\rho)}{1-a^2b}a\partial_\mu D(x-x'),\\
&[B(x), B(x')] = i\delta D(x-x') + i\varepsilon\left(b - \frac{1}{a^2}\right)\rho\tilde{D}(x-x'),\\
&[B(x), B_0(x')] = i\rho D(x-x'),\\
&[B_0(x), B_0(x')] = 0\,,
\end{aligned} \tag{A·11}$$

where $\alpha$, $\beta$, $\delta$ and $\rho$ remain arbitrary.

Canonical quantization determines these coefficients uniquely. From the Lagrangian density $L_{e=0}$ in (2·1), we have

$$\left.\begin{aligned}
&[B(x), \varepsilon\dot{B}_0(x')] = i\delta(\boldsymbol{x}-\boldsymbol{x}'),\\
&[B_0(x), \varepsilon\dot{B}(x') - \frac{\varepsilon}{a}A_0(x')] = i\delta(\boldsymbol{x}-\boldsymbol{x}'),\\
&[B(x), B(x')] = [B_0(x), B_0(x')]\\
&\quad = [A_\mu(x), B(x')] = [A_\mu(x), B_0(x')] = 0
\end{aligned}\right\}\text{for } x_0 = x_0', \tag{A·12}$$

and the usual relations among the electromagnetic field components. It follows from these canonical relations and (3·9) that

$$\alpha=\beta=\delta=0\,,\qquad \rho=\varepsilon\,. \tag{A·13}$$

Substitution of (A·13) into (A·11) leads to (3·6).

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