Impurity Pinning of Spin Density Wave

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The collective excitations from the spin density wave pinned by some impurities are investigated with the self-consistent harmonic approximation applied for the phase Hamiltonian. It is found that the pinning-depinning transition due to the quantum fluctuation in the ground state cannot occur in two-dimensional nor three-dimensional isotropic system. In contrast, it has already been revealed that the quantum depinning (melting) can occur in one dimension. It suggests that the critical dimension of the quantum localization-delocalization phase transition is one, within the present approximation. The present result is also available for the charge density wave pinned by some impurities.

§1. Introduction

The impurity pinning of collective motions of the spin density wave (SDW) or charge density wave (CDW) is one of interesting topics in the statistical physics of disordered systems. The pinning-depinning transition, namely localized-delocalized transition due to the quantum fluctuation in one-dimensional systems was investigated by Suzumura and Fukuyama[1)] in the framework of the self-consistent harmonic approximation.[2),3)] As a result, it was revealed that for sufficiently large kinetic energy of the phase excitation the pinned SDW or CDW would melt. In higher dimension, however, the possibility of such a quantum melting transition has not been researched so much. In the present paper, the pinning-depinning transition of SDW or CDW in two and three dimensions is investigated within the self-consistent harmonic approximation.

§2. Model

Assuming the simplest isotropic pattern of SDW or CDW in higher dimension, namely the specific wave vectors are $(Q,\ Q)$ and $(Q,\ Q,\ Q)$ in two and three dimensions, respectively, the phase excitation of the collective modes in d dimension is expected to be described by the following phase Hamiltonian:

$$\begin{aligned} H &= H_0 + H_1, \\ H_0 &= \frac{L^d}{(2\pi)^d}\int d^d q \sum_{\alpha=1}^{d}[CP_\alpha(\vec{q})P_\alpha(-\vec{q}) + A\vec{q}^2\phi_\alpha(\vec{q})\phi_\alpha(-\vec{q})], \\ H_1 &= V_0\sum_{\alpha=1}^{d}\sum_i \cos[Qr_{\alpha,i} + \phi_\alpha(\vec{r}_i)], \end{aligned} \tag{2·1}$$

where $\phi_\alpha(\vec{q})$ is the phase, $P_\alpha(\vec{q})$ the conjugate momentum, V_0 the strength of the pinning potential, and $\vec{r}_i$ the position of the i-th impurity. It is a straightforward generalization of the Hamiltonian previously given in one[1)] and two dimensions.[4)] In

one dimension, with the self-consistent harmonic approximation including the zero-point quantum fluctuation, the pinning-depinning transition occurs at some critical value of $\frac{C}{A}$. We apply the same approximation for higher dimensions in the following sections.

§3. Localization length

Introducing the localization length L_0[5)] within which the system is coherent, the impurity pinning energy H_1 is rewritten with the self-consistent harmonic approximation as

$$H_1 \sim -V_0 \left(\frac{n_i}{L_0^d}\right)^{\frac{1}{2}} \gamma \frac{L^d}{(2\pi)^d} \int d^d q \left[1 - \frac{1}{2}\phi(\vec{q})\phi(-\vec{q})\right]. \tag{3·1}$$

Here

$$\gamma = e^{-\frac{1}{2}\langle\phi^2\rangle}, \tag{3·2}$$

where γ is determined by the self-consistent equation

$$\gamma = \exp\left(-\frac{1}{2}\langle\phi^2\rangle\right) = \exp\left[-\frac{C}{(2\pi)^d}\int d^d q T \sum_l \frac{1}{\omega_l^2 + \omega_t^2}\right] \tag{3·3}$$

and

$$\begin{aligned} \omega_l &= 2\pi l T, \\ \omega_t &= [4\pi A C(\vec{q}^2 + q_c^2)]^{\frac{1}{2}}, \\ q_c^2 &\equiv \frac{V_0 n_i^{\frac{1}{2}} \gamma}{2A L_0^{\frac{d}{2}}}. \end{aligned} \tag{3·4}$$

In the ground state $T \to 0$ it is reduced to

$$\gamma = \exp\left[-\frac{1}{2(2\pi)^d}\left(\frac{C}{A}\right)^{\frac{1}{2}} \int_{-\frac{1}{\alpha_0}}^{\frac{1}{\alpha_0}} d^d q \frac{1}{(\vec{q}^2 + q_c^2)}\right], \tag{3·5}$$

where α_0 is a cutoff length. After the q-integration, $\langle\phi^2\rangle$ becomes

$$\langle\phi^2\rangle = \frac{1}{4\pi}\left(\frac{C}{A}\right)^{\frac{1}{2}} \frac{1}{\alpha_0}[(1 + \alpha_0^2 q_c^2)^{\frac{1}{2}} - \alpha_0 q_c] \tag{3·6}$$

in two dimensions and

$$\langle\phi^2\rangle = \frac{1}{8\pi^2}\left(\frac{C}{A}\right)^{\frac{1}{2}} \frac{1}{\alpha_0^2}(1 + \alpha_0^2 q_c^2)^{\frac{1}{2}} + \alpha^2 q_c^2 \log\left[\frac{\alpha_0 q_c}{1 + (1 + \alpha^2 q_c^2)^{\frac{1}{2}}}\right] \tag{3·7}$$

in three dimensions, respectively.

The localization length should be determined by optimizing the following excess energy:

$$\frac{\Delta E}{L^d} = \frac{A}{bL_0^2} - \frac{V_0(n_i)^{\frac{1}{2}}}{L_0^{\frac{d}{2}}}\gamma(1-\log\gamma) + \frac{\Delta E_0}{L^d}, \tag{3·8}$$

where b is a constant ~ 1 and n_i is the density of impurities.[1)] ΔE_0 is the zero-point energy given by

$$\begin{aligned}\Delta E_0 &= \frac{L^d}{(2\pi)^d}\int d^d q \frac{1}{2}(\omega_t^2 - \omega_0^2) \\ &= \frac{L^d}{(2\pi)^d}(AC)^{\frac{1}{2}}\int_{-\frac{1}{\alpha_0}}^{\frac{1}{\alpha_0}} d^d q[(\vec{q}^2 + q_c^2)^{\frac{1}{2}} - |\vec{q}|],\end{aligned} \tag{3·9}$$

where

$$\omega_0 = (4AC\vec{q}^2)^{\frac{1}{2}}. \tag{3·10}$$

After the q-integration we get the forms

$$\Delta E_0 = \frac{L^2}{6\pi^2}(AC)^{\frac{1}{2}}\frac{1}{\alpha_0^3}[(1+\alpha_0^2 q_c^2)^{\frac{3}{2}} - \alpha_0^3 q_c^3 - 1] \tag{3·11}$$

in two dimensions and

$$\begin{aligned}\Delta E_0 = \frac{L^3}{16\pi^2}(AC)^{\frac{1}{2}}\frac{1}{\alpha_0^4}\Bigg[&(2+\alpha_0^2 q_c^2)(1+\alpha_0^2 q_c^2)^{\frac{1}{2}} \\ &+ \alpha_0^4 q_c^4 \log\left(\frac{\alpha_0 q_c}{1+(1+\alpha_0^2 q_c^2)^{\frac{1}{2}}}\right) - 2\Bigg]\end{aligned} \tag{3·12}$$

in three dimensions, respectively. In order to determine the localization length L_0, we at first solve the self-consistent equation (3·3) with respect to γ, and optimize the excess energy with respect to L_0. In general it should be done by some numerical calculation except for one-dimensional systems.

§4. Weak pinning

Without taking the zero-point quantum fluctuation into account, L_0 is always finite for $V_0 \neq 0$ and the SDW or CDW is pinned in the ground state.[1),4)] If we consider whether the depinning transition due to quantum fluctuation occurs or not, we have only to investigate the weak pinning limit $V_0 \to 0$. The lowest order term in $\Delta E/L^d$ is

$$-2\gamma\frac{(n_i\alpha_0)^{\frac{1}{2}}V_0}{2A}\left(\frac{\alpha_0}{L_0}\right)^{\frac{d}{2}} \tag{4·1}$$

for $d = 2$ and 3. The term is always negative independently of the ratio $\frac{C}{A}$. It leads to a finite L_0, as far as $V_0 \neq 0$. Thus the quantum depinning cannot occur in two

or three dimensions, in contrast to one dimension. The same term for $d = 1$ has the form

$$-2\gamma \frac{(n_i \alpha_0)^{\frac{1}{2}} V_0}{2A} \left(\frac{\alpha_0}{L_0}\right)^{\frac{1}{2}} \left(1 - \frac{1}{4\pi}\left(\frac{C}{A}\right)^{\frac{1}{2}}\right). \tag{4·2}$$

Thus the pinning-depinning transition should occur at the critical value of the ratio $\frac{C}{A}$.

§5. Summary

Within the self-consistent harmonic approximation it was revealed that the quantum pinning-depinning transition of SDW or CDW due to impurities does not occur in two nor three dimensions, because the quantum fluctuation is too small, in comparison with one dimension. It suggests that the critical dimension of the quantum localization-delocalization phase transition is one, within the present approximation. The present result is based on the isotropic SDW or CDW pattern, namely the wave vector is (Q, Q) or (Q, Q, Q). It would be more interesting to study on anisotropic cases.

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