

Neutrino Radiation-Hydrodynamics: General Relativistic versus Multidimensional Supernova Simulations

Matthias LIEBENDÖRFER,¹ Tobias FISCHER,² Matthias HEMPEL,³
 Roger KÄPPELI,¹ Giuseppe PAGLIARA,³ Albino PEREGO,¹ Irina SAGERT,⁴
 Jürgen SCHAFFNER-BIELICH,³ Simon SCHEIDEGGER,¹
 Friedrich-Karl THIELEMANN¹ and Stuart C. WHITEHOUSE¹

¹*Physics Department, University of Basel, Klingelbergstr. 82,
 4056 Basel, Switzerland*

²*GSI Helmholtzzentrum für Schwerionenforschung,
 Planckstr. 1, 64291 Darmstadt, Germany*

³*Institut für Theoretische Physik, Ruprecht-Karls-Universität,
 Philosophenweg 16, 69120 Heidelberg, Germany*

⁴*Institut für Theoretische Physik, Goethe-Universität,
 Max-von-Laue-Str. 1, 60438 Frankfurt, Germany*

Recently, simulations of the collapse of massive stars showed that selected models of the QCD phase transitions to deconfined quarks during the early postbounce phase can trigger the supernova explosion that has been searched for over many years in spherically symmetric supernova models. Using sophisticated general relativistic Boltzmann neutrino transport, it was found that a characteristic neutrino signature is emitted that permits to falsify or identify this scenario in the next Galactic supernova event. On the other hand, more refined observations of past supernovae and progressing theoretical research in different supernova groups demonstrated that the effects of multidimensional fluid instabilities cannot be neglected in global models of the explosions of massive stars. We point to different efforts where neutrino transport and general relativistic effects are combined with multidimensional fluid instabilities in supernovae. With those, it will be possible to explore the gravitational wave emission as a potential second characteristic observable of the presence of quark matter in new-born neutron stars.

§1. Introduction

Nuclear burning in a massive star comes to an end when iron-group nuclei with a large binding energy per nucleon accumulate at the center of the star. The decaying source of thermal pressure in combination with the soft equation of state of relativistic electrons lead to a sudden gravitational collapse of the inner stellar core. The collapse proceeds until nuclear matter density is reached, where the equation of state becomes much stiffer. An initially stable proto-neutron star (PNS) of about 50 km is formed at the center. The layers outside the iron core fall in with delay and accumulate on the surface of the PNS. The conversion of their kinetic energy to thermal energy provides sufficient heat to fully dissociate the nuclei into protons and neutrons. A large part of this thermal energy is radiated away by the emission of neutrinos: Electron neutrinos stem from electron captures on protons, electron antineutrinos from positron captures on neutrons, and all neutrino flavours are additionally produced by pair annihilation and bremsstrahlung.¹⁾ This collapse scenario is consistent with the few neutrinos that have been observed from SN1987A.²⁾ The

hot layer around the PNS is bordered at a radius of few 100 km by a standing accretion shock that adjusts its position according to the neutrino cooling efficiency at the surface of the PNS and the accretion rate. The physical cause for the ensuing supernova explosion has not yet been unambiguously identified. Most plausible is a transfer of energy from the emanating neutrinos onto the hot matter enclosed by the standing accretion shock.^{3),4)} Alternatively, magnetic fields could play an important role in combination with the rotation of the inner stellar core.⁵⁾ More recent suggestions involve shock-heating by accretion-induced oscillations of the PNS⁶⁾ or a second shock wave that is launched after a phase transition deep in the PNS.^{7),8)} While first computer models show promising results for all these mechanisms, the very time-consuming and complicated calculations have not yet demonstrated the necessary convergence between the results of different groups to draw any firm conclusions. An unambiguous understanding of the supernova mechanism is crucial for the prediction of nucleosynthetic yields in the supernova ejecta that provide heavy elements as source of the Galactic evolution.

§2. General relativistic neutrino transport in spherical symmetry

Very detailed nuclear input physics can be treated in a supernova model that is restricted to spherical symmetry. If we choose comoving orthogonal coordinates,⁹⁾ a line element is described by

$$ds^2 = -\alpha dt^2 + \left(\Gamma^{-1} \frac{dr}{da} \right)^2 da^2 + r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2), \quad (2.1)$$

where t is the coordinate time, a is the enclosed baryon number, and $d\vartheta + \sin \vartheta d\varphi$ the element of a sphere. The metric is given by the lapse function α , the Lorentz factor $\Gamma = (1 + u^2 - \frac{2m}{r})$, and the area $4\pi r^2$ of the sphere.^{*}) A ‘velocity’ can be defined as the change of r with proper time, $u = \alpha^{-1} dr/dt$, and m is the enclosed energy. The hydrodynamics equations for the evolution of the stellar structure are then given in Eqs. (6)–(11) of Ref. 10), while the general relativistic Boltzmann equation for neutrino transport becomes

$$\begin{aligned} & \frac{\partial F_i}{\alpha \partial t} + \frac{\mu}{\alpha} \frac{\partial}{\partial a} (4\pi r^2 \alpha \rho F_i) + \Gamma \left(\frac{1}{r} - \frac{1}{\alpha} \frac{\partial \alpha}{\partial r} \right) \frac{\partial}{\partial \mu} [(1 - \mu^2) F_i] - \mu \Gamma \frac{1}{\alpha} \frac{\partial \alpha}{\partial r} \frac{1}{E^2} \frac{\partial}{\partial E} (E^3 F_i) \\ & + \left[\mu^2 \left(\frac{\partial \ln \rho}{\alpha \partial t} + \frac{3u}{r} \right) - \frac{u}{r} \right] \frac{1}{E^2} \frac{\partial}{\partial E} (E^3 F_i) + \left(\frac{\partial \ln \rho}{\alpha \partial t} + \frac{3u}{r} \right) \frac{\partial}{\partial \mu} [\mu (1 - \mu^2) F_i] \\ & = \frac{j}{\rho} - \chi F_i + \left[\frac{1}{\rho} - F_i(\mu, E) \right] \Sigma_j \int E'^2 dE' d\mu' R_{ij}^{\text{in}}(\mu, \mu', E, E') F_j(\mu', E) \\ & - F_i(\mu, E) \Sigma_j \int E'^2 dE' d\mu' R_{ij}^{\text{out}}(\mu, \mu', E, E') \left[\frac{1}{\rho} - F_j(\mu', E') \right]. \end{aligned} \quad (2.2)$$

The principal quantity is the specific neutrino distribution function $F_i(t, a, \mu, E)$ for neutrino species i . Its dependency on the momentum phase space is expressed by the neutrino energy E and the angle θ between the neutrino propagation direction

^{*}) The units are chosen such that $c = G = h = 1$.

and the radius. We prefer using $\mu = \cos \theta$ instead of the angle θ itself. The left-hand side specifies an interaction-free geodesic motion of the neutrinos. The right-hand side describes the collision integral with an isotropic neutrino emissivity j and an absorptivity χ . As an example for scattering reactions, we add the scattering kernels R^{in} for neutrinos that are scattered into the beam and R^{out} for neutrinos that are scattered out of the beam. This transport equation can be solved numerically.¹¹⁾ The results of independent groups agree: In spherical symmetry there are no explosions obtained except for one model of a ONeMg progenitor star.^{12),13)} Even with the absence of self-consistent explosions, spherically symmetric models with spectral neutrino transport are very useful to explore the collapse phase,¹⁴⁾ to model the formation of a stellar mass black hole^{10),15),16)} to study the effect of neutrinos on the nucleosynthesis in explosions with parameterised explosion energies,^{17),18)} and to carry the models through an extended postbounce evolution phase that includes the formation of the neutrino wind and the onset of the long-term cooling of the PNS.^{13),19)}

§3. Exploring the QCD phase transition in the postbounce phase

The central density in the PNS reaches few times saturation density at bounce and during the early postbounce evolution phase. Currently, there are no indications from heavy ion collision experiments that a phase transition from hadronic to quark matter occurs at such densities. In supernova models, the occurrence of quark matter can be forced by choosing very massive progenitor stars that lead to much higher densities already shortly after bounce.²⁰⁾ However, one has also to bear in mind, that the conditions in a PNS differ from the conditions of heavy ion collisions with respect to two important aspects:²¹⁾ Firstly, the matter in the PNS with an electron fraction $Y_e < 0.3$ is significantly more neutron rich than in heavy ions. The high asymmetry energy makes the hadronic state of matter less favourable. Secondly, the conditions in the PNS remain stationary over milliseconds and longer, giving weak interactions time to produce strangeness and to establish weak equilibrium. The additional degree of freedom of strange quarks makes states of deconfined matter more favourable. In order to explore the phase transition under astrophysical conditions, the Shen et al.²²⁾ equation of state (EOS) was linked to a quark matter EOS that is based on the MIT bag model. A mixed phase is constructed using the Gibbs conditions for phase equilibrium. Low bag constants between $B^{1/4} = 162$ and $B^{1/4} = 165$ turned out to be especially interesting when implemented in spherically symmetric supernova models with general relativistic Boltzmann neutrino transport:^{8),23)} For models with main sequence masses between 11 and 15 M_\odot , the soft mixed phase of hadronic and quark matter is already reached at bounce. After few 100 ms of postbounce accretion the central matter in the mixed phase becomes too large to support the PNS. An inner part of the PNS collapses until the stiffer pure quark phase is reached. Similar to the previous collapse to nuclear density, an accretion shock forms inside the PNS and propagates outward through the mixed phase. The high temperature of this second shock pushes infalling hadronic matter into the mixed phase. However, as soon as the second shock reaches the steep density gradient

at the surface of the PNS, the density of infalling hadronic matter becomes too low to make the transition to the mixed phase. At this point, the second shock accelerates to large outward velocities, catches up with the standing first shock and triggers an energetic explosion in spherical models that would have failed to explode with a conventional hadronic EOS. It is well-known that the first shock leads to a very characteristic sharp peak in the neutrino emission when it breaks through the neutrino sphere. It is caused by the many electron captures that are required to establish the low electron fraction dictated by β -equilibrium. The second shock has a similar characteristic signature: When it crosses the neutrinospheres—now sitting in deleptonised degenerate matter—the electron degeneracy is reduced and the equilibrium electron fraction is raised by a burst of positron captures. This leads to a similarly sharp millisecond peak emission of electron antineutrinos that should clearly be identifiable in the Superkamiokande or Icecube neutrino detectors.^{8),24)} It is an exciting example where the physics of dense matter in astrophysical events can be probed under conditions that are difficult to reach experimentally.

§4. Multidimensional supernova models

Independent from the specific explosion mechanisms, it has become clear that supernova models must include more than just the radial dimension. The reasons are manifold: Early models showed that convection in the PNS^{25),26)} and in the hot layer enclosed by the standing accretion shock^{27),28)} significantly enhances the heating efficiency. During the last decade, additional important fluid instabilities have been investigated in the supernova context, for example the magneto-rotational instability,²⁹⁾ the standing accretion shock instability (SASI),^{30)–33)} and the low T/W-instability,^{34),35)} which open new perspectives for the detection of gravitational waves from core-collapse supernovae.³⁶⁾

The dilemma is that the Boltzmann transport equation for the crucial spectral neutrino transport, Eq. (2.2), and a general relativistic space-time as in Eq. (2.1) are very difficult to implement in multidimensional supernova models that can treat all the above-listed fluid instabilities. An investigation of the QCD phase transition at high density necessitates even finer spatial resolution and a careful treatment of general relativistic effects. Nevertheless, large progress has been made: For the first time, it is possible to perform two-dimensional general relativistic models with sophisticated spectral neutrino transport.³⁷⁾ Alternative approaches start from the other end and improve the neutrino physics approximations for efficient three-dimensional models. Using this approach in our 3D magneto-hydrodynamics code FISH⁴⁰⁾ and implementing the isotropic diffusion source approximation⁴¹⁾ and a leakage scheme for the μ - and τ -neutrinos, we performed 3D supernova models for three different progenitor stars. The trajectories of the shock in Fig. 1 show a pronounced progenitor-dependence of the postbounce evolution. In these models, the shock perturbations are mainly caused by the neutrino heating-driven convection, while the SASI rather appears as a side effect. Although the position of the standing accretion shock is quickly pushed out in radius, net matter outflow does not establish, except for the 11 M_{\odot} progenitor star at the very end of the simulation.

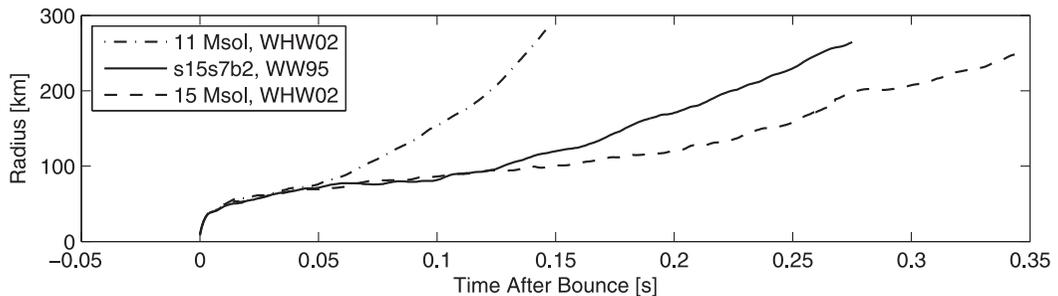


Fig. 1. Spherically averaged shock position as a function of time in three-dimensional supernova models with spectral neutrino transport for different solar metallicity progenitor stars taken from Refs. 38) and 39).

Hence, it is not yet clear whether these models will lead to an explosion. We are looking forward to investigate this question with further improved models.

§5. Conclusions

Investigations of supernova dynamics with computer models were traditionally split into models with sophisticated input physics and restricted dimensionality on the one hand and multidimensional models with severe neutrino physics approximations and/or neglect of general relativistic effects on the other hand. Both branches made significant progress: Situations were discovered where spherically symmetric models do lead to explosions, e.g. with a specific ONeMg progenitor star or a QCD phase transition to quark matter. With respect to the investigation of important fluid instabilities, first axisymmetric models with accurate neutrino transport become available while approximations for efficient three-dimensional models have been improved. The continuously increasing availability of computational resources will allow these two branches to join over the next years so that hopefully a convergence with respect to the physics implications can be approached.

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