

## Pentaquarks in QCD Sum Rules

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QCD sum rules for pentaquarks are studied. We point out that naive pentaquark correlations function include two-hadron-reducible contributions, which are given by convolution of baryon and meson correlation functions and have nothing to do with the pentaquark. We show that the two-hadron-reducible contributions are large in the operator product expansion of the correlation functions. Instead, we propose to use the two-hadron-irreducible correlation function, which is obtained by subtracting the two-hadron-reducible contribution from the naive correlation function. How large the two-hadron-reducible contributions are in the OPE is demonstrated through the application to spin-1/2 and 3/2 states of the pentaquark.

The report by LEPS collaboration<sup>1)</sup> to suggest the existence of the pentaquark has triggered an intense experimental and theoretical activity to clarify the quantum numbers and to understand the structure of the pentaquark. In this talk, we focus on the application of the QCD sum rule<sup>2)</sup> to the pentaquark and discuss an issue which is characteristic for exotic hadrons. We point out that the naive pentaquark correlation functions include two-hadron-reducible contributions, which are due to noninteracting propagation of the three-quark baryon and the meson and therefore have nothing to do with the pentaquark.<sup>3)</sup> These contributions exist in the correlation function only for exotic hadrons and are potentially large. Instead, we propose to use the two-hadron-irreducible correlation function, which is obtained by subtracting the two-hadron-reducible contribution from the naive correlation function. We show how large the two-hadron-reducible contributions are in the OPE through the application to the spin  $J = 1/2$  and  $3/2$  states of the pentaquark.

The basic object of the study is the correlation function of baryon interpolating field  $\eta$ ,

$$\Pi(p) = -i \int d^4x e^{ipx} \langle 0 | T(\eta(x) \bar{\eta}(0)) | 0 \rangle. \quad (1)$$

The spectral function,  $\rho(p_0) = -\text{Im}\Pi(p_0 + i\epsilon)/\pi$ , in the rest frame  $\vec{p} = 0$  can be

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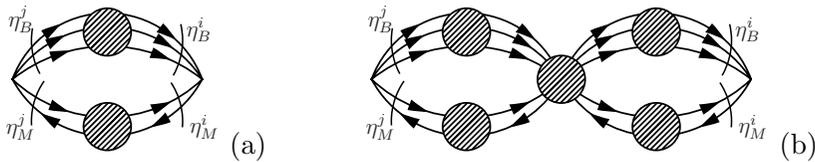


Fig. 1. (a) Two-Hadron-Reducible (2HR) diagram. (b) Two-Hadron-Irreducible (2HI) diagram.

written as

$$\rho(p_0) = P_+ \rho_+(p_0) + P_- \rho_-(p_0), \quad (2)$$

where  $P_{\pm} = (\gamma_0 \pm 1)/2$ . In  $\rho_{\pm}(p_0)$ , there exists not only positive but negative parity states contributions. For example, positive parity states contribute to  $\rho_+(p_0 > 0)$  and negative parity states to  $\rho_+(p_0 < 0)$ . This is because the interpolating field couples to positive and negative parity states. On the other hand, in the deep Euclid region,  $p_0^2 \rightarrow -\infty$ ,  $\rho_{\pm}(p_0)$  can be evaluated by an OPE. Using the analyticity we obtain the QCD sum rule as

$$\int_{-\infty}^{\infty} dp_0 \rho_{\pm}^{\text{OPE}}(p_0) W(p_0) = \int_{-\infty}^{\infty} dp_0 \rho_{\pm}(p_0) W(p_0), \quad (3)$$

where  $W(p_0)$  is an analytic function of  $p_0$ .  $\rho_{\pm}(p_0)$  are parameterized by a pole plus continuum contribution,

$$\rho_{\pm}(p_0) = |\lambda_{\pm}|^2 \delta(p_0 - m_{\pm}) + |\lambda_{\mp}|^2 \delta(p_0 + m_{\mp}) + [\theta(p_0 - \omega_{\pm}) + \theta(-p_0 - \omega_{\mp})] \rho^{\text{OPE}}(p_0), \quad (4)$$

where  $m_{\pm}$  and  $\omega_{\pm}$  are the masses and the continuum threshold parameters of positive (negative) parity states, respectively. Substituting this equation to the right-hand side of the sum rule and using the Borel weight,  $W(p_0) = p_0^n \exp(-p_0^2/M^2)$ , we obtain the Borel sum rules for positive and negative parity baryons.<sup>4)</sup>

Now we consider the pentaquark sum rule. A remarkable feature of the pentaquark is that it can be decomposed into a color-singlet three-quark state, baryon, and a color-singlet quark-antiquark state, meson. (Hereafter, we use the term baryon when its minimal quark-content is  $qqq$ .) Therefore, the interpolating field for the pentaquark can be expressed as a sum of the product of baryon and meson interpolating fields:

$$\eta_P(x) = \sum_i \eta_B^i(x) \eta_M^i(x), \quad (5)$$

where  $\eta_B^i(x)$  and  $\eta_M^i(x)$  are color-singlet baryon and meson interpolating fields, respectively. Due to this separability, the pentaquark correlation function has a part in which the baryon and the meson propagate independently without interacting each other. We define this part as the two-hadron-reducible (2HR) part and the rest of the correlation function as the two-hadron-irreducible (2HI) part. Diagrammatically, the 2HR and 2HI parts are represented as Figs. 1(a) and (b), respectively. Clearly, the 2HR part is completely determined by the baryon and meson correlation functions and has nothing to do with the pentaquark.

Let us next look at the separation of the 2HR and 2HI parts in the spectral function. We suppose that the lowest states generated by  $\eta_B$  and  $\eta_M$  are spin-1/2

baryon  $B$  and spin-0 meson  $M$ , respectively. Consider only the contribution of the  $BM$  scattering states in the spectral function for  $p^0 > 0$ ,  $\rho_P^{BM}(p)$ , just for simplicity, which is given by

$$\rho_P^{BM}(p) = \sum_s \int d^3k d^3q (2\pi)^3 \delta^4(p - k - q) \langle 0 | \eta_P(0) | kqsout \rangle \langle kqsout | \bar{\eta}_P(0) | 0 \rangle, \quad (6)$$

where  $q$  and  $k$  are the momentum of the baryon and the meson, respectively, and  $s$  the spin projection of the baryon.  $|kqsout\rangle$  denote the baryon-meson scattering states with *out* boundary condition. One can separate  $\rho_P^{BM}(p)$  into two parts by means of the reduction formula,<sup>3)</sup>

$$\begin{aligned} \rho_P^{BM}(p) = & -\frac{1}{\pi} |\lambda_B|^2 |\lambda_M|^2 \text{Im} \left\{ i \int \frac{d^4q}{(2\pi)^4} \frac{1}{(p-q)^2 - m^2 + i\eta} \frac{1}{\not{q} - M + i\eta} \right. \\ & - \int \frac{d^4q}{(2\pi)^4} \frac{d^4q'}{(2\pi)^4} \frac{1}{(p-q)^2 - m^2 + i\eta} \frac{1}{\not{q} - M + i\eta} \\ & \left. \times T(p-q, q, p-q', q') \frac{1}{\not{q}' - M + i\eta} \frac{1}{(p-q')^2 - m^2 + i\eta} \right\}, \quad (7) \end{aligned}$$

where  $\lambda_{B(M)}$  is the coupling strength of the interpolating field to the baryon (meson) state and  $M(m)$  is the baryon (meson) mass.  $T(k, q, k', q')$  is the  $T$ -matrix for the process  $kq \rightarrow k'q'$ . In Eq. (7), the first term in the curly bracket is the 2HR contribution due to the trivial noninteracting contribution of the  $BM$  intermediate states. The second term is related with the  $T$ -matrix for the  $BM$  scattering and corresponds to the 2HI contribution. If the pentaquark is a bound state in the  $BM$  channel the spectral function has an additional contribution from the bound state, while if the pentaquark is a resonance in the  $BM$  channel the effect of the pentaquark lies in the  $BM$   $T$ -matrix as a pole at a complex pentaquark energy. In any case, the 2HR contribution is not related to the pentaquark.

Some comments are in order here. The 2HR contributions discussed here exist commonly in the correlation functions for exotic hadrons but not for ordinary hadrons. Crucial assumption here is confinement. Namely, we assume that only color-singlet states contribute to the spectral function. Therefore, the separability of the pentaquark into color-singlet baryon and meson is the origin of the existence of the 2HR contribution. Another point to be mentioned is that by removing the 2HR part we do not take out all the baryon-meson continuum contributions. We just remove noninteracting contributions. The continuum contributions included in the 2HI part cannot be separated from the pentaquark contribution.

Let us turn to the separation of the 2HR and 2HI parts in the OPE. We calculated the 2HR parts of the correlation functions for interpolating fields used in Refs. 5) and 6). We found that the 2HR part of the correlation function is large in general at least of the same order as the 2HI part. For both of  $J = 1/2$  and  $3/2$  interpolating field, the Wilson coefficients of the operators in the 2HR part are  $4/3$  of the total up to dimension 6 except for the operator,  $\frac{\alpha_s}{\pi} G^2$ .<sup>7)</sup>

Some final comments are in order here. (i) An important aspect in the sum rules for pentaquarks worth mentioning is that the convergence of the OPE is probably

very slow since the dimension of the interpolating field is large.<sup>8)</sup> Therefore, in order to draw any definite conclusion, it may be necessary to include the higher dimension terms in OPE. The present work, where only the terms up to dimension six are taken into account, may be intolerable. (ii) Logically, there is nothing wrong to use the total correlation function. It is much better if the background can be exactly separated, which is what we proposed in this talk. By considering the 2HI correlation function of pentaquark in which the OPE is carried out up to sufficiently high dimension, we can estimate the mass and specify the parity of the pentaquark reliably. (iii) Also, we would like to make a brief comment on the lattice study of the pentaquark. In the lattice calculation, the mass of the pentaquark is extracted from the long-time behavior of the total correlation function, which contains large two-hadron-reducible contribution. It is necessary to remove this two-hadron-reducible contribution in order to obtain information on the pentaquark. It would be interesting to see the relevance of the present work in the context of the lattice study of the pentaquark.

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