

Chiral-Super Interplay in QCD

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On the basis of the Ginzburg-Landau free energy with QCD symmetry, the interplay between the chiral and diquark condensates is investigated. We demonstrate that the axial anomaly drives a new critical point in the phase diagram and leads to a smooth crossover between the hadronic and color superconducting phases. We also discuss the continuity of these phases in the excited states.

§1. Introduction

Elucidating the aspects of QCD in the intermediate baryon density region between the hadronic and deconfined phases is one of the most interesting challenges. Phenomenologically, the phase structure of this region is relevant to the matter inside neutron and possible quark stars and also to the dynamics of moderate energy heavy-ion collisions in the future, e.g., at GSI.

Recently in 1), we have shown the existence of a new critical point and an associated smooth crossover at intermediate density region, using a model-independent Ginzburg-Landau (GL) analysis of the phase structure. The appearance of this smooth crossover is intimately connected to the idea of the continuity between hadronic and quark phases, proposed in 2). In our study, the coupling between the chiral condensate $\Phi \sim \langle \bar{q}q \rangle$ and the diquark condensate $d \sim \langle qq \rangle$ plays a crucial role.

§2. Ginzburg-Landau potential

The guiding principle in constructing the GL potential describing chiral and diquark condensates is the preservation of the QCD symmetry, $\mathcal{G} \equiv SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_A \times SU(3)_C$, where $U(1)_A$ is explicitly broken down to Z_6 by the axial anomaly. We define the chiral condensate (Φ) and diquark condensate (d) in the $J^P = 0^+$ channel by $\Phi_{ij} \sim -\langle \bar{q}_R^j q_L^i \rangle$ and $\langle (q_L)_b^j C (q_L)_c^k \rangle \sim \epsilon_{abc} \epsilon_{ijk} [d_L^\dagger]_{ai}$ (plus $L \leftrightarrow R$), where i, j, k and a, b, c are the flavor and color indices, and $C = i\gamma^2\gamma^0$ is the charge conjugation operator. The order parameters transform under \mathcal{G} as $\Phi \rightarrow e^{-2i\alpha_A} V_L \Phi V_R^\dagger$, $d_L \rightarrow e^{2i\alpha_B + 2i\alpha_A} V_L d_L V_C^\dagger$, where the $V_{L,R,C}$ are $SU(3)_{L,R,C}$ rotations and the $\alpha_{A,B}$ are $U(1)_{A,B}$ rotations. Given these transformations, and assuming that the order parameters are small enough to write a power series, we

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can construct the most general GL potential $\Omega(\Phi, d)$ invariant under \mathcal{G} except for $U(1)_A$. For three massless flavors, after making an ansatz of maximal flavor symmetry ($\Phi = \text{diag}(\sigma, \sigma, \sigma)$ and $d_L = -d_R = \text{diag}(d, d, d)$), the general GL potential assumes the simple form up to fourth order in Φ and d ,

$$\Omega_{3F} = \left(\frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4 \right) + \left(\frac{\alpha}{2}d^2 + \frac{\beta}{4}d^4 \right) - \gamma d^2\sigma + \lambda d^2\sigma^2, \quad (2.1)$$

where all the coefficient terms depend on temperature, T , and chemical potential, μ . Since $|\beta| \gg |\lambda|$, as can be shown from microscopic theories such as weak coupling QCD and the Nambu–Jona-Lasinio (NJL) model, we treat the λ -term as a small perturbation in the three-flavor case. On the other hand, γ and c , which both originate from the axial anomaly, are not negligible. Indeed, a positive c is essential for making the η' meson much heavier than the pion; it also leads to a first-order chiral phase transition at finite T with $\mu = 0$. In Eq. (2.1), the γ -term acts as an external field for σ and leads to a new critical point as we see below.

In principle, the system can have four possible phases: normal (NOR) ($\sigma = d = 0$), CSC ($\sigma = 0, d \neq 0$), NG ($\sigma \neq 0, d = 0$), and coexistence (COE) ($\sigma \neq 0, d \neq 0$). We locate the phase boundaries and the order of the phase transitions within the GL formalism by comparing the potential minima in these phases.

§3. Phase structure

Figure 1 shows the phase structure in the a - α plane for the massless three-flavor case with and without the γ -coupling.*) The figure shows the case for $b > 0$; the structure for $b < 0$ is not qualitatively different as long as we introduce a σ^6 -term to stabilize the system. Without the γ -coupling, the critical lines of the chiral and the CSC transitions simply cross (left panel of Fig. 1). With the γ -coupling, the phase structure undergoes major modifications. Firstly, the first order line between the CSC and COE phases, which originally continued all the way down terminates at a critical point; as a result, the CSC and COE phases are continuously connected. This is because the γ -term acts as an external field for σ , smoothing out the first order chiral transition for large γd^2 . Moreover, the second order CSC transition splits into two, since σ varies discontinuously across the first order line between the NOR and COE phases.

Mapping the coordinates from (a, α) to (T, μ) , which requires a microscopic model, is beyond the scope of the present Ginzburg-Landau approach. Assuming that at low temperature, the NG phase appears at low densities and the CSC phase at high densities we obtain the schematic phase diagram for massless three-flavor shown in the left panel of Fig. 2. Analyzing the massless two-flavor case similarly, we find the phase diagram in the right panel of Fig. 2.

The phase structure with realistic quark masses (light up and down quarks and a medium-heavy strange quark) corresponds to a situation intermediate to the two

*) For $\gamma > (c/3)\sqrt{\beta/b}$, a tricritical point emerges on the boundary between the NG and COE phases.

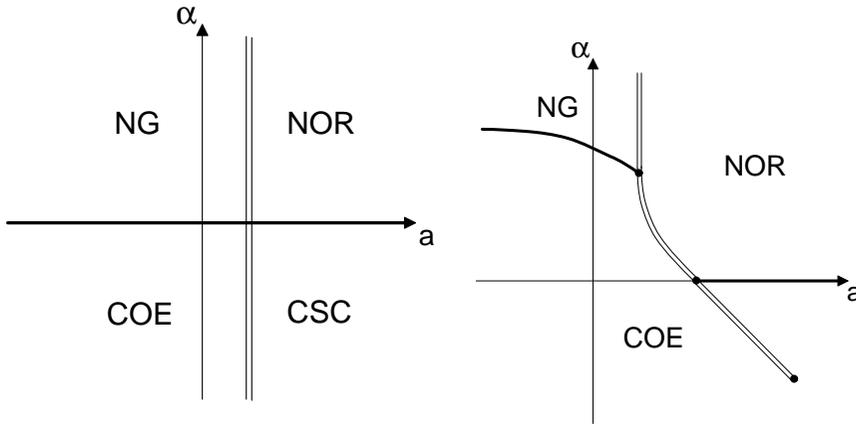


Fig. 1. Phase structure in the a - α plane in the massless three-flavor case without γ -coupling (left) and with (right). Phase boundaries with a first order transition are denoted by a double line and second order by a single line.

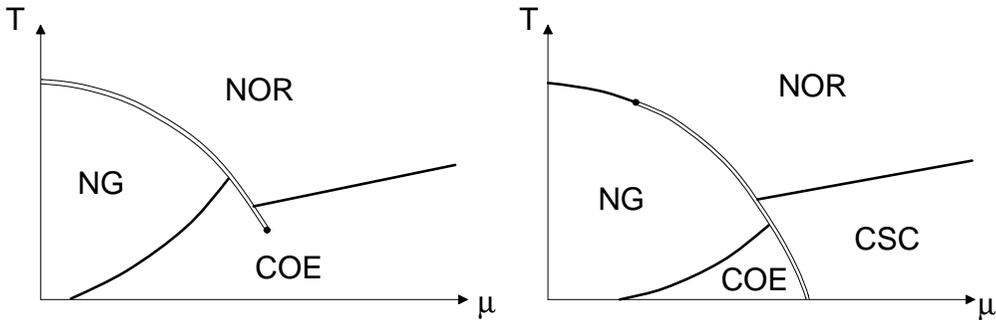


Fig. 2. Schematic phase structure in the T - μ plane for the massless three-flavor case (left) and massless two-flavor case (right).

cases in Fig. 3. There are two critical points in the diagram. The first one, near the vertical axis originally found by Asakawa and Yazaki,³⁾ is driven by the quark mass which washes out the second order transition in high T region. The one near the horizontal axis is our new critical point originating from the axial anomaly. The existence of the new critical point indicates that a smooth crossover takes place between the COE and CSC phases.*)

§4. Excited states

Furthermore we can study the excited states such as pions and the H boson in the intermediate density region for three flavors. The coupling between the chiral and diquark condensates via the axial anomaly plays an important role.⁵⁾ As a consequence, we obtain a *generalized* Gell-Mann–Oakes–Renner (GOR) relation

*) A similar critical point has been pointed out in 4) from the two-flavor NJL model. However, its origin is quite different from ours, since the axial anomaly does not produce the σd^2 coupling in two-flavor case.

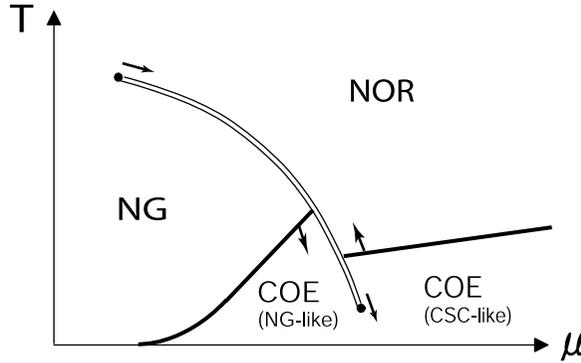


Fig. 3. Schematic phase structure with realistic quark masses (light up and down quarks and a medium-heavy strange quark). The arrows show how the critical point and the phase boundaries move as the strange-quark mass increases toward the two-flavor limit.

between the masses of pseudoscalar bosons and the magnitude of the chiral and diquark condensates as follows:

$$(f_{\pi}^2 + f_{\tilde{\pi}}^2)m_{\Pi}^2 = m_q(A_0\sigma + \Gamma_1 d^2), \quad (4.1)$$

where f_{π} and $f_{\tilde{\pi}}$ are the decay constants for excited states around the chiral and diquark condensates, respectively. The parameters A_0 and Γ_1 are GL coefficients.

§5. Conclusions

Using the model-independent GL approach, we have studied the competition or the interplay between the chiral and diquark condensates. We found that a new critical point driven by the axial anomaly emerges and leads to a smooth crossover between the hadronic phase and the color superconducting phase. Furthermore, we found that there is a smooth continuity of the excited states, e.g., the ordinary pion ($\bar{q}q$) in the NG phase and an exotic pion ($\bar{q}\bar{q}qq$) in the CSC phase are mixed together and it leads us to a generalized GOR relation in the moderate density region. Determination of the precise location of the new critical point with the help of phenomenological models and lattice QCD simulations is a fascinating and important open problem.

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