

Dynamics with Vector Condensates at Finite Density in QCD and Beyond

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I describe the dynamics in recently revealed phases with vector condensates of gauge fields in dense QCD (gluonic phase) and other gauge models. In this case, the Higgs mechanism is provided by condensates of gauge (or gauge plus scalar) fields. Because most of the initial symmetries in such systems are spontaneously broken, their dynamics is very rich. In particular, by using the Ginzburg-Landau approach, the existence of a gluonic phase with both the rotational symmetry and the electromagnetic $U(1)$ being spontaneously broken was established. In other words, this phase describes an anisotropic superconductor. In the dual (confinement) description of this dynamics in dense two-flavor QCD, there are light exotic vector hadrons in the spectrum, some of which condense. There are also vortex-like and roton-like excitations in these phases.

§1. Introduction

In this review, I summarize results obtained by our group in the description of the dynamics in recently revealed phases with vector condensates of gauge fields in dense two-flavor QCD (gluonic phase)¹⁾⁻⁵⁾ and in other gauge models.⁶⁾⁻⁹⁾ Let me at once emphasize that the vector condensates of gluon fields in dense QCD are very different from the conventional gluon condensate in vacuum QCD.¹⁰⁾ While the latter is Lorentz invariant, the former are not. In fact, since vacuum expectation values of spatial components of vector fields break the rotational symmetry, it is natural to have a spontaneous breakdown both of external and internal symmetries in this case. Dynamics in such systems are very rich.

It is expected that at sufficiently high baryon density, cold quark matter should be in a color superconducting state (for reviews, see Ref. 11)). On the other hand, it was suggested long ago that quark matter might exist inside the central region of compact stars.¹²⁾ This is one of the main reasons why the dynamics of the color superconductivity has been intensively studied.

Bulk matter in compact stars should be in β -equilibrium, providing by weak interactions, and be electrically and color neutral. The electric and color neutrality conditions play a crucial role in the dynamics of quark pairing.¹³⁾⁻¹⁷⁾ Also, in the dense quark matter, the strange quark mass cannot be neglected. These factors lead to a mismatch $\delta\mu$ between the Fermi momenta of the pairing quarks.

As was revealed in Refs. 18) and 19), the gapped (2SC) and gapless (g2SC) two-flavor color superconducting phases¹⁵⁾ suffer from a chromomagnetic instability

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connected with the presence of imaginary Meissner masses of gluons. While the 8th gluon has an imaginary Meissner mass only in the g2SC phase, with the diquark gap $\bar{\Delta} < \delta\mu$ (an intermediate coupling regime), the chromomagnetic instability for the 4-7th gluons appears also in a strong coupling regime, with $\delta\mu < \bar{\Delta} < \sqrt{2}\delta\mu$. Later a chromomagnetic instability was also found in the three-flavor gapless color-flavor locked (gCFL) phase.²⁰⁾

Meissner and Debye masses are screening (and not pole) ones. It has been recently revealed in Ref. 3) that the chromomagnetic instabilities in the 4-7th and 8th gluonic channels correspond to two very different tachyonic spectra of plasmons. It is noticeable that while (unlike the Meissner mass) the (screening) Debye mass for an electric mode remains real for all values of $\delta\mu$ both in the 2SC and g2SC phases,^{18),19)} the tachyonic plasmons occur both for the magnetic and electric modes.³⁾ The latter is important since it clearly shows that this instability is connected with vectorlike excitations: Recall that two magnetic modes correspond to two transverse components of a plasmon, and one electric mode corresponds to its longitudinal component. This form of the plasmon spectrum leads to the unequivocal conclusion about the existence of vector condensates of gluons in the ground state of two flavor quark matter with $\bar{\Delta} < \sqrt{2}\delta\mu$, thus supporting the scenario with gluon condensates (gluonic phase) proposed in Ref. 1). While the analysis in the paper¹⁾ was done only in the vicinity of the critical point $\delta\mu \simeq \bar{\Delta}/\sqrt{2}$, a numerical analysis of the gluonic phase far away of the scaling region was considered in Refs. 21) and 22). It confirms the general picture suggested in Ref. 1).

§2. Renormalizable model for dynamics with vector condensates

Since a dynamics with vector condensates is a rather new “territory”, it would be important to have an essentially soluble model which would play the same role for such a dynamics as the linear σ models play for the conventional dynamics with spontaneous symmetry breaking with condensates of scalar fields. Fortunately, such a model exists: it is the gauged linear $SU(2)_L \times U(1)_Y$ σ -model (without fermions) with a chemical potential for hypercharge Y considered in Ref. 6). Let me describe this model. It will be very useful for better understanding the dynamics in the gluonic phase.

2.1. Vacuum solution as anisotropic superconducting medium

The Lagrangian density of this model reads (the metric $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ is used):

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^{(a)}F^{\mu\nu(a)} - \frac{1}{4}F_{\mu\nu}^{(Y)}F^{\mu\nu(Y)} + [(D_\nu - i\mu_Y\delta_{\nu 0})\Phi]^\dagger(D^\nu - i\mu_Y\delta^{\nu 0})\Phi - m^2\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2, \quad (2.1)$$

where the covariant derivative $D_\mu = \partial_\mu - igA_\mu - (ig'/2)B_\mu$, Φ is a complex doublet field $\Phi^T = (\varphi^+, \varphi_0)$, and the chemical potential μ_Y is provided by external conditions (to be specific, we take $\mu_Y > 0$). Here $A_\mu = A_\mu^{(a)}\tau^a/2$ are $SU(2)_L$ gauge fields (τ^a are three Pauli matrices) and the field strength $F_{\mu\nu}^{(a)} = \partial_\mu A_\nu^{(a)} - \partial_\nu A_\mu^{(a)} + g\epsilon^{abc}A_\mu^{(b)}A_\nu^{(c)}$.

B_μ is a $U_Y(1)$ gauge field with the field strength $F_{\mu\nu}^{(Y)} = \partial_\mu B_\nu - \partial_\nu B_\mu$. The hypercharge of the doublet Φ equals +1. This model has the same structure as the electroweak theory without fermions and with the chemical potential for hypercharge Y . Henceforth we will omit the subscript L , allowing various interpretations of the $SU(2)$.*)

The model is renormalizable and for small coupling constants g , g' and λ , the tree approximation is reliable there. Because the chemical potential explicitly breaks the Lorentz symmetry, the symmetry of the model is $SU(2) \times U(1)_Y \times SO(3)_{\text{rot}}$. As was shown in Ref. 6), for sufficiently large values of the chemical potential μ_Y , the condensates of both the scalar doublet Φ and the gauge field A_μ occur. The ground state solution is given by

$$\langle \Phi^T \rangle = (0, v_0), \quad |\langle W_z^{(-)} \rangle|^2 \equiv |C|^2 = \frac{\mu_Y v_0}{\sqrt{2}g} - \frac{v_0^2}{4}, \quad \langle A_0^{(3)} \rangle \equiv D = \frac{v_0}{\sqrt{2}}, \quad (2.2)$$

where

$$v_0 = \frac{\sqrt{(g^2 + 64\lambda)\mu_Y^2 - 8(8\lambda - g^2)m^2 - 3g\mu_Y}}{\sqrt{2}(8\lambda - g^2)}, \quad (2.3)$$

$W_\mu^{(\mp)} = \frac{1}{\sqrt{2}}(A_\mu^{(1)} \pm iA_\mu^{(2)})$, $\Phi^T = (\varphi^+, \varphi_0)$, and the vacuum expectation values of all other fields are equal to zero.**)

It is clear that this solution implies that the initial symmetry $SU(2) \times U(1)_Y \times SO(3)_{\text{rot}}$ is spontaneously broken down to $SO(2)_{\text{rot}}$, i.e., the corresponding medium is anisotropic. It is noticeable that the electromagnetic $U(1)_{\text{em}}$, with electric charge $Q_{\text{em}} = I_3 + Y/2$, is spontaneously broken by the condensate of W bosons, i.e., electric superconductivity takes place in this anisotropic medium.

Because the dynamics in this model is under control for small g , g' and λ , the model provides a proof that the dynamics with vector condensate is a real thing. Moreover, this dynamics is very rich. In particular, as was shown in Ref. 7), there are three types of topologically stable vortices in model (2.1), which are connected either with photon field or hypercharge gauge field, or with both of them. In Ref. 6), gapless Nambu-Goldstone excitations and roton-like excitations were revealed in this phase.

*) Note that because the $U(1)_Y$ symmetry is local, for a nonzero chemical potential μ_Y one should introduce a source term $B_0 J_0$ in Lagrangian density (2.1) in order to make the system neutral with respect to hypercharge Y (the Gauss law). The value of the background hypercharge density J_0 (representing heavy particles) is determined from the requirement that $B_0 = 0$ is a solution of the equation of motion for B_0 .⁶⁾ There exists an alternative description of this dynamics in which a background hypercharge density J_0 is considered as a free parameter and μ_Y is taken to be zero. Then the Gauss law will define the vacuum expectation value $\langle B_0 \rangle$. It is not difficult to check that these two approaches are equivalent if the chemical potential μ_Y in the first approach is taken to be equal to the value $\frac{g'}{2} \langle B_0 \rangle$ from the second one.

**) Here "sufficiently large values of μ_Y " means the following: When $m^2 \geq 0$, μ_Y should be larger than the critical value $\mu_Y^{(\text{cr})} = m$, and for $m^2 < 0$, μ_Y should be larger than $\mu_Y^{(\text{cr})} = g|m|/2\sqrt{\lambda}$ (the critical value $g|m|/2\sqrt{\lambda}$ coincides with the mass of W boson in the vacuum theory with $\mu_Y = 0$ and $m^2 < 0$).

It is noticeable that solution (2.2) describes a nonzero field strength $F_{\mu\nu}^{(a)}$ which corresponds to the presence of *non-abelian* constant “chromoelectric”-like condensates in the ground state. Choosing the vacuum with the condensate $\langle W_z^{(-)} \rangle$ to be real, we find from Eq. (2.2)

$$E_3^{(2)} = F_{03}^{(2)} = g\sqrt{2} \langle W_z^{(-)} \rangle \langle A_0^{(3)} \rangle = gv_0 \sqrt{\frac{\mu_Y v_0}{\sqrt{2}g} - \frac{v_0^2}{4}}. \quad (2.4)$$

We emphasize that while an abelian constant electric field in different media always leads to an instability,^{*)} non-abelian constant chromoelectric fields do not in many cases. For a discussion of the stability problem for constant non-abelian fields, see Refs. 1) and 23). On a technical side, this difference is connected with that while a vector potential corresponding to a constant abelian electric field depends on spatial and/or time coordinates, a constant non-abelian chromoelectric field is expressed through constant vector potentials, as takes place in our case, and therefore momentum and energy are good quantum numbers in the latter.

As we will see below, the dynamics in the gluonic phase strikingly resembles the dynamics in this renormalizable model being however much more complicated.

2.2. Vector condensates, gauge invariance and Gauss law constraint

Condensates of vector fields are of course not gauge invariant. What is a gauge invariant description of the physics they manifest? In the Higgs phase, as that described in the previous subsection, the simplest way to achieve this is to use a unitary gauge. The important point is that in the unitary gauge, all auxiliary (gauge dependent) degrees of freedom are removed. *Therefore in this gauge the vacuum expectations values (VEVs) $\langle A_\mu^{(a)} \rangle$ of vector fields are well-defined physical quantities.* The unitary gauge in the renormalizable model in Eq. (2.1) is given by the constraint

$$\Phi^T = (0, \phi_0), \quad (2.5)$$

where ϕ_0 is a real field. The unitary gauge in the gluonic phase in dense QCD, used in Refs. 1) and 5), will be described below in §3.

In the case when a Higgs field is assigned to the fundamental representation of the gauge group, there exists a dual, gauge invariant, description in terms of composite gauge singlet fields. The structure of these composites for the electroweak theory (and, therefore, for the present model) is explicitly given in Ref. 25). In particular, in this description, the condensate of a gauge $W_z^{(\mp)}$ field is replaced by a condensate of a gauge invariant vector composite.

There is another subtlety in the description of the dynamics with vector condensates, which is connected with the derivation of a *physical* effective potential, whose minima correspond to stable or metastable vacua. The point is that although the gauge symmetry is gone in the unitary gauge, the theory still has constraints. In

^{*)} In metallic and superconducting media, such an instability is classical in its origin. In semiconductors and insulators, this instability is manifested in creation of electron-hole pairs through a quantum tunneling process.

fact, it is a system with second-class constraints, similar to the theory of a free massive vector field A_μ described by the Proca Lagrangian (for a thorough discussion of systems with second-class constraints, see §2.3 in Ref. 24)). In such theories, while the Lagrangian formalism can be used without introducing a gauge, the physical Hamiltonian is obtained by explicitly resolving the constraints. In our case, this implies that to obtain the physical effective potential V_{phys} one has to impose the Gauss law constraints on the conventional effective potential V .

This feature is intimately connected with the presence of time-like components of vector fields associated with would be negative norm states. The Gauss constraints amount to integrating out the time-like components. In tree approximation, this can be done by using their equations of motion. In particular, one can show that solution (2.2) is a minimum of the physical potential V_{phys} .⁶⁾ Is it the global minimum and are there other minima? This question has been recently addressed in Refs. 8) and 9) and we will consider it in the next subsection.

2.3. Landscape of vacua

The analysis of minima of the physical potential V_{phys} in this model is quite nontrivial. Indeed, there are 10 physical fields in the model, and the problem is equivalent of studying the geometry of a ten dimensional hypersurface corresponding to V_{phys} . Very recently, this analysis has been done for the special case with the quartic coupling constant λ and the mass of the scalar field Φ chosen to be zero.^{8),9)} This case, retaining richness of the dynamics, simplifies the analysis of the structure of the vacuum manifold. This allowed to establish that the anisotropic superconducting vacuum described in §2.1 above is the *global* vacuum in the model. Besides that, there are some metastable vacua. Among them, the metastable vacua with an abnormal number of Nambu-Goldstone bosons were identified.*) The $SO(2)$ symmetry of these vacua corresponds to locking gauge, flavor, and spin degrees of freedom. There are also metastable $SO(3)$ rotationally invariant vacua. Thus, the landscape of vacua in the model is rich. These results encourage studies of the vacuum landscape in dense QCD (see Ref. 5)).

§3. Gluonic phase in dense two-flavor QCD

Both the chromomagnetic^{18),19)} and plasmon³⁾ instabilities for the 4-7th gluons in the 2SC phase at $\bar{\Delta} < \sqrt{2}\delta\mu$ suggest a condensation of these gluon fields. Because the chromomagnetic instability develops in the magnetic channel, it is naturally to expect that some spatial components of these fields have a nonzero VEVs. The latter implies that the rotational symmetry should be spontaneously broken. This led to the suggestion that the underlying dynamics is similar to that in the gauged σ model considered in §2 above.²⁷⁾ Such a solution (the gluonic phase) was revealed in Ref. 1). A detailed description of this phase has been recently given in Ref. 5).

*) It is the same phenomenon as that found in a non-gauge relativistic field model at finite density in Ref. 26).

3.1. Condensates in gluonic phase

At intermediate energy scales of the order of the diquark condensate $\bar{\Delta} \sim \mathcal{O}(50 \text{ MeV})$, the analysis of QCD dynamics is very hard. Hence the phenomenological Nambu-Jona-Lasinio (NJL) model plays a prominent role in the analysis in dense quark matter.^{(11), (15)–(17)} The NJL model is usually regarded as a low-energy effective theory in which massive gluons are integrated out. The situation with dense quark matter is however quite different from that in the vacuum QCD. In our analysis,^{(1), (5)} we introduce gluonic degrees of freedom into the NJL model because in the 2SC/g2SC phase the gluons of the unbroken $SU(2)_c$ subgroup of the color $SU(3)_c$ are left as massless, and, near the critical point $\delta\mu = \bar{\Delta}/\sqrt{2}$, plasmons in the 4-7th gluon channels are also very light.⁽³⁾ This yields the gauged NJL model. Our analysis was done in the framework of the Ginzburg-Landau (GL) approach. Here I will present a physical picture underlying the gluonic phase. As we will see, it strikingly resembles that corresponding to the gauged σ model considered in §2.

The major “players” in the dynamics in the gluonic phase are the following. The first one is the chemical potential matrix $\hat{\mu}_0$ for up and down quarks. In the β -equilibrium, it is

$$\hat{\mu}_0 = \mu - \mu_e Q_{\text{em}} + \mu_8 Q_8, \quad (3.1)$$

where $Q_{\text{em}} \equiv \text{diag}(2/3, -1/3)_f$ is the electric charge, $Q_8 \equiv \text{diag}(1/3, 1/3, -2/3)_c$ is the color charge, and μ , μ_e and μ_8 are the quark, electron and color chemical potentials, respectively (the baryon chemical potential μ_B is $\mu_B \equiv 3\mu$ and the mismatch $\delta\mu$ between the Fermi momenta of the pairing quarks is $\delta\mu = \mu_e/2$). Here the subscripts f and c mean that the corresponding matrices act in the flavor and color spaces. The second player is the diquark field $\Phi^\alpha \sim i\bar{\psi}^C \varepsilon \varepsilon^\alpha \gamma_5 \psi$, with $(\varepsilon) \equiv \varepsilon^{ij}$ and $(\varepsilon^\alpha) \equiv \varepsilon^{\alpha\beta\gamma}$ being the totally antisymmetric tensors in the flavor and color spaces, respectively. The last players are seven gluon fields $A_\mu^{(1)} - A_\mu^{(7)}$, which are light near the critical point $\delta\mu = \bar{\Delta}/\sqrt{2}$.

Let us start from the 2SC/g2SC phase, with subcritical values of $\delta\mu < \bar{\Delta}/\sqrt{2}$. In this case, the only condensate is that of the diquark field. Without loss of generality, the diquark condensate in the 2SC/g2SC phase can be chosen along the anti-blue direction: $\langle \Phi^r \rangle = 0$, $\langle \Phi^g \rangle = 0$, $\bar{\Delta} \equiv \langle \Phi^b \rangle \neq 0$.

The gap $\bar{\Delta}$ breaks the color $SU(3)_c$ down to $SU(2)_c$. The octet of gluons is decomposed with respect to $SU(2)_c$ as

$$\mathbf{8} = \mathbf{3} \oplus \mathbf{2} \oplus \bar{\mathbf{2}} \oplus \mathbf{1}, \quad \text{i.e.,} \quad \{A_\mu^a\} = (A_\mu^{(1)}, A_\mu^{(2)}, A_\mu^{(3)}) \oplus \phi_\mu \oplus \phi_\mu^* \oplus A_\mu^{(8)}, \quad (a = 1, 2, \dots, 8). \quad (3.2)$$

Here we defined the complex doublets of the matter (with respect to the gauge $SU(2)_c$) fields:

$$\phi_\mu \equiv \begin{pmatrix} \phi_\mu^r \\ \phi_\mu^g \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} A_\mu^{(4)} - iA_\mu^{(5)} \\ A_\mu^{(6)} - iA_\mu^{(7)} \end{pmatrix}, \quad \phi_\mu^* \equiv \begin{pmatrix} \phi_\mu^{*r} \\ \phi_\mu^{*g} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} A_\mu^{(4)} + iA_\mu^{(5)} \\ A_\mu^{(6)} + iA_\mu^{(7)} \end{pmatrix}. \quad (3.3)$$

Because of the chromomagnetic instability, one should expect that a spatial component of the complex doublet ϕ_μ has a nonzero VEV for the supercritical values

of $\delta\mu > \bar{\Delta}/\sqrt{2}$. By using the rotational symmetry $SO(3)_{\text{rot}}$, one can take $\langle\phi_z\rangle \neq 0$. And because of the $SU(2)_c$ symmetry, without loss of generality, we can choose $\langle A_z^{(6)}\rangle \neq 0$. Acting as a Higgs field, ϕ_z breaks the $SU(2)_c$ down to nothing. Besides that, it also breaks the $SO(3)_{\text{rot}}$ down to $SO(2)_{\text{rot}}$.

The analysis^(1),5) shows that there indeed exists a stable (at least locally) solution in which, besides the diquark condensate $\langle\Phi^b\rangle = \bar{\Delta}$, there are the following condensates for gluon fields:

$$\langle\phi_z^T\rangle = \frac{1}{\sqrt{2}}(0, \langle A_z^{(6)}\rangle \equiv v'_0), \quad \langle A_z^{(\pm)}\rangle \equiv C', \quad \langle A_0^{(3)}\rangle \equiv D', \quad (3.4)$$

where $A_\mu^{(\pm)} = 1/\sqrt{2}(A_\mu^{(1)} \pm iA_\mu^{(2)})$. These VEVs lead to chromoelectric field strength condensates, similar to that in model (2.1) (see Eq. (2.4)).

Note that VEVs (3.4) correspond to choosing the following unitary gauge in the gluonic phase:

$$\Phi^T = (0, 0, \Phi^b \equiv \Delta), \quad \phi_z^T = \frac{1}{\sqrt{2}}(0, A_z^{(6)}) \quad (3.5)$$

with the fields Δ and $A_z^{(6)}$ being real (compare with Eq. (2.5)).

3.2. Symmetry breaking structure in the gluonic phase

Comparing Eq. (3.4) with Eq. (2.2), one can see that the condensates v'_0 , C' , and D' in the gluonic phase play the same role and the condensates v_0 , C , and D in the model (2.1). In particular, as in that case, the $SO(3)_{\text{rot}}$ and the electromagnetic $U(1)_{\text{em}}$ are spontaneously broken in the gluonic phase, i.e., this phase describes an anisotropic superconducting medium. Let us describe this feature in more detail.

As is well known,⁽¹¹⁾ in the 2SC/g2SC phase, the generator of the unbroken $\tilde{U}(1)_{\text{em}}$ is $\tilde{Q} = Q - \frac{1}{\sqrt{3}}T^8$, where Q is the electric charge in the vacuum QCD and T^a are the $SU(3)_c$ generators. The modified baryon charge is $\tilde{B} = 2(\tilde{Q} - I_3)$, where I_3 is the flavor isospin generator.

In the gluonic phase, the VEV $v'_0 = \langle A_z^{(6)}\rangle$ breaks $SU(2)_c$, but a linear combination of the generator T^3 from the $SU(2)_c$ and \tilde{Q} , $\tilde{\tilde{Q}} = \tilde{Q} - T^3 = Q - \frac{1}{\sqrt{3}}T^8 - T^3$, determines the unbroken $\tilde{\tilde{U}}(1)_{\text{em}}$ (the new baryon charge is $\tilde{\tilde{B}} = 2(\tilde{\tilde{Q}} - I_3)$). However, because T^1 does not commute with T^3 , the VEV $C' = \langle A_3^{(1)}\rangle$ breaks $\tilde{\tilde{U}}_{\text{em}}(1)$. The baryon charge is also broken. The gluonic phase is an anisotropic superconductor indeed.

3.3. Exotic hadrons in gluonic phase

It is easy to check that the electric charge $\tilde{\tilde{Q}}_{\text{em}}$ and the baryon number $\tilde{\tilde{B}}$ are integer both for all gluons and all quarks (see Tables I and II in Ref. 5)). Do they describe hadronic-like excitations? We believe that the answer is “yes”.^(1),5) The point is that because both the Higgs fields Φ and ϕ_z are assigned to (anti-) fundamental representations of the corresponding gauge groups ($SU(3)_c$ and $SU(2)_c$, respectively), there should exist a dual, gauge invariant (or confinement), description of this dynamics.⁽²⁵⁾

Such a description for the gluonic phase has been recently considered in Ref. 5). It was shown there that all the gluonic and quark fields can indeed be replaced by colorless composite ones in the confinement picture. The flavor quantum numbers of these composite fields are described by the conventional electric and baryon charges Q_{em} and \mathcal{B} . They are integer and coincide with those the charges \tilde{Q}_{em} and $\tilde{\mathcal{B}}$ yield for gluonic and quark fields (see Tables III and IV in Ref. 5)).

A very interesting point is that vector hadrons corresponding to $A_{\mu}^{(\pm)}$ gluons carry *both* electric and baryon charges: $Q_{\text{em}} = \pm 1$, $\mathcal{B} = \pm 2$. In other words, they are exotic hadrons (in vacuum QCD, bosons carry of course no baryon charge). The origin of these exotic quantum numbers is connected with (anti-) diquarks, which are constituents of these hadrons (see Table IV in Ref. 5)). Indeed, (anti-) diquarks are bosons carrying the baryon charge $\pm 2/3$ and therefore are exotic themselves. This feature has a dramatical consequence for the gluonic phase. Since in the Higgs description of this phase $A^{(\pm)}$ gluons condense (leading to the spontaneous $\tilde{U}(1)_{\text{em}}$ breakdown), we conclude that in the confinement picture this corresponds to a condensation of *exotic* charged vector hadrons. In this regard, it is appropriate to mention that some authors speculated about a possibility of a condensation of vector ρ mesons in dense baryon matter.²⁸⁾ The dynamics in the gluonic phase yields a scenario even with a more unexpected condensation.

§4. Conclusion

The dynamics in the gluonic phase is extremely rich. The existence of exotic hadrons there is especially intriguing. What could be directions for future studies in gluonic-like phases? It is evident that it would be interesting to consider the spectrum of light collective excitations there. It is also clear that it would be worth to figure out whether phases with vector condensates of gluons could exist in dense matter with three quark flavors. It would be also interesting to examine a landscape of (stable and metastable) vacua with different types of gluon condensates, some of which have been pointed out in Ref. 5). Last but not least, it would be important to study manifestations of gluonic-like phases in compact stars.

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