

Numerical Studies of Hydraulic Jump Phenomena with Largely Deformed Interfaces

Kensuke YOKOI^{1,2,*} and Feng XIAO²

¹*Division of Mathematics, and Research Institute for Electronic Science
Hokkaido University, Sapporo 060-0812, Japan*

²*Computational Science Division, RIKEN, Wako 351-0198, Japan*

(Received October 11, 1999)

A computational model for interfacial flows including the effect of surface tension force has been constructed based on the C-CUP (Cubic interpolated propagation, Combined Unified Procedure) method, the level set method and the CSF (Continuum Surface Force) model. The computational model can simulate largely deformed interfaces as those found in hydraulic jump phenomena. By using this computational model, we performed axisymmetric (r - z) simulations to clarify the structure formation of the circular hydraulic jump. The transition from a type I to a type II jump, which was induced by changing the depth of the fluid far away from the jet in the laboratory experiment, was investigated numerically. We found that the transition is associated with a rise in pressure beneath the surface immediately after the hydraulic jump. This result shows that the hydrostatic assumption used in most of the theoretical studies may not be appropriate for the formation of a type II jump.

The schematic figure of a circular hydraulic jump can be shown in Fig. 1(a). The vertical liquid jet impinges on the horizontal plate and spreads out radially, and the circular hydraulic jump is then formed. The phenomenon can also be observed in a kitchen sink and has been so far investigated experimentally and theoretically by many researchers.

In some experiments,¹⁾ the depth on the outside of the jump was controlled by varying the height of a circular wall d as shown in Fig. 1. Experimental results show that a circular hydraulic jump has two kinds of steady states that are reached by changing d .¹⁾ When d is small or 0, a steady state with the eddy on the bottom is formed as in Fig. 1(a). This flow structure is called type I. On increasing d the jump

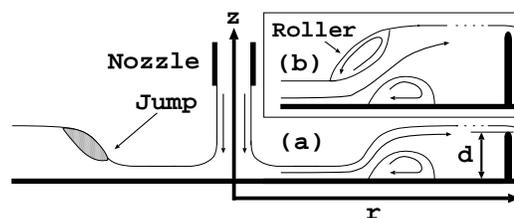


Fig. 1. Schematic figures of the circular hydraulic jump. The radius of the wall is much larger than the radius of the jump. The flow from the nozzle is constant. In experiment, a high viscous liquid is used for controlling the instability of flow pattern. (a) and (b) are called Type I and Type II respectively.

*) kensuke@atlas.riken.go.jp

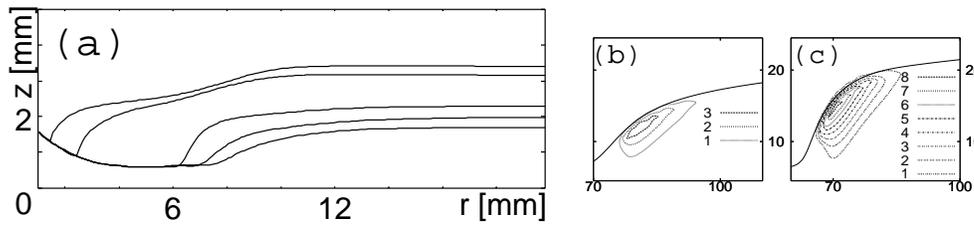


Fig. 2. (a) shows the surface profiles for different d . $Q = 5.6$ ml/s, $\nu_l = 7.6 \times 10^{-6}$ m²/s and $\sigma = 4.5 \times 10^{-2}$ N/m are used. The dynamic pressure [Pa] contours and the surface profiles around the jump of the second and the third from the lowest are shown in (b) and (c) respectively.

becomes steep until a critical d_c is reached. If d becomes larger than d_c , the liquid outside the jump topples. Then, another steady state with not only the eddy on the bottom but also on the surface is formed as in Fig. 1(b). This flow structure is called type II and the eddy on the surface is called a roller.

In this work, numerical simulations on circular hydraulic jumps were conducted by using the C-CUP method,²⁾ the level set method³⁾ and CSF model.⁴⁾ We investigated the transition from a type I jump to a type II jump. Non-hydrostatic pressure distributions in the gravitational direction were observed in our simulations. In our studies, we call ‘dynamic pressure’ the net amount of the pressure resulting from extracting the hydrostatic pressure from the actual pressure. We found that the dynamic pressure around the jump, which has been neglected in most of the theoretical studies to date, is important for the transition.⁵⁾ In a type I jump, a steeper jump is always associated with a higher wall height (Ref. 1) and Fig. 2(a)). Thus, as d is increased, the curvature of the interface immediately after the jump becomes larger, then the surface tension is strengthened, because the surface tension is proportional to the curvature. In order to counteract this surface tension and keep the jump surface steady, a larger rise in pressure is required (Fig. 2(b),(c)). If the wall height is increased over the critical d_c , the reverse pressure gradient generated by the dynamic pressure becomes stronger than the flow from below and a transition occurs.

In summary, axis-symmetrical simulations were carried out for the circular hydraulic jump. The transition from type I to type II was reproduced by changing d . We found that a type II jump formation depends on the establishment of a high pressure region after the jump.

Numerical computation in this work was partially carried out at the Computer Information Center, RIKEN and the Yukawa Institute for Theoretical Physics, Kyoto University.

References

- 1) T. Bohr et al., *Physica* **B228** (1996), 1.
- 2) T. Yabe and P. Y. Wang, *J. Phy. Soc. Jpn.* **60** (1991), 2105.
- 3) M. Sussman, P. Smereka and S. Osher, *J. Comput. Phys.* **114** (1994), 146.
- 4) J. U. Brackbill, D. B. Kothe and C. Zemach, *J. Comput. Phys.* **100** (1992), 335.
- 5) K. Yokoi and F. Xiao, *Phys. Lett.* **A257** (1999), 153.