

Hadrons in Medium — Theory Confronts Experiment

Fabian EICHSTAEDT, Stefan LEUPOLD, Ulrich MOSEL^{*)} and Pascal MUEHLICH

Institut fuer Theoretische Physik, Universitaet Giessen, Giessen, Germany

In this talk we briefly summarize our theoretical understanding of in-medium selfenergies of hadrons. With the special case of the ω meson we demonstrate that earlier calculations that predicted a significant lowering of the mass in medium are based on an incorrect treatment of the model Lagrangian; more consistent calculations lead to a significant broadening, but hardly any mass shift. We stress that the experimental reconstruction of hadron spectral functions from measured decay products always requires knowledge of the decay branching ratios which may also be strongly mass-dependent. It also requires a quantitatively reliable treatment of final state interactions which has to be part of any reliable theory.

§1. Introduction

The study of in-medium properties of hadrons has attracted quite some interest among experimentalists and theorists alike because of a possible connection with chiral symmetry restoration in hot and/or dense matter. Experiments using ultrarelativistic heavy ions reach not only very high densities, but connected with that also very high temperatures. In their dynamical evolution they run through various — physically quite different — states, from an initial high-nonequilibrium stage through a very hot stage of — possibly — a new state of matter (QGP) to an equilibrated ‘classical’ hadronic stage at moderate densities and temperatures. Any observed signal necessarily represents a time-integral over all these physically quite distinct states of matter. On the contrary, in experiments with microscopic probes on cold nuclei one tests interactions with nuclear matter in a well-known state, close to cold equilibrium. Even though the density probed is always smaller than the nuclear saturation density, the expected signals are as large as those from ultrarelativistic heavy-ion collisions.^{1),2)}

In this talk we discuss as an example the theoretical situation concerning the ω meson in medium and use it to point out various essential points both in the theoretical framework as well as in the interpretation of data (for further references see the reviews in 3)–5)).

§2. In-medium properties: Theory

The interest in in-medium properties arose suddenly in the early 90’s when several authors^{6),7)} predicted a close connection between in-medium masses and chiral symmetry restoration in hot and/or dense matter. This seemed to establish a direct link between nuclear properties on one hand and QCD symmetries on the other. Later on it was realized that the connection between the chiral condensates of QCD and hadronic spectral functions is not as direct as originally envisaged. The only

^{*)} Speaker, E-mail: mosel@physik.uni-giessen.de

strict connection is given by QCD sum rules which restrict only an integral over the hadronic spectral function by the values of the quark and gluon condensates which themselves are known only for the lowest twist configurations. Indeed a simple, but more realistic analysis of QCD sum rules showed that these do not make precise predictions for hadron masses or widths, but can only serve to constrain hadronic spectral functions.⁸⁾⁻¹¹⁾ Thus hadronic models are needed for a more specific prediction of hadronic properties in medium.

For example, in the past a lively discussion has been going on about a possible mass shift of the ω -meson in a nuclear medium. While there seems to be a general agreement that the ω acquires a certain width of the order of 40–60 MeV in the medium, the mass shift is not so commonly agreed on. While some groups have predicted a dropping mass,¹²⁾⁻¹⁴⁾ there have also been suggestions for a rising mass¹⁵⁾⁻¹⁸⁾ or even a structure with several peaks.^{19),20)} In this context a recent experiment by the CBELSA/TAPS collaboration is of particular interest, since it is the first indication of a downward shift of the mass of the ω -meson in a nuclear medium.²¹⁾ Since Klingl et al.¹³⁾ were among the first to predict such a downward shift it is worthwhile to look into their approach again.

The central quantity that contains all the information about the properties of an ω meson in medium is the spectral function

$$A_{med}(q) = -\frac{1}{\pi} \text{Im} \frac{1}{q^2 - (m_\omega^0)^2 - \Pi_{vac}(q) - \Pi_{med}(q)}, \quad (2.1)$$

with the bare mass m_ω^0 of the ω . The vacuum part of the ω selfenergy Π_{vac} is dominated by the decay $\omega \rightarrow \pi^+ \pi^0 \pi^-$.²²⁾ For the calculation of the in-medium part one can employ the low-density-theorem^{13),19),20)} which states that at sufficiently small density of the nuclear medium one can expand the selfenergy in orders of the density ρ

$$\Pi_{med}(\nu, \vec{q} = 0; \rho) = -\rho T(\nu), \quad (2.2)$$

where $T(\nu)$ is the ω -nucleon forward-scattering amplitude. We note that a priori it is not clear up to which densities this low-density-theorem is reliable.²³⁾

To obtain the imaginary part of the forward scattering amplitude via Cutkosky's Cutting Rules Klingl et al.^{12),13)} used an effective Lagrangian that combined chiral $SU(3)$ dynamics with VMD. The ω selfenergy was evaluated on tree-level which needs as input the inelastic reactions $\omega N \rightarrow \pi N$ (1π channel) and $\omega N \rightarrow 2\pi N$ (2π channel) to determine the effective coupling constants. The amplitude $\omega N \rightarrow \pi N$ is more or less fixed by the measurable and measured back reaction.²⁵⁾ This is in contrast to the reaction $\omega N \leftrightarrow \rho N$ which — in the calculations of Refs. 12) and 13) — is not constrained by any data and which dominates the 2π channel. Furthermore, Klingl et al.^{12),13)} employed a heavy baryon approximation (HBA) to drop some of the tree-level diagrams generated by their Lagrangian. All the calculations were made for isospin-symmetric nuclear matter at temperature $T = 0$. The scattered ω was taken to have $\vec{q} = 0$ relative to the nuclear medium.

We have repeated these calculations without, however, invoking the HBA.*)

*) For further details of the present calculations we refer to Ref. 26).

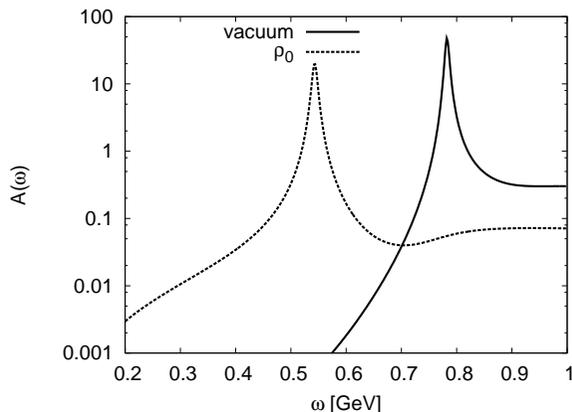


Fig. 1. Spectral function of the ω meson in the vacuum and at normal nuclear density.

For the 2π channel which decides about the in-medium mass shift of the ω in the calculations of Refs. 12) and 13) we find considerable differences — up to one order of magnitude in the imaginary part of the selfenergy — when comparing calculations using the full model with those using the HBA.²⁶⁾ We thus have to conclude already at this point that the HBA is unjustified for the processes considered here and leads to grossly incorrect results.

We show our resulting in-medium spectral function of the ω (where HBA was not employed) in Fig. 1. Note that in the medium the peak is shifted to 544 MeV which is due to the large effects of a relativistic, full treatment of the imaginary and real parts of the amplitudes obtained in the present model. This has to be compared with the results obtained by Klingl et al.¹²⁾ Since Klingl et al. find an in-medium peak at about 620 MeV it is obvious that in the relativistic calculation the physical picture changes drastically. It is also obvious that the correct treatment of the same Lagrangian as used in Ref. 12) on tree-level leads to an unrealistic lowering of the ω spectral function.

It is, therefore, worthwhile to look into another method to calculate the ω selfenergy that takes experimental constraints as much as possible into account and — in contrast to the tree-level calculations of Ref. 12) — respects unitarity. A first study in this direction has been performed by Lutz et al.¹⁹⁾ who solved the Bethe-Salpeter equation with local interaction kernels. These authors found a rather complex spectral function with a second peak at lower energies due to a coupling to nucleon resonances with masses of about ≈ 1500 MeV. We have recently used a large-scale K-matrix analysis of all available γN and πN data^{27)–29)} that does respect unitarity and thus constrains the essential 2π channel by the inelasticities in the 1π channel.²⁰⁾ By consistently using the low-density-approximation we have obtained the result shown in Fig. 2. Figure 2 clearly exhibits a broadened ω spectral function with only a small (upwards) shift of the peak mass. In agreement with the calculations of Lutz et al.,¹⁹⁾ although with less strength, it also exhibits a second peak at masses around 550 MeV that is due to a coupling to a $N^*(1535)$ -nucleon hole configuration. Such a resonance-hole coupling is known to play also a major role

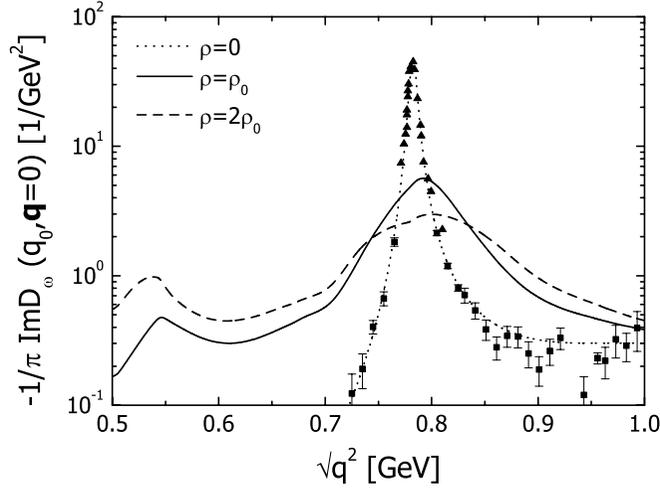


Fig. 2. ω spectral function for an ω meson at rest, i.e. $q_0 = \sqrt{q^2}$ (from Ref. 20)). The appropriately normalized data points correspond to the reaction $e^+e^- \rightarrow \omega \rightarrow 3\pi$ in vacuum. Shown are results for densities $\rho = 0$, $\rho = \rho_0 = 0.16 \text{ fm}^{-3}$ (solid) and $\rho = 2\rho_0$ (dashed).

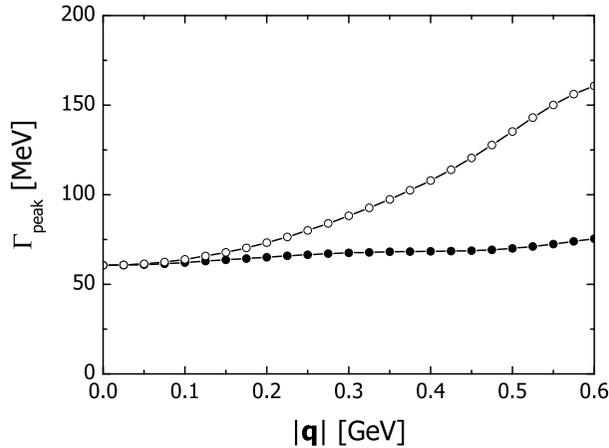


Fig. 3. On-shell width of the ω in nuclear matter at nuclear matter density ρ_0 (from Ref. 20)). The open (solid) points give the width for the transverse (longitudinal) degree of freedom.

in the determination of the ρ meson spectral function;^{23),24)} in the context of QCD sum rules it has been examined in Ref. 17). It is obviously quite sensitive to the detailed coupling strength of this resonance to the ωN channel which energetically opens up only at much higher masses.

As mentioned earlier, there is general consensus among different theories, that the on-shell width of the ω meson in medium reaches values of about 50 MeV at saturation density. To illustrate this point we show in Fig. 3 the width as a function of omega momentum relative to the nuclear matter restframe both for the transverse and the longitudinal polarization degree of freedom. It is clearly seen that the transverse width increases strongly as a function of momentum. At values of about

500 MeV, i.e. the region, where CBELSA/TAPS measures, the transverse width has already increased to about 125 MeV and even the polarization averaged width amounts to 100 MeV.

§3. Spectral functions and observables

Apart from invariant mass measurements, there is another possibility to experimentally constrain the in-medium broadening of the ω -meson. The total width plotted in Fig. 3 is the sum of elastic and inelastic widths. In general, the inelastic width alone is determined by the imaginary part of the selfenergy and the latter determines the amount of reabsorption of ω mesons in the medium. In a Glauber approximation the cross section for ω production on a nucleus reads

$$\frac{d\sigma_{\gamma+A\rightarrow\omega+X}}{d\Omega} = \int d^3x \rho(\vec{x}) \frac{d\sigma_{\gamma+N\rightarrow\omega+X}}{d\Omega} \exp \left[- \int_z^\infty dz' \left(-\frac{1}{p} \Im \Pi(p, \rho(\vec{x}')) \right) \right]. \quad (3.1)$$

The ratio of this cross section on the nucleus to that on the nucleon then determines the nuclear transmission T which depends on the imaginary part of the omega selfenergy $\Im \Pi$

$$T(A) \approx \int d^3x \rho(\vec{x}) \exp \left[- \int_z^\infty dz' \left(-\frac{1}{p} \Im \Pi(p, \rho(\vec{x}')) \right) \right]. \quad (3.2)$$

Using in addition the low-density-approximation

$$\Im \Pi(p, \vec{x}) = -p\rho(\vec{x})\sigma_{\omega N}^{\text{inel}}, \quad (3.3)$$

one obtains the usual Glauber result

$$T(A) = \int d^3x \rho(\vec{x}) \exp \left[- \int_z^\infty dz' \rho(\vec{x}') \sigma_{\omega N}^{\text{inel}} \right]. \quad (3.4)$$

We show the calculated transmission T in Fig. 4 together with the data obtained by CBELSA/TAPS. The measured cross section dependence on massnumber A is reproduced very well³⁰⁾ if the inelastic ωN cross section is increased by 25% over the usually used parametrization. This may indicate a problem with the usually used cross section, or — more interesting — it may indicate a breakdown of the low-density-approximation.

It is, furthermore, important to realize that the spectral functions themselves are not observable. What can be observed are the decay products of the meson under study. It is thus obvious that even in vacuum the invariant mass distribution of the decaying resonance ($V \rightarrow X_1 X_2$), reconstructed from the four-momenta of the decay products (X_1, X_2), involves a product of spectral function and partial decay width into the channel being studied

$$\frac{dR_{V\rightarrow X_1 X_2}}{dq^2} \sim A(q^2) \times \frac{\Gamma_{V\rightarrow X_1 X_2}(q^2)}{\Gamma_{\text{tot}}(q^2)}. \quad (3.5)$$

Since in general the branching ratio also depends on the invariant mass of the decaying resonance this dependence may distort the observed invariant mass distribution

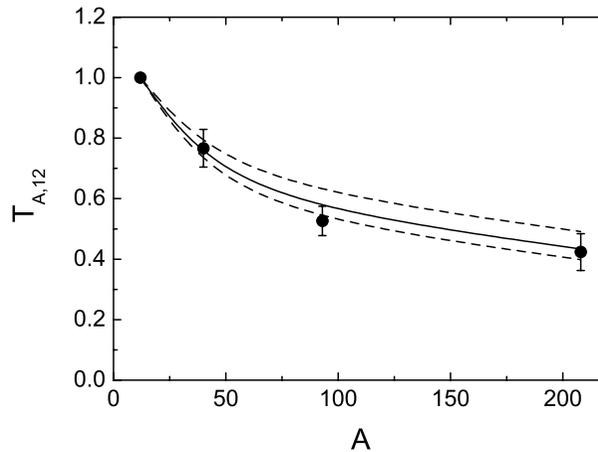


Fig. 4. Transparency of nuclei for ω production. Calculations and preliminary data are normalized to ^{12}C . Dashed lines reflect error estimates obtained from the spread of the data. Data are from CBELSA/TAPS.³¹⁾

compared with the spectral function itself. This effect is obviously the more important the broader the decaying resonance is and the stronger the widths depend on q^2 .

While these branching ratios are usually well known in vacuum there is considerable uncertainty about their value in the nuclear medium. This uncertainty is connected with the lack of knowledge about the in-medium vertex corrections, i.e. the change of coupling constants with density. Even if we assume that these quantities stay the same, then at least the total width appearing in the denominator of the branching ratio has to be changed, consistent with the change of the width in the spectral function. This point has only rarely been discussed so far, but it has far-reaching consequences.

For example, for the ρ meson the partial decay width into the dilepton channel goes like

$$\Gamma_{\rho \rightarrow e^+e^-} \sim \frac{1}{M^4} M = \frac{1}{M^3}, \quad (3.6)$$

where the first factor on the rhs originates in the photon propagator and the last factor M comes from phase-space. On the other hand, the total decay width of the ρ meson in vacuum is given by (neglecting the pion masses for simplicity)

$$\Gamma_{\text{tot}} \approx \Gamma_{\rho \rightarrow \pi\pi} \sim M, \quad (3.7)$$

so that the branching ratio in vacuum goes like

$$\frac{\Gamma_{\rho \rightarrow e^+e^-}}{\Gamma_{\text{tot}}} \sim \frac{1}{M^4}. \quad (3.8)$$

This strong M -dependence distorts the spectral function, in particular, for a broad resonance such as the ρ meson. This effect is contained and clearly seen in theoretical simulations of the total dilepton yield from nuclear reactions (see, e. g., Figs. 8 – 10

in Ref. 32)); it leads to a considerable shift of strength in the dilepton spectrum towards lower masses.

For the semileptonic decay channel $\pi^0\gamma$ that has been exploited in the CBELSA TAPS experiment again a strong mass-dependence of the branching ratio shows up because just at the resonance the decay channel $\omega \rightarrow \rho\pi$ opens up.

In both of these cases the in-medium broadening changes the total widths in the denominator of the branching ratios even if the partial decay width stays the same as in vacuum. Such an in-medium broadening of the total width, which should be the same as in the spectral function, will tend to weaken the M -dependence of the total width and thus the branching ratio as a whole. In medium another complication arises: the spectral function no longer depends on the invariant mass alone, but — due to a breaking of Lorentz-invariance because of the presence of the nuclear medium — in addition also on the three-momentum of the hadron being probed. Again, this p -dependence of the vector meson selfenergy has only rarely been taken into account (see, however, Refs. 20), 24) and 33). In addition, final state interactions do affect hadronic decay channels. A quantitatively reliable treatment of these FSI thus has to be integral part of any trustable theory that aims at describing these data.

§4. Conclusions

QCD sum rules establish a very useful link between the chiral condensates, both in vacuum and in medium, but their connection to hadronic spectral functions is indirect. The latter can thus only be constrained by the QCDSR, but not be fixed; for a detailed determination hadronic models are needed. We have pointed out in this talk that the low-density-approximation nearly always used in these studies does not answer the question up to which densities it is applicable. First studies²³⁾ have shown that this may be different from particle to particle.

While the in-medium properties of all vector mesons ρ , ω , and ϕ are the subject of intensive experimental and theoretical research, in this talk we have concentrated on the ω meson for which recent experiments indicate a lowering of the mass by about 60 MeV in photon-produced experiments on nuclei. A tree-level calculation, based on an effective Lagrangian, that predicted such a lowering, has been shown to be incorrect because of the heavy-baryon approximation used in that calculation. A correct tree-level calculation with the same Lagrangian gives strong contributions from the $\omega \rightarrow 2\pi N$ channel, which, however, is unconstrained by any data; in effect, the spectral function is softened by an unreasonable pole mass shift. This problem might partially be based on the fact that all the inelastic processes $\omega N \rightarrow \pi N$ and $\omega N \rightarrow 2\pi N$ are only treated at tree-level. Here an improved calculation is needed, which incorporates coupled-channels and rescattering, e.g. a Bethe-Salpeter¹⁹⁾ or a K-matrix approach.^{27), 34), 35)}

We have indeed shown that a better calculation that again starts out from an effective Lagrangian and takes unitarity, channel-coupling and rescattering into account yields a significantly different in-medium spectral function in which the pole mass hardly changes, but a broadening of about 60 MeV at nuclear saturation density

takes place, which increases with momentum, primarily in the transverse channel.

Finally, we have pointed out that any measurement of the spectral function necessarily involves also a branching ratio into the channel being studied. The experimental in-medium signal thus contains changes of both the spectral function and the branching ratio.

Acknowledgements

The authors acknowledge discussions with Norbert Kaiser and Wolfram Weise. They have also benefitted a lot from discussions with Vitaly Shklyar. This work has been supported by DFG through the SFB/TR16 ‘‘Subnuclear Structure of Matter’’.

References

- 1) U. Mosel, in *QCD Phase Transitions, Proc. Int. Workshop, Hirschegg, 1997*, GSI, Darmstadt, p. 201; nucl-th/9702046.
- 2) U. Mosel, in *Hadrons in Dense Matter, Proc. Int. Workshop, Hirschegg, 2000*, GSI, Darmstadt, p. 11; nucl-th/0002020.
- 3) T. Falter, J. Lehr, U. Mosel, P. Muehlich and M. Post, *Prog. Part. Nucl. Phys.* **53** (2004), 25.
- 4) L. Alvarez-Ruso, T. Falter, U. Mosel and P. Muehlich, *Prog. Part. Nucl. Phys.* **55** (2005), 71.
- 5) R. Rapp and J. Wambach, *Adv. Nucl. Phys.* **25** (2000), 1.
- 6) G. E. Brown and M. Rho, *Phys. Rev. Lett.* **66** (1991), 2720.
- 7) T. Hatsuda and S. H. Lee, *Phys. Rev. C* **46** (1992), 34.
- 8) S. Leupold, W. Peters and U. Mosel, *Nucl. Phys. A* **628** (1998), 311.
- 9) S. Leupold and U. Mosel, *Phys. Rev. C* **58** (1998), 2939.
- 10) S. Leupold, *Phys. Rev. C* **64** (2001), 015202.
- 11) S. Leupold and M. Post, *Nucl. Phys. A* **747** (2005), 425.
- 12) F. Klingl, T. Waas and W. Weise, *Nucl. Phys. A* **650** (1999), 299.
- 13) F. Klingl, N. Kaiser and W. Weise, *Nucl. Phys. A* **624** (1997), 527.
- 14) T. Renk, R. A. Schneider and W. Weise, *Phys. Rev. C* **66** (2002), 014902.
- 15) A. K. Dutt-Mazumder, R. Hofmann and M. Pospelov, *Phys. Rev. C* **63** (2001), 015204.
- 16) M. Post and U. Mosel, *Nucl. Phys. A* **699** (2002), 169.
- 17) B. Steinmueller and S. Leupold, *Nucl. Phys. A* **778** (2006), 195.
- 18) S. Zschocke, O. P. Pavlenko and B. Kampfer, *Phys. Lett. B* **562** (2003), 57.
- 19) M. F. M. Lutz, G. Wolf and B. Friman, *Nucl. Phys. A* **706** (2002), 431 [Errata; **765** (2006), 495].
- 20) P. Muehlich, V. Shklyar, S. Leupold, U. Mosel and M. Post, *Nucl. Phys. A* **780** (2006), 187.
- 21) D. Trnka et al. (CBELSA/TAPS Collaboration), *Phys. Rev. Lett.* **94** (2005), 192303.
- 22) F. Klingl, N. Kaiser and W. Weise, *Z. Phys. A* **356** (1996), 193.
- 23) M. Post, S. Leupold and U. Mosel, *Nucl. Phys. A* **741** (2004), 81.
- 24) W. Peters, M. Post, H. Lenske, S. Leupold and U. Mosel, *Nucl. Phys. A* **632** (1998), 109.
- 25) B. Friman, nucl-th/9801053.
- 26) F. Eichstaedt, Diploma Thesis, Institut fuer Theoretische Physik, JLU Giessen, 2006, <http://theorie.physik.uni-giessen.de/documents/diplom/eichstaedt.pdf>
- 27) G. Penner and U. Mosel, *Phys. Rev. C* **66** (2002), 055211.
- 28) G. Penner and U. Mosel, *Phys. Rev. C* **66** (2002), 055212.
- 29) V. Shklyar, H. Lenske, U. Mosel and G. Penner, *Phys. Rev. C* **72** (2005), 015210.
- 30) P. Muehlich and U. Mosel, *Nucl. Phys. A* **773** (2006), 156.
- 31) M. Kotulla, nucl-ex/0609012.
- 32) M. Effenberger, E. L. Bratkovskaya and U. Mosel, *Phys. Rev. C* **60** (1999), 044614.
- 33) M. Post, S. Leupold and U. Mosel, *Nucl. Phys. A* **689** (2001), 753.
- 34) T. Feuster and U. Mosel, *Phys. Rev. C* **59** (1999), 460.
- 35) V. Shklyar, H. Lenske, U. Mosel and G. Penner, *Phys. Rev. C* **71** (2005), 055206 [Errata; **72** (2005), 019903].