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Coordinate-Dependent Mass and the Validity of the WKB Approximation in Fission Barrier Penetration Calculations

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It is shown that the quasi-classical condition for the validity of the WKB approximation is satisfied although the mass associated with the spontaneous fission of ^{240}Pu varies by a factor of 12.

It has been known for quite some time that the mass (or mass parameter) associated with collective nuclear motion may vary substantially with the collective coordinates. For instance, the microscopically calculated[1)] diagonal matrix elements of the five-dimensional mass-matrix of ^{152}Sm vary by as much as a factor of 7 as the quadrupole deformation β varies from 0.0 to 0.5 and as the axial asymmetry variable γ varies from 0° to 60°. In a recent calculation [2)] of the fission lifetimes of transuranic and of superheavy nuclei, even larger variations (by factors of about 12) have been found. Such large variations are not very model-dependent. They arise from the general variation of Nilsson-type single-particle levels[3),4)] some of which go up as deformation increases while the others go down. Consequently, the energy denominator appearing in the cranking-type expression for the mass parameter varies rapidly with deformation, especially near level crossings.

Such variations of the collective mass by factors of 12, as the nucleus fissions, seem rather large from the point of view of using the WKB approximation to the barrier penetration probability. Hence , the quasi-classical condition necessary for the validity of the WKB approximation is examined below.

The WKB approximation is expected to be valid when the quasi-classical condition is satisfied. This condition is given by[5)]

$$|d\lambda\!\!\!^{-}/dr| \ll 1\,, \tag{1}$$

where $\lambda\!\!\!^{-}$ is the de-Broglie wavelength (divided by $2\pi\ i$) associated with the penetration process and is defined as

$$\lambda\!\!\!^{-} = \hbar/(ip) = \hbar\{2\mu[V(r)-E]\}^{-1/2}\,. \tag{2}$$

The barrier penetrability is then given by

$$p_f = \exp[-2\int(dr/\lambda\!\!\!^{-})]\,. \tag{3}$$

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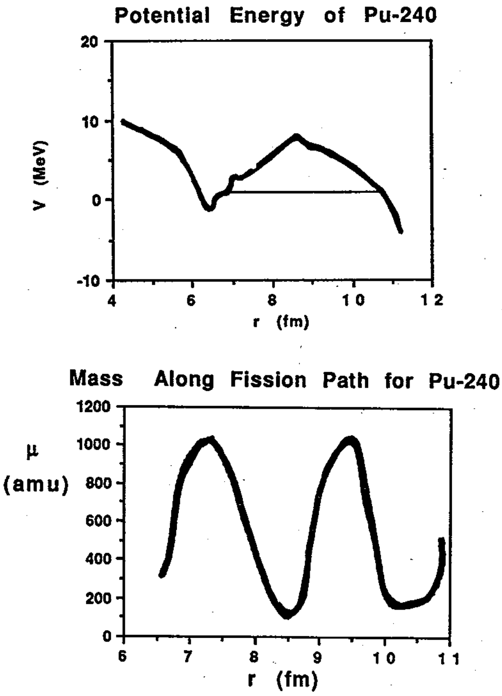


Fig. 1. Calculated potential energy and mass along the fission path of ^{240}Pu.

The actual coordinate-dependent masses and potentials were used[2] in a numerical integration of the action integral on the right-hand side of Eq. (3). Although, the method used was justified on the basis of good agreement with the experimental data for structure (level energies, $B(E2)$ values, quadrupole moments, magnetic moments,$\cdots$) as well as fission (barriers, lifetimes, mass asymmetry, heavy-ion emission, α-decay,$\cdots$), without any local parameter-fitting, it is legitimate to ask whether the numerical WKB method used is valid even when the mass depends strongly on the fission coordinates.

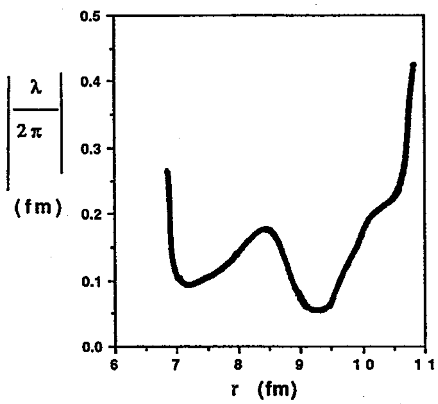
Since details of the coordinate-dependent mass were not given previously, we present in Fig. 1 the calculated potential energy and the fission-associated mass of ^{240}Pu. Details of the Dynamic Deformation Model used for the calulation have been given.[2] Main steps of the calculation are listed below.

In the first step, effects of deformations on the single-particle levels and matrix elements are taken into account. Empirical values are used for the energies of the single-particle states. The deforming potential is the difference between the potential energies of a triaxial oscillator and a spherical oscillator. Complete mixings are included in a large basis of eleven major shells. Scaling methods make this calculation independent of Z and of A. However, the steps discussed below are performed for each nucleus.

In the second step, the residual pairing interactions are taken into account in an improved BCS theory where the particle-hole terms are treated on an equal footing with the particle-particle and the hole-hole terms.

In the third step, the potential energy, three mass parameters (associated with β-γ-pairing-vibrations), three rotational moments of inertia, and various intrinsic electromagnetic matrix elements are calculated. The shell-correction method is used for the main part of the potential energy, while a time-dependent-mean-field theory is used for the rest.

In the fourth step, the quantized form of the collective Hamiltonian is solved by employing a non-linear, numerical integration method such that the rotation-vibration couplings and the K-mixings are taken into account in a very general theory of the collective bands of even-even nuclei. In particular, the energy of zero-point motion is calculated in a straightforward way as the difference between the ground state energy and the potential minimum energy. The ground state location, as well as the fission path, is indicated in Fig. 1 by a horizontal straight line which cuts the potential



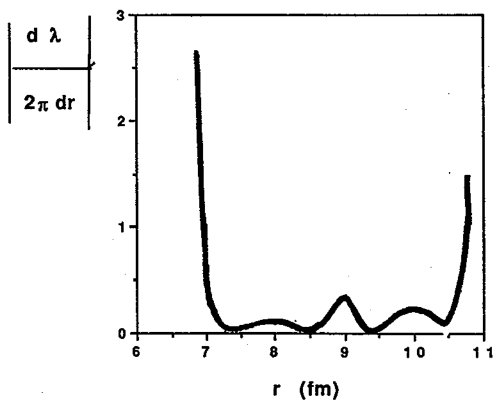


Fig. 2. Calculated de-Broglie wavelength and its derivative along the fission path of ^{240}Pu.

curve at $E=V$. This line lies at 1.41 MeV (= zero-point energy) above the ground state potential minimum.

In the fifth step, the fission penetrability and half-life are calculated. The potential energy calculated in step 3 is matched first to that for the Hill-Wheeler[6] y-family shapes and then to a nucleus-nucleus potential which depends directly on a fragment-dependent fission channel (Z_1, A_1, Z_2, A_2). For each fission channel, the fission path is determined by minimizing the action integrand between any two successive points in the β-γ-plane. Effects of the three mass parameters are projected onto a single mass μ along the fission coordinate r, which is defined as the separation distance between the two separated or nascent fragments. During the prescission stage, this distance equals (3/4) R_3 which is the separation distance between the centers of mass of the two nuclear halves, while R_3 is the semimajor axis length or half of the nuclear elongation along the fission axis.

The potential energy and the mass plotted in Fig. 1 are for the most probable fission channel (^{106}Mo+^{134}Te). Note that the fission-related mass of ^{240}Pu varies by a factor of 12 as r varies from 6.58 to 10.9 fm over the region of the action integral. In order to test the validity of the WKB approximation, we plot the de-Broglie wavelength of Eq. (2) and its derivative in Fig. 2. The quasi-classical condition (1) is not expected to be satisfied near the entrance and the exit points of the region of integration, but it is satisfied in the middle quite well.

Hence, we conclude that the use of the WKB approximation can be justified even when the mass depends substantially on the fission coordinates.

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