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Field Theory in $2+\varepsilon$ Dimensions and Critical Dimension in the String Model. II

Yūichi CHIKASHIGE and Yasunori FUJII*

Institute for Nuclear Study, University of Tokyo, Tanashi, Tokyo 188
and
**Institute of Physics, University of Tokyo-Komaba, Meguro-ku, Tokyo 153*

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The critical dimension in the string model is analysed in terms of the field theory in the two-dimensional Minkowski space-time which is obtained by transforming the basic variables τ and σ. It is proposed to allow the dimensionality of this internal space-time to be continuous and go to the limit of two dimensions. The theory is then invariant under the Lorentz transformation in our ordinary space-time with any dimensionality including the realistic value four.

§ 1. Introduction and summary

In the preceding paper[1)] (hereafter referred to as I) we studied the commutation relation of the energy-momentum tensor in the field theory in the N-dimensional Minkowski space-time. We were concerned mainly with the vacuum expectation value (VEV) called the anomalous term. In the model of a massless free scalar field this anomaly calculated for $N=2$ goes over to the anomaly of the Virasoro algebra (VA) of the string model when we change the variables and make certain analytic continuation (the Wick rotation). We showed that the anomaly disappears if we calculate VEV first with $N\neq 2$ and then take the limit $N\to 2$ from the larger side.

In this analysis we found that the anomaly obtained for the fixed value $N=2$ is a consequence of the "quasi Nambu-Goldstone boson" which is a special configuration of the two massless particles moving in the same direction. This observation suggests that the limit $N\to 2$ will be smooth giving the value for $N=2$ chosen at the beginning unless the above "parallel configuration" occurs in the two- or more-particle intermediate state. In § 2 of the present paper we confirm this in the example of the "normal term" of the commutator.

We then extend the analysis in I to the argument of the critical dimension in the string model. We rewrite in § 3 the relevant equations in the string model in terms of the equations in the N-dimensional Minkowski space-time. The crucial question in the light-like gauge is whether or not the condition[2),3)]

$$[M^{i-}, M^{j-}]=0 \tag{1·1}$$

holds true, where M^{i-} $(i=1, 2, \cdots, D-2)$ is one of the generators of the rotations in the ordinary space-time. Should the condition (1·1) be violated, the theory would be no longer invariant under the ordinary Lorentz transformation.

Exactly as in the anomaly of VA, it turns out again that there occurs a subtle situation in the two-particle intermediate state. The detailed analysis in § 3·1 shows that for the fixed value $N=2$ the one-particle matrix element of (1·1) acquires the anomaly due to the emergence of a quasi Nambu-Goldstone boson, resulting in a spontaneous breaking of the ordinary Lorentz invariance. Since, however, the anomaly is a sum of several terms, one of which is essentially the anomalous term of VA, there is a chance of cancellation among the terms. The cancellation does occur for $D=26$; the well-known result.[2),3)]

In § 3·2 we now apply the method of the continuous dimension in a manner completely parallel to that in § 3 of I. Although some complications arise because many invariant functions are involved, our final result is simple: Each of the terms that contribute to (1·1) disappears *separately* in the limit $N\to 2+\varepsilon$. The Lorentz invariance of the theory in the ordinary space-time (not the fictitious space-time of N dimensions) follows for *any* dimensionality including the realistic value $D=4$.

The final section 4 is devoted to the concluding remarks. In the three appendices some of the mathematical details are presented.

§ 2. Normal terms

We consider a set of real massless free scalar field $\phi^i(x)$ in the N-dimensional Minkowski space-time (called the internal space-time, IST, in I); the suffix i is for the external space-time (EST, our ordinary space-time) and runs from 1 to $D-2$ in the light-like gauge with D being the EST dimensionality. For the fixed value $N=2$ the equal-t commutation relation is given by[1),4)]

$$[T_\pm(x), T_\pm(0)]_{t=0} = \pm i[T_\pm(x) + T_\pm(0)]\delta'(x_1) + [(D-2)/24\pi]\delta'''(x_1). \tag{2·1}$$

Here we define the canonical energy-momentum tensor $T_{\mu\nu}$ by

$$T_{\mu\nu}(x) = \sum_{i=1}^{D-2} :[\partial_\mu\phi^i\partial_\nu\phi^i - \tfrac{1}{2}\eta_{\mu\nu}\partial_\lambda\phi^i\partial^\lambda\phi^i]: \tag{2·2}$$

and

$$T_\pm = \tfrac{1}{2}(T_{11} \pm T_{10}). \tag{2·3}$$

The metric is diagonal; $-\eta_{00}=\eta_{11}=+1$. The first term of (2·1) proportional to $\delta'(x_1)$ is the normal term which gives the conformal algebra, while the second term proprotional to $\delta'''(x_1)$ is the anomalous term. We check if the normal term remains unaffected if we let N be continuous and take the limit $N\to 2$.

Since $T_{\mu\nu}$ is bilinear in ϕ^i, it suffices to examine the one-particle matrix element

$$\langle q^a|[T_{\mu\nu}(x), T_{\rho\sigma}(0)]|q^b\rangle, \tag{2·4}$$

where $|q^b\rangle$, for example, is the one-particle state of the field $\phi^b(x)$. Notice that the zero-frequency part which is not well defined in two-dimensions does not contribute to (2·4) since $\phi^i(x)$ appears in $T_{\mu\nu}$ always with a derivative.

It is obvious that only the one-particle intermediate state contributes to (2·4);

$$\langle q^a|T_{\mu\nu}(x)T_{\rho\sigma}(0)|q^b\rangle_1 = \int d\boldsymbol{k}\langle q^a|T_{\mu\nu}(x)|k^k\rangle\langle k^k|T_{\rho\sigma}(0)|q^b\rangle$$

$$=\delta^{ab}(2\pi)^{-n}(2\omega)^{-1}e^{-iqx}G^{(+)}_{\mu\nu,\rho\sigma}(q;x), \tag{2·5a}$$

where a bold-face letter stands for a vector in $n=N-1$ dimensions. Also

$$\omega=q^0=|\boldsymbol{q}|, \tag{2·5b}$$

and

$$G^{(+)}_{\mu\nu,\rho\sigma}(q;x)=(2\pi)^{-n}\int d\boldsymbol{k}\frac{1}{2E}e^{ikx}\mathcal{H}_{\mu\nu,\rho\sigma} \tag{2·5c}$$

with

$$\mathcal{H}_{\mu\nu,\rho\sigma}=[q_\mu k_\nu+q_\nu k_\mu-\eta_{\mu\nu}(qk)][q_\rho k_\sigma+q_\sigma k_\rho-\eta_{\rho\sigma}(qk)], \tag{2·5d}$$

$$E=k^0=|\boldsymbol{k}|. \tag{2·5e}$$

From the form of $\mathcal{H}_{\mu\nu,\rho\sigma}$, the relations

$$G^{(+)}_{\mu\nu,\rho\sigma}(q;x)=G^{(+)}_{\mu\nu,\rho\sigma}(-q;x), \tag{2·6a}$$

$$=G^{(+)}_{\nu\mu,\rho\sigma}(q;x), \tag{2·6b}$$

$$=G^{(+)}_{\rho\sigma,\mu\nu}(q;x) \tag{2·6c}$$

follow immediately.

We must add the crossing symmetric term which is obtained from (2·5a) by $q\to-q$ and $a\leftrightarrow b$. By virtue of (2·6) the total contribution is found to be

$$\langle q^a|T_{\mu\nu}(x)T_{\rho\sigma}(0)|q^b\rangle=\delta^{ab}(2\pi)^{-n}\omega^{-1}\cos qx\cdot G^{(+)}_{\mu\nu,\rho\sigma}(q;x).$$

The matrix element of the commutator is then obtained:

$$\langle q^a|[T_{\mu\nu}(x),T_{\rho\sigma}(0)]|q^b\rangle=\delta^{ab}(2\pi)^{-n}\omega^{-1}\cos qx$$

$$\times[G^{(+)}_{\mu\nu,\rho\sigma}(q;x)-G^{(+)}_{\mu\nu,\rho\sigma}(q;-x)], \tag{2·7}$$

where use has been made of (2·6c).

We decompose $G^{(+)}_{\mu\nu,\rho\sigma}(q;x)$ into the invariant functions $g_A{}^{(+)}(x)$;

$$G^{(+)}_{\mu\nu,\rho\sigma}(q;x)=\sum_{A=21}^{32}\Pi^{(A)}_{\mu\nu,\rho\sigma}g_A{}^{(+)}(x). \tag{2·8}$$

By taking into account the symmetry properties (2·6) and the fact that (2·5d) is bilinear in q, the tensors $\Pi^{(A)}_{\mu\nu,\rho\sigma}$ are given by (A·4) in Appendix A. Not all of the twelve tensors are independent of each other if $N=2$. It is convenient to introduce the tensors with primed indices which vanish as $N\to2$. They are defined in (A·5). These are substituted for the tensors with the corresponding unprimed indices. The coefficients a_{AB} are defined by

$$\Pi^{\mu\nu,\rho\sigma(A)}\Pi^{(B)}_{\mu\nu,\rho\sigma}=(q\partial)^4 a_{AB}\,,$$

which are tabulated on Table II in Appendix A.

Multiplying $\Pi^{\mu\nu,\rho\sigma(A)}$ by $\mathcal{H}_{\mu\nu,\rho\sigma}$ yields

$$\Pi^{\mu\nu,\rho\sigma(A)}\mathcal{H}_{\mu\nu,\rho\sigma}=(qk)^4\beta_A\,, \tag{2·9}$$

where

$$\begin{aligned}&\beta_{30}=-\beta_{27'}=-\beta_{29'}=-(1/2)\beta_{31'}=2(N-2),\\&\beta_{32}=-1\,,\\&\beta_{28'}=-(N-2)^2,\\&\text{other } \beta\text{'s}=0\,.\end{aligned} \tag{2·10}$$

Combining (2·5c) and (2·9) we get

$$\Pi^{\mu\nu,\rho\sigma(A)}G^{(+)}_{\mu\nu,\rho\sigma}(q;x)_{\mathrm{RHS}}=\beta_A(q\partial)^4\Delta^{(+)}(x;0), \tag{2·11}$$

where use has been made of (2·5e).

This suggests that the function $g_A{}^{(+)}(x)$ is also of the form

$$g_A{}^{(+)}(x)=\alpha_A\Delta^{(+)}(x;0), \tag{2·12}$$

where the constants α_A are determined by

$$\sum_B a_{AB}\alpha_B=\beta_A\,. \tag{2·13}$$

In the method described in detail in Appendix B, the solution in the limit $N\to 2$ is obtained;

$$\alpha_{21}=1/2\,,\quad \alpha_{24}=-1\,,\quad \alpha_{25}=1/2\,,\quad \text{other } \alpha\text{'s}=0\,. \tag{2·14}$$

Substituting (2·12) and (2·14) into (2·8), and making use of

$$\Pi^{(21)}_{\mu\nu,\rho\sigma}\Delta(x;0)=0\,,$$

we obtain

$$\lim_{N\to 2}G^{(+)}_{\mu\nu,\rho\sigma}(q;x)=\Pi^{(g)}_{\mu\nu,\rho\sigma}\Delta^{(+)}(x;0), \tag{2·15a}$$

where

$$\Pi^{(g)}_{\mu\nu,\rho\sigma}=-\Pi^{(24)}_{\mu\nu,\rho\sigma}+\tfrac{1}{2}\Pi^{(25)}_{\mu\nu,\rho\sigma}\,. \tag{2·15b}$$

This is now to be substituted into (2·7). Recalling that $\Pi^{(A)}_{\mu\nu,\rho\sigma}$ is an even function of the derivatives, we finally arrive at

$$\lim_{N\to 2}\langle q^a|[T_{\mu\nu}(x),T_{\rho\sigma}(0)]|q^b\rangle=i\delta^{ab}(2\pi)^{-1}(\cos qx/\omega)\Pi^{(g)}_{\mu\nu,\rho\sigma}\Delta(x;0). \tag{2·16}$$

In deriving the result (2·16) we find no abrupt change at $N=2$. We in fact confirm that (2·16) agrees with the result obtained if we choose $N=2$ at the beginning.

For $t=0$ we find

$$\Pi^{(g)}_{11,11}(x;0)=\Pi^{(g)}_{10,10}\Delta(x;0)=-2\omega q_1\delta'(x_1),$$
$$\Pi^{(g)}_{11,10}(x;0)=2\omega^2\delta'(x_1),$$

from which it follows that

$$\lim_{N\to 2}\langle q^a|[T_\pm(x),T_\pm(0)]|q^b\rangle=\pm i\delta^{ab}(2\pi)^{-1}(\omega\mp q_1)\cos qx\cdot\delta'(x_1). \quad (2\cdot17)$$

It is easy to show that the r.h.s. of (2·17) are the one-particle matrix elements of the normal terms of (2·1).

§ 3. Critical dimension

In the theory of the string model it is necessary to specify the gauge condition. This makes it nontrivial to prove the covariance of the theory under the Lorentz transformation in EST. If we choose the light-like gauge, most crucial is to demonstrate that

$$[M^{i-},M^{j-}]=0, \quad (3\cdot1)$$

where M^{i-} ($i=1,2,\cdots,D-2$) is the generator of the rotation in the EST plane of X^i and $X^-=(X^{D-1}-X^0)/\sqrt{2}$. It has been shown that the condition (3·1) is fulfilled only for $D=26$ and $\alpha(0)=1$.[2),3),*)] In § 3·1, we reproduce the conclusion $D=26$ in the field theory in the fixed dimensionality $N=2$, and subsequently in § 3·2 apply the method of analytic continuation in N. Before going into the main part of the discussion,we present some preliminary considerations together with the array of the formulas valid for the arbitrary dimensionality N.

In the light-like gauge the generator M^{i-} is given by

$$M^{i-}=M_+{}^{i-}=\tfrac{1}{2}(M^{i-}_{11}+M^{i-}_{10}), \quad (3\cdot2a)$$

where[**)]

$$M^{i-}_{\mu\nu}=\int\mathcal{M}^i_{\mu\nu}(x)\,d\boldsymbol{x}, \quad (3\cdot2b)$$

$$\mathcal{M}^i_{\mu\nu}(x)=:\phi^i(x)T_{\mu\nu}(x):. \quad (3\cdot2c)$$

Notice that $\mathcal{M}^i_{\mu\nu}$ is *trilinear* in the field.

It is easy to see that (3·1) does not contribute to VEV; $\langle 0|M^{i-}M^{j-}|0\rangle$ is obviously ij symmetric. Among the other matrix elements of (3·1), only the connected one-particle and two-particle matrix elements are nontrivial. From the

*) $\alpha(0)=2$ for a closed string.

**) In the string model it is necessary to add to the r.h.s. some other terms involving the zero-frequency part of $\phi^i(x)$. Those terms, however, do not affect the discussion on the critical dimension.

discussion in § 2 on the normal term of the Virasoro algebra, it is expected that the two-particle matrix element is of no interest because then only the one-particle intermediate state contributes resulting in no anomaly for $N=2$. An explicit calculation shows that there is no two-particle matrix element indeed. For these reasons we hereafter confine ourselves to the one-particle matrix element alone.

Let us define

$$J^{ij,ab}_{\mu\nu,\rho\sigma}(q)=\langle q^a|M^{i-}_{\mu\nu}M^{j-}_{\rho\sigma}|q'^b\rangle$$
$$=(2\pi)^n\delta(\boldsymbol{q}-\boldsymbol{q}')\int G^{ij,ab}_{\mu\nu,\rho\sigma}(q;x)\,dx\,, \tag{3·3a}$$

where

$$G^{ij,ab}_{\mu\nu,\rho\sigma}(q;x)=\langle q^a|\mathcal{M}^i_{\mu\nu}(x)\,\mathcal{M}^j_{\rho\sigma}(0)\,|q^b\rangle_{\rm conn}\,. \tag{3·3b}$$

Obviously only the two-particle intermediate state contributes to (3·3);

$$G^{ij,ab}_{\mu\nu,\rho\sigma}(q;x)_2=e^{-iqx}\omega^{-1}\mathcal{G}^{ij,ab}_{\mu\nu,\rho\sigma}(q;x)\,, \tag{3·4a}$$

where

$$\mathcal{G}^{ij,ab}_{\mu\nu,\rho\sigma}(q;x)=\frac{1}{2}\int d\boldsymbol{k}_1\int d\boldsymbol{k}_2 e^{ipx}$$
$$\times\langle q^a|\mathcal{M}^i_{\mu\nu}(0)\,|k_1{}^ck_2{}^d\rangle\langle k_1{}^ck_2{}^d|\mathcal{M}^i_{\rho\sigma}(0)\,|q^b\rangle\,. \tag{3·4b}$$

It is necessary to add the crossing symmetric term corresponding to the zig-zag diagram. This implies that $G^{ij,ab}_{\mu\nu,\rho\sigma}$ in (3·3a) consists of two parts;

$$G^{ij,ab}_{\mu\nu,\rho\sigma}(q;x)=G^{ij,ab}_{\mu\nu,\rho\sigma}(q;x)_2+G^{ij,ba}_{\mu\nu,\rho\sigma}(-q;x)_2\,. \tag{3·5}$$

In terms of the quantities introduced above the one-particle matrix element of the commutator is given by

$$J^{[i,j],ab}_{(\mu\nu,\rho\sigma)}(q)=\langle q^a|\,[M^{i-}_{\mu\nu},M^{j-}_{\rho\sigma}]\,|q^b\rangle$$
$$=J^{ij,ab}_{\mu\nu,\rho\sigma}(q)-J^{ji,ab}_{\rho\sigma,\mu\nu}(q)\,. \tag{3·6}$$

The expression (3·2) allows us to put this into the form

$$J^{[ij],ab}_{(\mu\nu,\rho\sigma)}(q)=(2\pi)^n\delta(\boldsymbol{q}-\boldsymbol{q}')\,\omega^{-1}\int d\boldsymbol{x}\,e^{-iqx}\mathcal{G}^{[ij],ab}_{(\mu\nu,\rho\sigma)}(q;x)\,, \tag{3·7a}$$

where

$$\mathcal{G}^{[ij],ab}_{(\mu\nu,\rho\sigma)}(q;x)=(1/16)\,(2\pi)^{-3n}\delta^{ij,ab}\mathcal{G}_{(\mu\nu,\rho\sigma)}(q;x) \tag{3·7b}$$

with

$$\delta^{ij,ab}=\delta^{ia}\delta^{jb}-\delta^{ja}\delta^{ib}\,. \tag{3·7c}$$

Here $\mathcal{G}_{(\mu\nu,\rho\sigma)}(q;x)$ without EST indices is the sum of the three terms;

$$\mathcal{G}_{(\mu\nu,\rho\sigma)}=\mathcal{G}^{I}_{(\mu\nu,\rho\sigma)}+\mathcal{G}^{II}_{(\mu\nu,\rho\sigma)}+\mathcal{G}^{III}_{(\mu\nu,\rho\sigma)}\,, \tag{3·8a}$$

where

$$\mathcal{G}^{I}_{(\mu\nu,\rho\sigma)}(q;x)=(D-2)\int d\boldsymbol{k}_1\int d\boldsymbol{k}_2\frac{e^{ipx}}{E_1E_2}\langle\!\langle 0|T_{\mu\nu}|k_1k_2\rangle\!\rangle\langle\!\langle k_1k_2|T_{\rho\sigma}|0\rangle\!\rangle\,, \tag{3·8b}$$

$$\mathcal{G}^{II}_{(\mu\nu,\rho\sigma)}(q;x)=2\int d\boldsymbol{k}_1\int d\boldsymbol{k}_2\frac{e^{ipx}}{E_1E_2}[\langle\!\langle 0|T_{\mu\nu}|k_1k_2\rangle\!\rangle\langle\!\langle k_2|T_{\rho\sigma}|q\rangle\!\rangle+(\mu\nu\leftrightarrow\rho\sigma)]\,, \tag{3·8c}$$

$$\mathcal{G}^{III}_{(\mu\nu,\rho\sigma)}(q;x)=-2\int d\boldsymbol{k}_1\int d\boldsymbol{k}_2\frac{e^{ipx}}{E_1E_2}\langle\!\langle q|T_{\mu\nu}|k_2\rangle\!\rangle\langle\!\langle k_1|T_{\rho\sigma}|q\rangle\!\rangle\,, \tag{3·8d}$$

with

$$\langle\!\langle 0|T_{\mu\nu}|k_1k_2\rangle\!\rangle=-k_{1\mu}k_{2\nu}-k_{2\mu}k_{1\nu}+\eta_{\mu\nu}(k_1k_2),\ \text{etc.} \tag{3·8e}$$

An inspection of (3·8b) gives immediately

$$\mathcal{G}^{I}_{(\mu\nu,\rho\sigma)}(q;x)=8(2\pi)^{2n}G^{(+)}_{\mu\nu,\rho\sigma}(x)\,, \tag{3·9a}$$

with $G^{(+)}_{\mu\nu,\rho\sigma}(x)$ defined in (2·5) and (2·11) of I for VEV;

$$G^{(+)}_{\mu\nu,\rho\sigma}(x)=(2\pi)^{-1}(1/6)(D-2)\partial_\mu\partial_\nu\partial_\rho\partial_\sigma\Delta^{(+)}(x;0)\,. \tag{3·9b}$$

We finally obtain

$$J^{[ij],ab}_{(+,+)}(q)=\langle q^a|[M_+{}^{i-},M_+{}^{j-}]|q'^b\rangle$$
$$=\delta^{ij,ab}(1/16)(2\pi)^{-2n}\delta(\boldsymbol{q}-\boldsymbol{q}')\omega^{-1}\int d\boldsymbol{x}\,e^{-iqx}\mathcal{G}_{(+,+)}(q;x)_{t=0}\,, \tag{3·10a}$$

where

$$\mathcal{G}_{(+,+)}=(1/4)[\mathcal{G}_{(11,11)}+2\mathcal{G}_{(10,11)}+\mathcal{G}_{(10,10)}]\,. \tag{3·10b}$$

3·1. *Exactly two dimensions*

From (3·9) we have

$$\mathcal{G}^{I}_{(\mu\nu,\rho\sigma)}(q;x)=2\pi(4/3)(D-2)\partial_\mu\partial_\nu\partial_\rho\partial_\sigma\Delta^{(+)}(x;0)\,. \tag{3·11}$$

The calculation of $\mathcal{G}^{II}$ goes as follows: A straightforward computation gives

$$\mathcal{G}^{II}_{(11,11)}(q;x)=-2\int d\boldsymbol{k}_1\int d\boldsymbol{k}_2\,e^{ipx}(p^0q^0+p^1q^1)(1+\zeta_1\zeta_2)\,.$$

As in (2·4) and (2·6) of I only the "parallel movement" contributes. Following the same procedure as in (2·6)～(2·10) of I we obtain

$$\mathcal{G}^{II}_{(11,11)}(q;x)=-8i(2\pi)(q_1\partial_1+q_0\partial_0)\partial_1{}^2\Delta^{(+)}(x;0)\,. \tag{3·12a}$$

The other components are given by

$$\mathcal{G}^{II}_{(10,10)}(q;x)=\mathcal{G}^{II}_{(11,11)}(q;x)\,, \tag{3·12b}$$

$$\mathcal{G}^{II}_{(11,10)}(q;x) = -8i(2\pi)(q_1\partial_0+q_0\partial_1)\partial_1{}^2\Delta^{(+)}(x;0). \tag{3·12c}$$

These are combined with the formula

$$\mathcal{G}^{II}_{(\mu\nu,\rho\sigma)}(q;x) = -8i(2\pi)\Box(\Pi^{(13)}-\Pi^{(16)})_{\mu\nu,\rho\sigma}\Delta^{(+)}(x;0), \tag{3·12d}$$

where $\Pi^{(13)}$ and $\Pi^{(16)}$ are defined in (A·3).

The result for $\mathcal{G}^{III}$ is similar;

$$\mathcal{G}^{III}_{(11,11)} = \mathcal{G}^{III}_{(10,10)} = 8(2\pi)(q_1\partial_1+q_0\partial_0)q_0\partial_0\Delta^{(+)}(x;0), \tag{3·13a}$$

$$\mathcal{G}^{III}_{(11,10)} = 8(2\pi)(q_1\partial_1+q_0\partial_0)q_0\partial_1\Delta^{(+)}(x;0), \tag{3·13b}$$

or

$$\mathcal{G}^{III}_{(\mu\nu,\rho\sigma)}(q;x) = 2(2\pi)(2\Pi^{(24)}-\Pi^{(25)})_{\mu\nu,\rho\sigma}\Delta^{(+)}(x;0), \tag{3·13c}$$

where $\Pi^{(24)}$ and $\Pi^{(25)}$ are also shown in (A·4). Obviously all the $\mathcal{G}$'s share the same origin; the quasi massless particle arising from the parallel movement of the two particles in the intermediate state.

We now calculate $\mathcal{G}_{(+,+)}$ on the basis of (3·11)~(3·13);

$$\mathcal{G}_{(+,+)}(q;x) = 4(2\pi)[(1/6)(D-2)\partial_1{}^2+(q_1-\omega)(q_1-i\partial_1)](\partial_1+\partial_0)\partial_1\Delta^{(+)}(x;0). \tag{3·14}$$

In substituting (3·14) into (3·10a) we use the formulas,

$$\int d\boldsymbol{x}e^{-iqx}(\partial_1)^m\Delta^{(+)}(x;0)_{t=0} = (iq_1)^m/(2\omega),$$

$$\int d\boldsymbol{x}e^{-iqx}(\partial_1)^m\partial_0\Delta^{(+)}(x;0)_{t=0} = -i(iq_1)^m/2\,.$$

We thus obtain

$$\int d\boldsymbol{x}\, e^{-iqx}\mathcal{G}_{(+,+)}(q;x)_{t=0} = \begin{cases} 0\,, & \text{if } q_1=+\omega\,, \\ (4\pi/3)\omega^3[(D-2)-24], & \text{if } q_1=-\omega\,, \end{cases} \tag{3·15}$$

which vanishes only if*)

$$D=26\,.$$

It is noticed that nowhere we have the intercept $\alpha(0)$ which is also known to be relevant to the EST covariance. This is due to our insufficiency in including in M^{i-} the contribution from the zero-frequency part of $\phi^i(x)$. Its complete inclusion, which is of vital importance in the string model but is inevitably complicated, adds some more terms to M^{i-} other than (3·2b). As a consequence another condition must be satisfied in addition to the vanishing of (3·15).**) Then follows

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*) Equation (3·15) vanishes for any D if $q_1=\omega$. For the positive q_1, however, another combination $\int d\boldsymbol{x}e^{-iqx}\mathcal{G}_{(-,-)}(q;x)_{t=0}$ fails to vanish.

**) Roughly speaking, (3·15) and the "second condition" correspond to the first and the second terms of (58) in Ref. 2), respectively.

the restriction on $\alpha(0)$. *It still remains that it is (3·15) that determines D*, in terms of which $\alpha(0)$ is determined subsequently by the "second condition". The complete discussion on this point will be reported elsewhere.

3·2. *N dimensions and the limit N→2*

From the result (3·16) of I combined with (3·9) follows immediately:

$$\lim_{N\to 2+\varepsilon} \mathcal{G}^{I}_{(\mu\nu,\rho\sigma)}(q;x)=0\,. \tag{3·16}$$

We turn to the calculation of $\mathcal{G}^{II}_{(\mu\nu,\rho\sigma)}$. Since this is linear in q, as seen from (3·8), it is decomposed into eight invariant functions;

$$\mathcal{G}^{II}_{(\mu\nu,\rho\sigma)}(q;x)=\sum_{A=11}^{18}\Pi^{(A)}_{\mu\nu,\rho\sigma}f_A{}^{II}(x)\,, \tag{3·17}$$

with $\Pi^{(11)}$ to $\Pi^{(18)}$ defined in (A·3). These tensors are subject to the symmetry conditions,

$$\Pi^{(A)}_{\mu\nu,\rho\sigma}=\Pi^{(A)}_{\nu\mu,\rho\sigma}=\Pi^{(A)}_{\rho\sigma,\mu\nu}\,, \tag{3·18}$$

in accordance with the same conditions as dictated from the r.h.s. of (3·8c). As in § 2 it is convenient to replace $\Pi^{(15)}$ and $\Pi^{(17)}$ by $\Pi^{(15')}$ and $\Pi^{(17')}$ which vanish in the limit $N\to 2$, as confirmed by examining each component explicitly;

$$\Pi^{(A)}_{\mu\nu,\rho\sigma}=O(N-2)\,. \quad (A=15', 17') \tag{3·19}$$

The coefficients a_{AB} are defined by

$$\Pi^{\mu\nu,\rho\sigma(A)}\Pi^{(B)}_{\mu\nu,\rho\sigma}=(q\partial)^2 a_{AB}\,,$$

as are tabulated in Table I in Appendix A. Multiplying $\Pi^{\mu\nu,\rho\sigma(A)}$ to (3·17) yields

$$\Pi^{\mu\nu,\rho\sigma(A)}\mathcal{G}^{II}_{(\mu\nu,\rho\sigma)}=(q\partial)^2\sum_B a_{AB}f_B{}^{II}\,. \tag{3·20}$$

The tensor $\Pi^{\mu\nu,\rho\sigma(A)}$ is also multiplied to the r.h.s. of (3·8c). The result is put into the spectral representation,

$$\Pi^{\mu\nu,\rho\sigma(A)}\mathcal{G}^{II}_{(\mu\nu,\rho\sigma)\mathrm{RHS}}=(q\partial)^2\int ds\,\sigma_A{}^{II}(s)\,\Delta^{(+)}(x;s)\,. \tag{3·21}$$

As shown in Appendix C, most of the spectral functions vanish except

$$\sigma^{II}_{12}(s)=-16\frac{N-2}{n2^N}(2\pi)^n V_n s^{(N/2)-1}\,. \tag{3·22}$$

Even this function vanishes as $N\to 2$. Including the other (zero) spectral functions we have the general result,

$$\sigma_A{}^{II}(s)=O(N-2)\,. \tag{3·23}$$

In conformity with (3·21), the functions $f_A{}^{II}(x)$ are also given the spectral representation,

$$f_A{}^{II}(x) = \int ds\, \rho_A{}^{II}(s)\, \Delta^{(+)}(x; s). \tag{3·24}$$

Combining (3·20), (3·21) and (3·24) results in the equation

$$\sum_B a_{AB}\rho_B{}^{II}(s) = \sigma_A{}^{II}(s). \tag{3·25}$$

We can solve (3·25) by taking (3·23) into account, and find

$$\lim_{N\to 2} \rho_A{}^{II}(s) = \begin{cases} \text{finite nonzero}, & \text{if } A=15',\ 17', \\ 0, & \text{otherwise.} \end{cases} \tag{3·26}$$

The detail of the derivation is given in Appendix B.

Notice that the only nonvanishing functions for $A=15'$ and $17'$ are multiplied by the tensors that vanish as $N\to 2$. We also find that the previous result (3·12) is nonanalytic in N as represented by the same infinite step function as (3·15) of I. It thus follows that

$$\lim_{N\to 2+\varepsilon} \mathcal{G}^{II}_{(\mu\nu,\rho\sigma)}(q; x) = 0, \tag{3·27}$$

as long as we go to the limit $N=2$ before the s integrations give rise to infinities.

We apply the same analysis to $\mathcal{G}^{III}$. As seen from (3·8d), $\mathcal{G}^{III}$ is bilinear in q. The decomposition into the invariant functions is of the same form as in (2·8).

$$\mathcal{G}^{III}_{(\mu\nu,\rho\sigma)}(q; x) = \sum_{A=21}^{32} \Pi_{\mu\nu,\rho\sigma} f_A{}^{III}(x). \tag{3·28}$$

Corresponding to (3·21) we obtain

$$\Pi^{\mu\nu,\rho\sigma(A)} \mathcal{G}^{III}_{(\mu\nu,\rho\sigma)\mathrm{RHS}} = (q\partial)^4 \int ds\, \sigma_A{}^{III}(s)\, \Delta^{(+)}(x; s). \tag{3·29}$$

The only nonvanishing spectral functions are

$$\sigma^{III}_{32}(s) = \frac{1}{4}\sigma^{III}_{31}(s) = 16\frac{N-2}{n2^N}(2\pi)^n V_n s^{(N/2)-2}. \tag{3·30}$$

See Appendix C for the details. Equation (3·23) holds also for $\sigma_A{}^{III}(s)$. Exactly the same arguments follow as (3·24)～(3·27), to result in

$$\lim_{N\to 2+\varepsilon} \mathcal{G}^{III}_{(\mu\nu,\rho\sigma)}(q; x) = 0. \tag{3·31}$$

Collecting (3·16), (3·27) and (3·31), we now arrive at our final conclusion

$$\lim_{N\to 2+\varepsilon} \mathcal{G}_{(\mu\nu,\rho\sigma)}(q; x) = 0 \tag{3·32a}$$

or

$$\lim_{N\to 2+\varepsilon} \langle q^a | [M^{i-}, M^{j-}] | q'^b \rangle = 0. \tag{3·32b}$$

Obviously this holds true for *any D*.

§ 4. Concluding remarks

We have translated the equations on the critical dimension in the string model to those in the field theory in two dimensions. Our finding is that the anomaly of the Virasoro algebra and the breaking of EST covariance share the same origin; the quasi Nambu-Goldstone boson which emerges only in two dimensions. We have then shown that these anomalous effects are removed, independent of the dimensionality of EST, in the limit $N=2+\varepsilon$, in other words in the limit where the hadrons are extended spatially in $1+\varepsilon$ dimensions. No other consequences of the free string model are affected by this limiting procedure; e.g. the mass-levels, the linear spin-squared mass relation. Extension to the continuous N is made only at the stage of quantization; the basic Lagrangian[5] need not be altered.

The argument is expected to remain unchanged if the interaction is introduced in a conformally invariant way.

We have failed to include the zero-frequency part of $\phi^i(x)$ in such a way that it corresponds correctly to that in the string model. It is this part that brings the intercept $\alpha(0)$ into the expression for (1·1). Our translation to the Minkowski field theory is not complete in this respect; the tachyon problem is left intact. Improving the formulation in this aspect is an important subject of the future study. We still believe that our result sheds a new light to possible attempts toward the final solution of the difficulties of the string model.

It is yet to be established how the calculation in terms of operators is to be modified in the dual resonance model, according to which eliminating the ghost requires the presence of the anomaly of VA together with $D=26$.

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Appendix A

We list the independent tensors which are made of $\eta_{\mu\nu}$, ∂_μ, q_μ and are subject to the symmetry conditions

$$\Pi^{(A)}_{\mu\nu,\rho\sigma}=\Pi^{(A)}_{\nu\mu,\rho\sigma}=\Pi^{(A)}_{\rho\sigma,\mu\nu}\,. \tag{A·1}$$

Tensors independent of q

$$\Pi^{(1)}_{\mu\nu,\rho\sigma}=\tfrac{1}{2}\eta_{\mu\nu}\eta_{\rho\sigma}\,,\quad \Pi^{(2)}_{\mu\nu,\rho\sigma}=\tfrac{1}{2}(\eta_{\mu\rho}\eta_{\nu\sigma}+\eta_{\mu\sigma}\eta_{\nu\rho})\,,$$

$$\Pi^{(3)}_{\mu\nu,\rho\sigma}=\tfrac{1}{2}\Box^{-1}(\eta_{\mu\nu}\partial_\rho\partial_\sigma+\eta_{\rho\sigma}\partial_\mu\partial_\nu)\,,$$

$$\Pi^{(4)}_{\mu\nu,\rho\sigma}=\tfrac{1}{2}\Box^{-1}(\eta_{\mu\rho}\partial_\nu\partial_\sigma+\eta_{\nu\rho}\partial_\mu\partial_\sigma+\eta_{\mu\sigma}\partial_\nu\partial_\rho+\eta_{\nu\sigma}\partial_\mu\partial_\rho)\,,$$

$$\Pi^{(5)}_{\mu\nu,\rho\sigma}=\tfrac{1}{2}\Box^{-2}\partial_\mu\partial_\nu\partial_\rho\partial_\sigma\,. \tag{A·2}$$

Tensors linear in q

$$\Pi^{(11)}_{\mu\nu,\rho\sigma}=\tfrac{1}{2}[\eta_{\mu\nu}(q_\rho\partial_\sigma+q_\sigma\partial_\rho)+\eta_{\rho\sigma}(q_\mu\partial_\nu+q_\nu\partial_\mu)],$$

$$\Pi^{(12)}_{\mu\nu,\rho\sigma}=\tfrac{1}{2}[\eta_{\mu\rho}(q_\nu\partial_\sigma+q_\sigma\partial_\nu)+\eta_{\nu\rho}(q_\mu\partial_\sigma+q_\sigma\partial_\mu)$$
$$+\eta_{\mu\sigma}(q_\nu\partial_\rho+q_\rho\partial_\nu)+\eta_{\nu\sigma}(q_\mu\partial_\rho+q_\rho\partial_\mu)],$$

$$\Pi^{(13)}_{\mu\nu,\rho\sigma}=\tfrac{1}{2}\Box^{-1}[\partial_\mu\partial_\nu(q_\rho\partial_\sigma+q_\sigma\partial_\rho)+\partial_\rho\partial_\sigma(q_\mu\partial_\nu+q_\nu\partial_\mu)],$$

$$\Pi^{(14)}_{\mu\nu,\rho\sigma}=(q\partial)\Pi^{(1)}_{\mu\nu,\rho\sigma}\,,\quad \Pi^{(15)}_{\mu\nu,\rho\sigma}=(q\partial)\Pi^{(2)}_{\mu\nu,\rho\sigma}\,,\quad \Pi^{(16)}_{\mu\nu,\rho\sigma}=(q\partial)\Pi^{(3)}_{\mu\nu,\rho\sigma}\,,$$

$$\Pi^{(17)}_{\mu\nu,\rho\sigma}=(q\partial)\Pi^{(4)}_{\mu\nu,\rho\sigma}\,,\quad \Pi^{(18)}_{\mu\nu,\rho\sigma}=(q\partial)\Pi^{(5)}_{\mu\nu,\rho\sigma}\,. \tag{A·3}$$

Tensors bilinear in q

$$\Pi^{(21)}_{\mu\nu,\rho\sigma}=\Box(\eta_{\mu\nu}q_\rho q_\sigma+\eta_{\rho\sigma}q_\mu q_\nu),$$

$$\Pi^{(22)}_{\mu\nu,\rho\sigma}=\Box(\eta_{\mu\rho}q_\nu q_\sigma+\eta_{\nu\rho}q_\mu q_\sigma+\eta_{\mu\sigma}q_\nu q_\rho+\eta_{\nu\sigma}q_\mu q_\rho),$$

$$\Pi^{(23)}_{\mu\nu,\rho\sigma}=\partial_\mu\partial_\nu q_\rho q_\sigma+\partial_\rho\partial_\sigma q_\mu q_\nu\,,$$

$$\Pi^{(24)}_{\mu\nu,\rho\sigma}=\partial_\mu\partial_\rho q_\nu q_\sigma+\partial_\nu\partial_\rho q_\mu q_\sigma+\partial_\mu\partial_\sigma q_\nu q_\rho+\partial_\nu\partial_\sigma q_\mu q_\rho\,,$$

$$\Pi^{(25)}_{\mu\nu,\rho\sigma}=2(q\partial)\Pi^{(11)}_{\mu\nu,\rho\sigma}\,,\quad \Pi^{(26)}_{\mu\nu,\rho\sigma}=2(q\partial)\Pi^{(12)}_{\mu\nu,\rho\sigma}\,,\quad \Pi^{(27)}_{\mu\nu,\rho\sigma}=2(q\partial)\Pi^{(13)}_{\mu\nu,\rho\sigma}\,,$$

$$\Pi^{(28)}_{\mu\nu,\rho\sigma}=2(q\partial)\Pi^{(14)}_{\mu\nu,\rho\sigma}\,,\quad \Pi^{(29)}_{\mu\nu,\rho\sigma}=2(q\partial)\Pi^{(15)}_{\mu\nu,\rho\sigma}\,,\quad \Pi^{(30)}_{\mu\nu,\rho\sigma}=2(q\partial)\Pi^{(16)}_{\mu\nu,\rho\sigma}\,,$$

$$\Pi^{(31)}_{\mu\nu,\rho\sigma}=2(q\partial)\Pi^{(17)}_{\mu\nu,\rho\sigma}\,,\quad \Pi^{(32)}_{\mu\nu,\rho\sigma}=2(q\partial)\Pi^{(18)}_{\mu\nu,\rho\sigma}\,. \tag{A·4}$$

Tensors with primed indices

$$\Pi^{(4')}=2\Pi^{(1)}-\Pi^{(2)}-2\Pi^{(3)}+\Pi^{(4)}\,,$$

$$\Pi^{(15')}=\Pi^{(11)}-\tfrac{1}{2}\Pi^{(12)}-2\Pi^{(14)}+\Pi^{(15)}\,,$$

$$\Pi^{(17')}=\Pi^{(11)}-\tfrac{1}{2}\Pi^{(12)}-2\Pi^{(16)}+\Pi^{(17)}\,,$$

$$\Pi^{(22')}=\Pi^{(21)}-\tfrac{1}{2}\Pi^{(22)}\,,$$

$$\Pi^{(26')}=-4\Pi^{(23)}+2\Pi^{(24)}-2\Pi^{(25)}+\Pi^{(26)}\,,$$

$$\Pi^{(27')}=-\Pi^{(23)}-\Pi^{(30)}+\Pi^{(27)}\,,$$

$$\Pi^{(28')}=\tfrac{1}{4}\Pi^{(22')}-\tfrac{1}{2}\Pi^{(25)}+\Pi^{(28)}\,,$$

$$\Pi^{(29')}=\tfrac{1}{2}\Pi^{(22')}-2\Pi^{(23)}+\Pi^{(24)}-\Pi^{(25)}+\Pi^{(29)}\,,$$

$$\Pi^{(31')}=-2\Pi^{(23)}+\Pi^{(24)}-2\Pi^{(30)}+\Pi^{(31)}\,. \tag{A·5}$$

These vanish as $N\to 2$.

The coefficients a_{AB} are defined by

$$\Pi^{\mu\nu,\rho\sigma(A)}\Pi^{(B)}_{\mu\nu,\rho\sigma}=\begin{cases}(q\partial)^2 a_{AB}\,, & (11\leqq A,\,B\leqq 18),\\ (q\partial)^4 a_{AB}\,, & (21\leqq A,\,B\leqq 32).\end{cases} \tag{A·6}$$

They are shown on Tables I and II. We use the symbol $\nu=N-2$.

Table I. Coefficients a_{AB} defined by $\Pi^{\mu\nu,\rho\sigma(A)}\Pi^{(B)}_{\mu\nu,\rho\sigma}=(q\partial)^2 a_{AB}$ for $A, B=11, \cdots, 18$; $15', 17'$. $\nu=N-2$. The elements in the left-lower half are omitted since $a_{AB}=a_{BA}$.

	11	12	13	14	16	18	15′	17′
11	$N+2$	4	3	N	$N+1$	1	$-\nu$	$-\nu$
12		$2(N+4)$	6	2	4	2	ν	ν
13			3	1	2	1	0	0
14				$\frac{1}{4}N^2$	$\frac{1}{2}N$	$\frac{1}{4}$	$-\frac{1}{2}\nu(N-1)$	0
16					$\frac{1}{2}(N+1)$	$\frac{1}{2}$	0	0
18						$\frac{1}{4}$	0	0
15′							$\frac{3}{2}\nu^2$	$-\frac{3}{2}\nu$
17′								$-\frac{3}{2}\nu$

Table II. Coefficients a_{AB} defined by $\Pi^{\mu\nu,\rho\sigma(A)}\Pi^{(B)}_{\mu\nu,\rho\sigma}=(q\partial)^4 a_{AB}$ for $A, B=21, \cdots, 32$; $22', \cdots, 31'$. $\nu=N-2$. The elements in the left-lower half are omitted since $a_{AB}=a_{BA}$, except in the off-diagonal part between the primed and the unprimed indices.

	21	23	24	25	30	32
21	0	0	0	0	$2N$	2
23		2	0	0	2	2
24			4	8	8	4
25				$4(N+2)$	$4(N+2)$	4
30					$2(N+1)$	2
32						1

	22′	26′	27′	28′	29′	31′
21	0	0	-2ν	0	0	-4ν
23	0	0	0	0	0	0
24	0	0	0	0	0	0
25	0	-8ν	-4ν	2ν	-4ν	-8ν
30	2ν	-8ν	-2ν	0	-4ν	-4ν
32	0	0	0	0	0	0

	22′	26′	27′	28′	29′	31′
22′	0	0	-2ν	0	0	-6ν
26′		24ν	8ν	-4ν	16ν	24ν
27′			2ν	0	4ν	4ν
28′				$\nu(N-1)$	0	ν
29′					$2\nu(N+5)$	14ν
31′						12ν

Appendix B

We consider the equation of the type

$$\sum_B a_{AB}(N)x_B(N)=y_A(N). \tag{B·1}$$

The $m\times m$ matrix a_{AB} defined by (A·6) has the structure

$$a(N)=\begin{pmatrix} b(N) & c(N) \\ c^T(N) & d(N)\end{pmatrix}, \tag{B·2}$$

where $b(N)$ is the $l\times l$ matrix corresponding to the unprimed indices while $d(N)$ is the $l'\times l'$ matrix corresponding to the primed indices $(l+l'=m)$. Likewise $c(N)$ is the matrix with l rows and l' columns. $c^T(N)$ is its transposition. Since the tensors of the primed indices vanish as $N\to 2$, it is appropriate to define tildered matrices by

$$c(N)=(N-2)\tilde{c}(N), \tag{B·3a}$$

$$c^T(N)=(N-2)\tilde{c}^T(N), \tag{B·3b}$$

$$d(N)=(N-2)\tilde{d}(N). \tag{B·3c}$$

The determinants will be denoted by

$$\Delta_a(N)=\det[a(N)], \text{ etc.}$$

The important conclusion from Tables I and II is that

$$\Delta_b(2)\neq 0\,,\quad \Delta_{\tilde{d}}(2)\neq 0\,. \tag{B·4}$$

In the calculation in § 3·2 of $\mathcal{G}^{II}$ and $\mathcal{G}^{III}$ it has been shown that

$$y_A(N)=(N-2)\tilde{y}_A(N),\ \text{for all}\ A\,. \tag{B·5}$$

In the calculation in § 2 of the normal terms of the Virasoro algebra, some of $y_A(2)$ do not vanish if A is the unprimed index. For the primed index, $y_A(2)$ vanishes in either case;

$$y_A(N)=(N-2)\tilde{y}_A(N),\ \text{for the primed}\ A\,. \tag{B·6}$$

On the basis of such information we will prove that

$$\lim_{N\to 2} x_A(N)=\begin{cases} 0\,, & \text{(Section 3·2)} \qquad \text{(B·7a)}\\ \text{finite}, & \text{(Section 2)} \qquad \text{(B·7b)}\end{cases}$$

if A is the unprimed index, while

$$\lim_{N\to 2} x_A(N)=\text{finite}, \tag{B·8}$$

if A is the primed index.

To prove this we introduce the following notations:

$b_A(N)$: the $l\times l$ matrix obtained from $b(N)$ by replacing the column A

by the upper l elements of the one-column matrix $y(N)$.

$c_A{}^T(N)$: the $l' \times l$ matrix obtained from $c^T(N)$ by replacing the column A by the lower l' elements of the one-column matrix $y(N)$.

$c_A(N)$: defined in the same way as in $b_A(N)$.

$d_A(N)$: defined in the same way as in $c_A{}^T(N)$.

We find from (B·3a) and (B·3c) that

$$\Delta_a(N) = (N-2)^{l'} \det\begin{pmatrix} b(N) & \tilde{c}(N) \\ c^T(N) & \tilde{d}(N) \end{pmatrix}.$$

Since $c^T(2)=0$, as seen from (B·3b) it follows

$$\Delta_a(2) = \lim_{N\to 2}(N-2)^{l'}\Delta_b(2)\Delta_{\tilde{d}}(2). \tag{B·9}$$

The coefficient $\Delta_b(2)\Delta_{\tilde{d}}(2)$ is nonvanishing due to (B·4).

The solution of (B·1) is given by

$$x_A(N) = D_A(N)/\Delta_a(N). \tag{B·10}$$

We first consider x_A with the unprimed A. $D_A(N)$ is given by

$$D_A(N) = \det\begin{pmatrix} b_A(N) & c(N) \\ c_A{}^T(N) & d(N) \end{pmatrix}. \tag{B·11}$$

If (B·5) holds we obtain

$$D_A(N) = (N-2)^{l'+1}\tilde{D}_A(N), \tag{B·12a}$$

where

$$\tilde{D}_A(N) = \det\begin{pmatrix} \tilde{b}_A(N) & \tilde{c}(N) \\ \tilde{c}_A{}^T(N) & \tilde{d}(N) \end{pmatrix}. \tag{B·12b}$$

Here $\tilde{b}_A(N)$ and $\tilde{c}_A{}^T(N)$ are obtained by substituting $\tilde{y}_A$ for y_A into b_A and $c_A{}^T$, respectively. By substituting (B·9) and (B·12) into (B·10) we conclude that

$$x_A(N) = O(N-2), \tag{B·13}$$

where (B·4) are crucial. This completes the proof of (B·7a).

If (B·6) alone holds, (B·12a) is replaced by

$$D_A(N) = (N-2)^{l'}\tilde{D}_A(N), \tag{B·14a}$$

where

$$\tilde{D}_A(N) = \det\begin{pmatrix} b_A(N) & \tilde{c}(N) \\ c_A{}^T(N) & \tilde{d}(N) \end{pmatrix}. \tag{B·14b}$$

$x_A(N)$ no longer vanishes as $N\to 2$, but stays finite.

We finally consider $D_A(N)$ for the primed A;

$$D_A(N)=\det\begin{pmatrix} b(N) & c_A(N) \\ c^T(N) & d_A(N)\end{pmatrix}. \tag{B·15}$$

From (B·3c) and (B·5) or (B·6) we find $d_A(N)$ vanishes as $N\to 2$;

$$d_A(N)=(N-2)\tilde{d}_A(N). \tag{B·16}$$

Combining this with (B·3b) we obtain

$$D_A(N)=(N-2)^{\nu'}\tilde{D}_A(N), \tag{B·17a}$$

where

$$\tilde{D}_A(N)=\det\begin{pmatrix} b(N) & c_A(N) \\ \tilde{c}^T(N) & \tilde{d}_A(N)\end{pmatrix}. \tag{B·17b}$$

The result (B·8) follows from (B·9) and (B·17) immediately.

Appendix C

From (3·8c) and (3·8e) we find

$$\Pi^{\mu\nu,\rho\sigma(12)}\mathcal{G}^{II}_{\mu\nu,\rho\sigma}=-8\int\frac{d\boldsymbol{k}_1}{E_1}\int\frac{d\boldsymbol{k}_2}{E_2}e^{ipx}p^2(k_1q)(k_2q). \tag{C·1}$$

We employ the formula (3·7) of I,

$$\int\frac{d\boldsymbol{k}_1}{E_1}\int\frac{d\boldsymbol{k}_2}{E_2}=2^{3-N}\int ds\, s^{(N/2)-2}\int d\Omega_k\int d^Np\,\theta(p^0)\,\delta(p^2+s). \tag{C·2}$$

The angular integration can be carried out in the center-of-mass frame of the two particles;

$$\int d\Omega_k(k_1q)(k_2q)=\int[\boldsymbol{k}^2\omega^2-(\boldsymbol{k}\cdot\boldsymbol{q})^2]d\Omega_k=\boldsymbol{k}^2\omega^2(1-1/n)V_n, \tag{C·3}$$

where we have made use of the isotropy of the k-space to replace k_ik_j by $(\boldsymbol{k}^2/n)\delta_{ij}$ with i, j the spatial indices. The factor $\boldsymbol{k}^2\omega^2$ in (C·3) is $(qp)^2/4$ in the general frame. Putting all these together in (C·1) we obtain

$$(\text{C·1})=-16\frac{N-2}{n2^N}(2\pi)^nV_n(q\partial)^2\int ds\, s^{(N/2)-1}\Delta^{(+)}(x;s), \tag{C·4}$$

from which (3·22) follows. Other σ's are also evaluated in the same way. They all vanish due to the conditions $q^2=k_1^2=k_2^2=0$.

From (3·8d) and (3·8e) we find

$$\Pi^{\mu\nu,\rho\sigma(32)}\mathcal{G}^{III}_{\mu\nu,\rho\sigma}=\frac{1}{4}\Pi^{\mu\nu,\rho\sigma(31)}\mathcal{G}^{III}_{\mu\nu,\rho\sigma}=2\int\frac{d\boldsymbol{k}_1}{E_1}\int\frac{d\boldsymbol{k}_2}{E_2}e^{ipx}(pq)^2(k_1q)(k_2q). \tag{C·5}$$

Using the result (C·3) we obtain

$$(\mathrm{C}\cdot 5) = 16\frac{N-2}{n2^N}(2\pi)^n V_n (q\partial)^4 \int ds\, s^{(N/2)-2} \Delta^{(+)}(x;s), \tag{C·6}$$

from which (3·30) follows.

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