Exotic Cluster Shapes in Neutron-Rich Be and C Isotopes

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(Received January 17, 2002)

The structure of C isotopes are investigated using a microscopic α+α+α+n+$n\cdots$ model based on the molecular orbit (MO) model. The stability of the linear-chain of 3α with respect to the breathing mode and the bending mode for various neutron configurations is investigated. The combination of the valence neutrons in the π- and the σ-orbit is promising to stabilize for these modes, and the excited state of ^{16}C with the $(3/2^-_\pi)^2(1/2^-_\sigma)^2$ configuration for the four valence neutrons is one of the most promising candidates for such structure. Furthermore, the equilateral-triangular shape of 3α surrounded by valence neutrons is suggested for ^{14}C based on a microscopic cluster model. The 3^-, 4^- and 5^- members of this rotational band appear around the ^{10}Be+α threshold, and these calculated states correspond to the experimentally observed 3^- state (9.80 MeV) and 4^- state (11.67 MeV). A positive-parity rotational band (0^+, 2^+, 4^+) also arises around this threshold energy, and these results suggest that the picture of inversion doublet structure works also in neutron-rich nuclei.

§1. Introduction

A survey of the molecule structure is one of the most challenging subjects in light neutron-rich nuclei. In Be isotopes, recently, decay into fragments of He isotopes (^{4}He, ^{6}He, ^{8}He) has been observed from the excited states of ^{10}Be and ^{12}Be and the presence of a two-center configuration is suggested. From the theoretical side, these states are studied by various models by which the molecular-orbital nature of the weakly bound neutrons around the two α-clusters has been revealed.[1),2)]

Recently, the discussions of the well-developed cluster structure are extended to the neutron-rich nuclei, and the role of valence neutrons which stabilize the linear-chain structure has been pointed out. For example, von Oertzen has extended his analyses for the molecular structure in Be isotopes[3)] to C isotopes, and the linear-chain state consisting of 3α and valence neurons around it has been speculated. We show that even if the 3α-system without valence neutrons (^{12}C) does not have a linear-chain structure, the valence neutrons around it are expected to increase the binding energy and stabilize the linear-chain state.[4)]

Furthermore, we investigate another kind of cluster structure in ^{14}C, that is an equilateral-triangular shape of the 3α core. The state with this symmetry is expected to appear much lower in energy than the linear-chain case. The presence of an equilateral-triangular shape of 3α has not been established in ^{12}C, partly because any other members of the rotational band starting from 3^-, have not been observed. In fact, the 3^- state is already above the 3α threshold by about 2 MeV, and a

large resonance width is expected for the 4^- and 5^- states, making their observation extremely difficult. On the contrary, we investigate neutron-rich C isotopes and show that this band structure is stabilized owing to the attractive interaction produced by the valence neutrons around the 3α core.

§2. Formulation

The total wave function of a microscopic $\alpha+\alpha+\alpha+n+n\cdots$ model is fully antisymmetrized and expressed by a superposition of basis states centered to different relative distances between the α-clusters (d) with various configurations of the valence neutrons ($c1$, $c2\cdots$) around the α-clusters:

$$\Phi^J_{MK} = \sum_{d,c1,c2\cdots} P^J_{MK}\mathcal{A}[\phi_1^{(\alpha)}\phi_2^{(\alpha)}\phi_3^{(\alpha)}(\phi_1^{c1}\chi_1)(\phi_2^{c2}\chi_2)\cdots]. \tag{2·1}$$

In the linear-chain case, the Gaussian centers of α-clusters are introduced on the z-axis, and the π-orbit with $\bar{K}^\pi = 3/2^-$ is described as a linear combination of p-orbits (ψ_x, ψ_y) centered at right- and left-hand side of the system.

$$|3/2^-_\pi\rangle = \{(\psi_x + i\psi_y)_{+a} + (\psi_x + i\psi_y)_{-a}\}|n\uparrow\rangle, \tag{2·2}$$

$$(\psi_x)_{\pm a} = G_{\pm a\vec{e}_z + b\vec{e}_x} - G_{\pm a\vec{e}_z - b\vec{e}_x}, \quad (\psi_y)_{\pm a} = G_{\pm a\vec{e}_z + b\vec{e}_y} - G_{\pm a\vec{e}_z - b\vec{e}_y}. \tag{2·3}$$

The $|1/2^-_\pi\rangle$ orbit where the spin-orbit interaction acts repulsively can also be defined by changing the spin-direction of $|3/2^-_\pi\rangle$.

$$|1/2^-_\pi\rangle = \{(\psi_x + i\psi_y)_{+a} + (\psi_x + i\psi_y)_{-a}\}|n\downarrow\rangle. \tag{2·4}$$

The distribution of the σ-orbit is fixed to the direction of the 3α axis, then it is introduced to have three nodes. $|1/2^-_\sigma\rangle$ is represented as a linear combination of three orbits with $\bar{K}^\pi = 1/2^-$, whose centers are $+a$, 0, and $-a$ on the z-axis.

$$|1/2^-_\sigma\rangle = \{(\psi_z)_{+a} - (\psi_z)_0 + (\psi_z)_{-a}\}|n\uparrow\rangle, \tag{2·5}$$

$$(\psi_z)_{\pm a} = G_{a\vec{e}_z + b\vec{e}_z} - G_{a\vec{e}_z - b\vec{e}_z}, \quad (\psi_z)_0 = G_{b\vec{e}_z} - G_{-b\vec{e}_z}. \tag{2·6}$$

These three orbits ($|3/2^-_\pi\rangle$, $|1/2^-_\pi\rangle$ and $|1/2^-_\sigma\rangle$) are the basic building blocks for the molecular-orbital structure. Also, $|-3/2^-_\pi\rangle$, $|-1/2^-_\pi\rangle$ and $|-1/2^-_\sigma\rangle$ orbits are introduced by taking the time reversal of $|3/2^-_\pi\rangle$, $|1/2^-_\pi\rangle$ and $|1/2^-_\sigma\rangle$ orbits, respectively.

The Hamiltonian and the effective nucleon-nucleon used is the same as in Ref. 2).

§3. Results

Based on the MO approach, we investigate polymer structure in C isotopes and show the stability of the linear-chain state for various configurations. The isotopes and configurations which we take into account are ^{12}C, ^{14}C$(3/2^-_\pi)^2$ (two n in the π-orbits), ^{14}C$(1/2^-_\sigma)^2$ (two n in the σ-orbits), ^{16}C$((3/2^-_\pi)^2(1/2^-_\pi)^2)$ (four n in the π-orbits) and ^{16}C$((3/2^-_\pi)^2(1/2^-_\sigma)^2)$ (two n in the π-orbits and two n in the σ-orbits).

As schematically shown in Fig. 1, two variational paths are introduced corresponding to the breathing (Fig. 1(a)) and the bending (Fig. 1(b)) degrees of freedom. The parameters d and θ stand for the α-α distance and the bending angle of the 3α-core, respectively.

Firstly, we discuss the 0^+ energy curves for the linear-chain structure with respect to the breathing-path. It is found that the energy pocket around $d = 3$ fm becomes deeper as the increase of number of valence neutrons in the π-orbit (^{12}C $\rightarrow$ ^{14}C$(3/2^-_\pi)^2$ $\rightarrow$ ^{16}C$((3/2^-_\pi)^2(1/2^-_\pi)^2)$). The 3α-system (^{12}C) has minimal energy around $d = 3.5$ fm, however, this is too shallow to conclude the stability of the linear-chain state. On the contrary, in ^{14}C$(3/2^-_\pi)^2$, there appears evident minimal energy around $d = 3$ fm. The energy (~ -82 MeV) is lower than ^{12}C by 11 MeV and the energy pocket is much deeper. After superposing states with different d-values, this energy corresponds to the excitation energy of 18 MeV from the ground state calculated with an equilateral-triangle configuration of the 3α-core, which is -101.2 MeV. The ^{16}C$((3/2^-_\pi)^2(1/2^-_\pi)^2)$ configurational state is most stable among states studied and has an energy pocket of ~ -86 MeV, where the α-α distance is $d = 2.5$ fm, shorter than those for ^{12}C and ^{14}C$(3/2^-_\pi)^2$. Therefore, the π-orbit is found to stabilize the linear-chain structure as the increase of valence neutrons.

We discuss the case where the valence neutrons occupy the σ-orbit. ^{14}C$(1/2^-_\sigma)^2$ has an excitation energy higher by about 14 MeV in comparison with ^{14}C$(3/2^-_\pi)^2$. It is rather surprising that the difference is only 14 MeV in spite of the fact that $3/2^-_\pi$ has only one node and $1/2^-_\sigma$ has three nodes. This is because the σ-orbit is along the α-α-α core: The higher nodal orbits along the symmetry axis become low-lying as a result of the clustering of the core. When two more valence neutrons occupy the π-orbit, although this minimal energy is higher by 9 MeV than that of ^{16}C$((3/2^-_\pi)^2(1/2^-_\pi)^2)$, ^{16}C$((3/2^-_\pi)^2(1/2^-_\sigma)^2)$ has the minimal 0^+ energy of ~ -77 MeV. The calculated energy pocket is deep enough to guarantee the stability for the breathing path.

Next, the stability of these linear-chain states with respect to the bending path is examined. The 0^+ energy curves of ^{12}C, ^{14}C$((3/2^-_\pi)^2)$, ^{14}C$((1/2^-_\sigma)^2)$, ^{16}C$((3/2^-_\pi)^2$ $(1/2^-_\pi)^2)$, and ^{16}C$((3/2^-_\pi)^2(1/2^-_\sigma)^2)$ with respect to the θ-value are shown in Fig. 2.

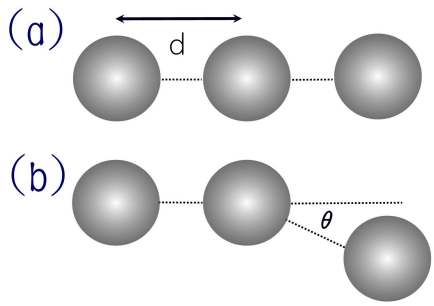

Fig. 1. The schematic figure for the breathing (a) and the bending (b) motion of the linear-chain state. The stability of the linear-chain state is examined for the α-α distance (d in (a)) and bending angle (θ in (b)).

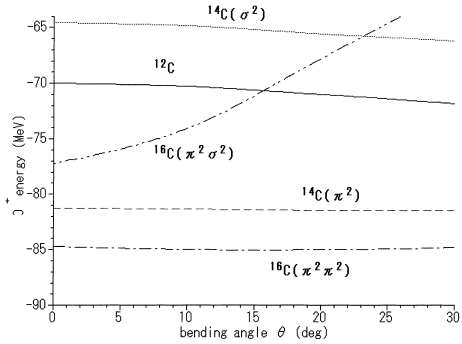

Fig. 2. The 0^+ energy curves with respect to the bending angle (θ) for ^{12}C (solid curve), ^{14}C($(3/2^-_\pi)^2$) (dashed curve), ^{14}C($(3/2^-_\sigma)^2$) (dotted curve), ^{16}C($(3/2^-_\pi)^2(1/2^-_\pi)^2$) (dash dotted curve) and ^{16}C($(3/2^-_\pi)^2(1/2^-_\sigma)^2$) (dash two-dotted curve). The 3α-threshold energy is calculated to be -82.5 MeV.

Except for the case of ^{16}C($(3/2^-_\pi)^2(1/2^-_\sigma)^2$), the curvature of these states is rather monotonic and the energy minimum does not clearly appear. In ^{14}C, the orthogonality between the linear-chain configuration and low-lying states with equilateral-triangle configuration of 3α is taken into account. This effect works but not sufficiently to push up the energy of the state with a finite bending angle. However, the ^{16}C($(3/2^-_\pi)^2(1/2^-_\sigma)^2$) case shows a sharp increase of the 0^+ energy as the increase of the bending angle and is found to be stable against the bending path. This feature is much different from ^{12}C, ^{14}C($(3/2^-_\pi)^2$), ^{14}C($(1/2^-_\sigma)^2$), and ^{16}C($(3/2^-_\pi)^2(1/2^-_\pi)^2$) cases. From the analysis above, the linear-chain configuration can be stabilized with respect to the breathing path by neutrons in the π-orbit (^{14}C($(3/2^-_\pi)^2$), ^{16}C($(3/2^-_\pi)^2(1/2^-_\pi)^2$), ^{16}C($(3/2^-_\pi)^2(1/2^-_\sigma)^2$)), but ^{16}C($(3/2^-_\pi)^2(1/2^-_\sigma)^2$) is only the case which is stable also against the bending path. It can be known that the drastic excitation as the increase of the bending angle is due to the increase of overlap between two neutrons in the π-orbit and two neutrons in the σ-orbit. The Pauli blocking effect increases the kinetic energy of system and stabilize the linear-chain configuration of ^{16}C.

Finally, we investigate equilateral configuration of 3α in ^{14}C using a microscopic α+α+α+n+n model. In ^{14}C, two valence neutrons rotate around the 3α-core symmetrically, just like covalent electrons in molecules. To describe such a density distribution, we introduce two kinds of basis states: shell-model-like and molecular-orbital basis states. In the shell-model-like basis state, the valence neutrons are described as shell-model orbits around the center of 3α. This basis state describes the neutron-density around the center, and is valid for the describing the yrast states with relatively small α-α distances. The lowest shell-model orbit of neutrons around 3α localized on the xy-plain is p_z ($\Phi(SM(p_z)^2)$). However, the two valence neutrons are not only localized around the center of the 3α-triangle, but they also rotate around each α-α pair. Therefore, we introduce another kind of the basis state, the molecular-orbital basis state ($\Phi(MO)$), where the orbit of the valence neutrons is described as a linear combination of orbits around each two-α pair. This basis state is important when the α-α distance becomes larger. The model space is

α+^{10}Be(α+α+2n).

Due to mixing of the $\Phi(SM(p_z)^2)$ and $\Phi(MO)$ configurations, in ^{14}C, there appear two 3^- states in the low energy region. From the second 3^- state, a rotational band structure comprised of the 3^-_2, 4^-_1, and 5^-_1 states is calculated just around the ^{10}Be+α threshold. Furthermore, it is significant that the corresponding states are experimentally observed in -2.21 MeV (3^-) and -0.34 MeV (4^-) with respect to the ^{10}Be+α threshold energy. This situation is very different from that of ^{12}C.

The calculation shows that a positive-parity rotational band with the cluster structure also appears around the ^{10}Be+α; the ground 0^+ state, the yrast 2^+ state, and the 4^+ state states (0^+_2, 2^+_2, 4^+_2) fit into the $J(J+1)$ rule. In this energy region, corresponding states are also experimentally observed. Therefore, both of the positive-parity (band head is 0^+) and the negative-parity (band head is 3^-) rotational bands appear around the ^{10}Be+α threshold energy. Although the neutron configurations are slightly different, these two rotational bands are considered to comprise an inversion doublet structure.

§4. Summary

It is summarized that the linear-chain structure of ^{16}C$((3/2^-_\pi)^2(1/2^-_\sigma)^2)$ with 3α core is the only case to have the simultaneous stabilities for the breathing-like break up path and for the bending-like path among ^{12}C, ^{14}C and ^{16}C. Other configurations, such as ^{14}C$((3/2^-_\pi)^2)$ and ^{16}C$((3/2^-_\pi)^2(1/2^-_\pi)^2)$ are stable against the breathing-like path but not stable against the bending-like path. A combination of the π- and the σ-orbits occupied by four neutrons plays doubly important roles to make a deep energy pocket for breathing-like path and to prevent the bending-like free motion of the system. The band head energy is calculated to be around 25 MeV in excitation, and is expected to form a rotational band with an energy slope of $\frac{\hbar^2}{2I} = 150$ keV.

The equilateral-triangular shape of 3α surrounded by valence neutrons is suggested for ^{14}C based on a microscopic cluster model. It is found that the attractive interaction between a valence neutron and a 3α particle stabilizes the rotational band structure built on top of the 3^- band head with an equilateral-triangular symmetry. The 3^-, 4^-, and 5^- members of this rotational band appear around the ^{10}Be+α threshold, and these calculated states correspond to the experimentally observed 3^- state (9.80 MeV) and 4^- state (11.67 MeV). A positive-parity rotational band (0^+, 2^+, 4^+) also arises around this threshold energy, and these results suggest that the picture of inversion doublet structure work also in neutron-rich nuclei.

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