Deuteron Clustering in Nuclear Matter

Hiroharu BANDO and Atsushi KURIYAMA*

Department of Physics, Kyoto University, Kyoto
*Department of Physics, Kyushu University, Fukuoka

(Received August 30, 1968)

In order to consider the problem of mutual transition between the shell structure and the cluster structure of nuclei, a model nuclear matter is treated in which deuteron is the most important unit of clustering. Hartree-Bogoliubov theory is applied to the problem of deuteron clustering of the whole system. The problem of a deuteron moving rapidly in the nuclear matter is also treated. The many-body effects on the clustering is discussed in two points: Pauli principle and the single particle potential.

§1. Introduction

In recent years the cluster structure of nuclei has been found extensively in light nuclei with the progress of experimental observations. Theoretical investigations have been also made¹⁾ and now it is one of the most interesting problem of nuclear structure.

The term "cluster structure" is used in various meanings; one of the examples is the α -particle model^{1a),(c)} in which free α -particles are assumed to exist with weak α - α or α -nuclear core interaction and the other typical example is the cluster coupling shell model^{1b),2)} in which the valence nucleons in the open shell correlate in a cluster type coupling scheme. We can consider these aspects of the cluster structure as the problem of composite particles within a many-body system. A composite particle feels the potential field from other particles and also the effect of Pauli principle. These two many-body effects tend to change the composite particle from the free one. If the many-body effects are relatively weak, the composite particle remains as the localized substructure of the many-body system, but as the many-body effects become strong, the composite particle can no longer retain its own localization and melts into the whole system, and its original properties remain only as the cluster coupling in shell model. This is, in other word, the problem of the mutual transition between the shell structure and the cluster structure.

In this paper we treat deuteron clustering in the nuclear matter as a model for the above-mentioned problem. In §2 the problem of deuteron clustering of the *whole* nuclear matter is qualitatively discussed. In §3 the problem is treated noting the relation between the deuteron clustering and

single particle potential. In $\S 4$ the problem of a cluster moving in nuclear matter is discussed.

§2. Qualitative discussions

—From the super-state to the cluster-state—

A deuteron in the nuclear matter is subjected to many-body effects in two points; one is the effective nucleon mass and the other Pauli principle. The effective nucleon mass in the nuclear matter is smaller than the free one.³⁾ Pauli principle reduces effectively the attractive force between neutron and proton by cutting off a part (mainly low momentum part) of the momentum components of wave function. Thus both the effects are unfavorable to the binding of a deuteron cluster in the nuclear matter. These many-body effects become more remarkable as the increase of the average density of nuclear matter.

If there can be a deuteron cluster in the nuclear matter at a certain density, all other n-p pairs will be also deuteron clusters. This is a many-body problem with a pair (n-p pair) interaction.* Such a problem can be, as is well-known, treated by a theory of superconductivity. For like nucleons such a problem has been already investigated as the problem of energy gap in the nuclear matter. Here we apply the theory to the case of the n-p interaction. Essential difference between the two cases is that the former has no bound state, while the latter has a bound state: a deuteron.

The so-called gap equations for $(\boldsymbol{k}, -\boldsymbol{k})$ pairs in B.C.S.-Bogoliubov theory are

$$\Delta(\mathbf{k}) = -\sum_{\mathbf{k}'} \langle \mathbf{k} | v | \mathbf{k}' \rangle \varphi(\mathbf{k}'), \qquad (2 \cdot 1)$$

$$\varphi(\mathbf{k}) = \frac{1}{2} \frac{\Delta(\mathbf{k})}{\sqrt{(E(\mathbf{k}) - \lambda)^2 + \Delta(\mathbf{k})^2}},$$
 (2·2)

$$\sum_{\mathbf{k}} \rho(\mathbf{k}) = \frac{1}{4} \rho_0 = \frac{4\pi}{3} k_F^3 \tag{2.3}$$

and

$$\rho(\mathbf{k}) = \frac{1}{2} \left\{ 1 - \frac{(E(\mathbf{k}) - \lambda)}{\sqrt{(E(\mathbf{k}) - \lambda)^2 + \Delta(\mathbf{k})^2}} \right\}, \qquad (2 \cdot 4)$$

where $\Delta(\mathbf{k})$, λ , ρ_0 and k_F are the energy gap, chemical potential, average density and Fermi momentum, respectively. Here v and $E(\mathbf{k})$ are the pair interaction under consideration and the single particle energies, and $\varphi(\mathbf{k})$ and $\rho(\mathbf{k})$ are the pair correlation function and the occupation density for the state of momentum \mathbf{k} .

^{*)} Actually it is fictious to consider only the n-p interaction neglecting the n-n and p-p interactions and also the interaction between pairs. In the real case there will occur α -particle clustering. It is noted that we treat a model in this paper.

It is known that the energy gap vanishes at higher density $(k_{\rm F} \gtrsim 1.4 \, {\rm fm}^{-1})$ because of the repulsive core in the case of the n-n (p-p) interaction and the nuclear matter becomes normal Fermi gas. This behavior will be similar for the n-p case. In the lower density the energy gap occurs due to the attractive nuclear interaction and the nuclear matter becomes a super-state with a diffused Fermi surface. As the density tends to zero $(k_{\rm F} \to 0)$ it is easily seen by Eq. $(2 \cdot 3)$ the nuclear matter follows either of two possible paths in Fig. 1. We know that for the n-n (p-p) interaction the case of

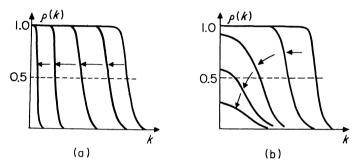


Fig. 1. Two possible paths of the occupation density $\rho(k)$ which are realized when average density of nuclear matter tends to zero.

Fig. 1(a) is realized and the nuclear matter tends to a normal Fermi gas again. In this case the energy gap tends to zero rapidly as follws, 4b)

$$\Delta(k_{\rm F}) \approx \frac{2\hbar^2}{M^*} k_{\rm F}^2 \exp\left(\frac{\pi}{2} \frac{\hbar^2}{M^*} \frac{\hbar^2}{k_{\rm F}} \frac{1}{\langle k_{\rm F} | R | k_{\rm F} \rangle}\right), \tag{2.5}$$

where $\langle k_{\rm F}|R|k_{\rm F}\rangle$ is the reaction matrix element which is negative at the Fermi momentum. On the other hand the case of Fig. 1 (b) is realized as seen in Eqs. (2·3) and (2·4) if λ tends to a negative value λ_0 and Δ to zero. The energy gap $\Delta(k)$ goes to zero as $k_{\rm F}^{3/2}$ less rapidly than in the case of Fig. 1 (a) (Eq. (2·5)). In the limit of $k_{\rm F} \rightarrow 0$ Eqs. (2·1) \sim (2·4) can be written as

$$R(\mathbf{k}) = -\sum_{\mathbf{k'}} \langle \mathbf{k} | v | \mathbf{k'} \rangle \frac{R(\mathbf{k'})}{2E(\mathbf{k'}) - 2\lambda_0}$$
 (2.6)

and
$$\sum_{\mathbf{k}} \left[\frac{R(\mathbf{k})}{2E(\mathbf{k}) - 2\lambda_0} \right]^2 = 1, \qquad (2.7)$$

where $R(\mathbf{k})$ is defined by $((4\pi/3)k_F^3)^{-1/2}\Delta(\mathbf{k})$. We see that these are the equation and normalization condition for a bound state of a pair in free space where $2\lambda_0$ is the binding energy and $[R(\mathbf{k})/(2E(\mathbf{k})-2\lambda_0)]$ is the wave function in momentum space. Even for finite density we may interprete the state

of negative λ as the bound state of pairs with the binding energy 2λ . Thus Fig. 1(b) means the path from a super-state (Fermi gas like) to a cluster-state (deuteron gas like, $\lambda < 0$), which occurs in the case of the n-p interaction.

The above process can be also interpreted in terms of the occupation density $\rho(\mathbf{k})$. Let us call the states with $\rho(\mathbf{k})$ larger (smaller) than 1/2 as the hole (particle) states. In the case of Fig. 1(b) the hole states decrease as the density becomes lower and finally at the density where $\rho(\mathbf{k}) < 1/2$ for all \mathbf{k} the concept of particle and hole (indispensable to independent particle picture) becomes quite meaningless. This is just the density where λ becomes negative (Eq. (2·4)). At such a low density it is better to interpret $\rho(\mathbf{k})$ as composed of deuteron clusters piled up in momentum space. The deuteron clusters have the wave-function more extended than the free deuteron and are degenerate in $\mathbf{K} = 0$ state as if they were bosons (\mathbf{K} : center of mass momentum of deuteron cluster). The effects of Pauli principle and the single particle potential produced by other particles are all included in the internal structure of the deuteron clusters.

Now we solve Eqs. $(2\cdot 1)\sim (2\cdot 4)$ in order to see at what density the nuclear matter goes to the cluster state. As the n-p interaction we use Yamaguchi potential, 5 central and separable;

$$\langle k | v(^{3}\text{E}) | k' \rangle = G_{t} \cdot g(k) \cdot g(k'),$$

$$g(k) = 1/(k^{2} + \mu^{2}),$$

$$G_{t} = \frac{\hbar^{2}}{2M} \frac{4}{\pi} \mu \left(\mu + \sqrt{2.226 / \frac{\hbar^{2}}{M}} \right)^{2},$$

$$\mu = 1.4488 \text{ fm}^{-1}.$$
(2·8)

The single particle energy $E(\mathbf{k})$ is taken here as the free kinetic energy $\hbar^2 k^2 / 2M$.*) The calculated results are shown in Fig. 2 (a) for chemical potential λ and in Fig. 2 (b) for the occupation density $\rho(k)$. λ tends to -1.1 MeV (1/2 of the deuteron binding energy) as expected in the limit of $k_{\rm F} \rightarrow 0$. λ is found to become negative, that is, the nuclear matter goes to the cluster state from the super-state, at the density corresponding to $k_{\rm F} \approx 0.3$ fm⁻¹. This value is interpreted as follows: a free deuteron wave function has a tail $e^{-\kappa r} (\kappa = \sqrt{M/\hbar^2)|E|} \approx 0.2$ fm⁻¹) and so its Fourier components $\sim 1/(k^2 + \kappa^2)$, that is, low momentum components $k \lesssim \kappa$ are necessary for a deuteron to be bound. If these components are cut off by Pauli principle, the deuteron can be no longer bound. The value $k_{\rm F} \approx 0.3$ fm⁻¹ roughly corresponds to this situation.

^{*)} This corresponds to taking account of only the effect of Pauli principle. The effect of the effective mass is discussed in §3.

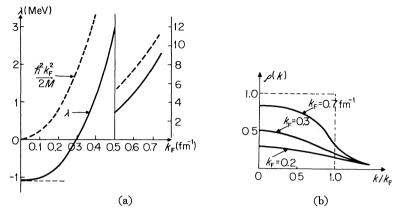


Fig. 2. (a) The chemical potential λ as a function of k_F (solid line). Fermi energy of normal Fermi gas is drawn for comparison (dotted line). (b) The occupation density $\rho(k)$. Here the effective mass is not taken into account.

§3. Deuteron clustering and single particle potential

In §2 we discuss qualitatively the problem of deuteron clustering of nuclear matter noticing only the effect of Pauli principle. Actually the structure of the system depends on not only the clustering energy but also the single particle potential energy. This problem is treated by considering self-consistently the deuteron-clustering in a single particle potential field and also the potential field produced by the clustering particles. We assume that in low density clustering correlation is important between n-p pairs and the n-n and p-p interaction only contribute to the single particle potential. We can treat the problem by Hartree-Bogoliubov method. We present here several important equations in the method.

Total energy perparticle E is given by

$$E = T + U + E_{c}, \tag{3.1}$$

where T, U and E_c mean respectively the kinetic energy, single particle potential energy and clustering energy of n-p pairs. They are given by

$$T = \int \frac{\hbar^2 k^2}{2M} \rho(\mathbf{k}) d\mathbf{k} / \int \rho(\mathbf{k}) d\mathbf{k} , \qquad (3.2)$$

$$U = \frac{1}{2} \int U(\mathbf{k}) \rho(\mathbf{k}) d\mathbf{k} / \int \rho(\mathbf{k}) d\mathbf{k}$$
 (3.3)

and
$$E_{c} = -\frac{1}{4} \int_{\sqrt{\overline{E(\mathbf{k})} - \lambda^{2} + \Delta(\mathbf{k})^{2}}} d\mathbf{k} / \int \rho(\mathbf{k}) d\mathbf{k}, \qquad (3 \cdot 4)$$

where $E(\mathbf{k})$ is the single particle energy $\hbar^2 k^2 / 2M + U(\mathbf{k})$. The single particle

potential $U(\mathbf{k})$ is obtained from the two-body interactions as follows:

$$U(\mathbf{k}) = 2 \times \frac{3}{4} \int \left\{ \left\langle \frac{\mathbf{k} - \mathbf{k'}}{2} | v(^{1}S) | \frac{\mathbf{k} - \mathbf{k'}}{2} \right\rangle + \left\langle \frac{\mathbf{k} - \mathbf{k'}}{2} | v(^{3}S) | \frac{\mathbf{k} - \mathbf{k'}}{2} \right\rangle \right\} \rho(\mathbf{k'}) d\mathbf{k'}, \tag{3.5}$$

where we take only the singlet S and triplet S (including effectively the triplet even tensor force) for simplicity. The energy gap $\Delta(\mathbf{k})$, the occupation density $\rho(\mathbf{k})$ and the chemical potential λ are obtained by solving the gap equations between n-p pairs:

$$\Delta(\mathbf{k}) = -\frac{1}{2} \int \langle \mathbf{k} | v(^{3}S) | \mathbf{k}' \rangle \frac{\Delta(\mathbf{k}')}{\sqrt{(E(\mathbf{k}') - \lambda)^{2} + \Delta(\mathbf{k}')^{2}}} d\mathbf{k}', \qquad (3.6)$$

$$\rho(\mathbf{k}) = \frac{1}{2} \left\{ 1 - \frac{E(\mathbf{k}) - \lambda}{\sqrt{(E(\mathbf{k}) - \lambda)^2 + \Delta(\mathbf{k})^2}} \right\}$$
(3.7)

and

$$\int \rho(\mathbf{k}) d\mathbf{k} = \frac{4\pi}{3} k_{\rm F}^3. \tag{3.8}$$

Equations $(3.5) \sim (3.8)$ should be solved self-consistently and then the total energy is given by Eqs. $(3.1) \sim (3.4)$.

We use again Yamaguchi potential in Eq. (2·8) but for singlet state $G_8 = 0.7G_t$ in place of G_t . Then from Eq. (3·6)

$$\Delta(k) = \Delta \cdot g(k), \tag{3.9}$$

and Eqs. $(3.5) \sim (3.8)$ are simplified and easily solved numerically. Calcu-

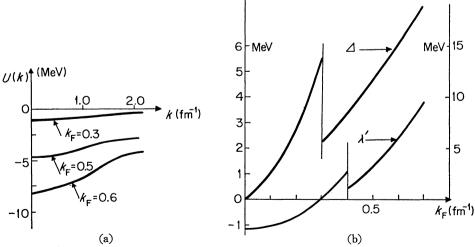


Fig. 3. (a) The single particle potential U(k). (b) The energy gap Δ and the chemical potential λ' ($\lambda' = \lambda - U(k=0)$). These are obtained by solving Hartree-Bogoliubov equations.

lated results of U(k), Δ and λ' are shown in Figs. 3 (a) and 3 (b). λ' is defined by

$$\lambda' = \lambda - U(k=0). \tag{3.10}$$

 λ' is a measure of deuteron clustering of n-p pairs. The single particle potential U(k) brings about an effective nucleon mass smaller than the free one, which makes difficult the clustering of n-p pairs. Because of this effect λ' in Fig. 3(b) is a little larger than λ in Fig. 2(a), though the effect is negligibly small at $k_F \lesssim 0.3 \, \mathrm{fm}^{-1}$. We see here again that λ' crosses zero at $k_F \approx 0.3 \, \mathrm{fm}^{-1}$. In Fig. 4 we show the results for E, T, U, E_c (solid

lines) and for comparison the values in the case of normal Fermi gas $(\rho(k)=1 \text{ for } k \leq k_F \text{ and } 0 \text{ for } k > k_F)$ (dotted lines). In higher density the single particle potential energy U is large and Fermi gas (independent particle) aspect becomes marked. In lower density where U is small the clustering energy E_c plays a principal role and the deuteron cluster aspect reveals itself. low density limit T and E_c correspond to the kinetic energy and the potential energy of a deuteron.*) The single particle potential energy U is reduced only a little from that The total of normal Fermi gas. energy per-particle E becomes positive, in the normal Fermi gas, at $k_{\rm F} \approx 0.7 \, {\rm fm^{-1}}$ because of predominance of the kinetic energy. In our case, however, E is always negative due to the clustering energy and tends to -1.1 MeV going over a very

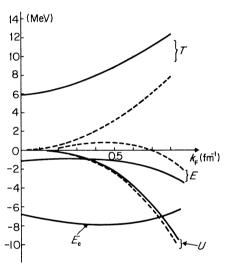


Fig. 4. Solid lines show the results from Hartree-Bogoliubov equations including the deuteron clustering effect. Dotted lines are those of normal Fermi gas with no clustering effects. E, T, U and E_c represent the total energy, kinetic energy, single particle potential energy and deuteron clustering energy, respectively, per-particle. $E = T + U + E_c$.

$$T \rightarrow \lambda + \int \{T(k) - \lambda\} \rho(k) dk / \int \rho(k) dk$$

$$\rightarrow \lambda + \int \{T(k) - \lambda\} \cdot \frac{1}{4} \frac{\Delta(k)^2}{(T(k) - \lambda)^2} dk / \int \rho(k) dk,$$

$$E_c \rightarrow -\frac{1}{4} \int \frac{\Delta(k)^2}{T(k) - \lambda} dk / \int \rho(k) dk.$$

Therefore $E \rightarrow \lambda$.

^{*)} When k_F tends to zero in Eqs. (3·1) \sim (3·4), it is easily seen that $U\rightarrow 0$ and so $E(k)\rightarrow T(k)$ and then

smooth peak. This peak, though very smooth, results from the fact that at $k_{\rm F}{\sim}0.3~{\rm fm^{-1}}$ the potential energy is already reduced and also the effect of Pauli principle still suppresses the clustering. The smoothness of the peak means that the structure of nuclear matter changes very gradually with density.

§4. A deuteron cluster moving in the nuclear matter

In the previous sections we consider the transition of the whole system from the super-state to the cluster-state (deuteron gas-like). There is another type of clustering such as a deuteron, alpha particle, etc. outside the core. This is related also to the case of reactions of deuteron, alpha particle, with nucleus.

In order to consider such a case, we investigate a deuteron moving with centre of mass momentum P in the nuclear matter. The equation of motion of the deuteron is given by

$$\begin{aligned}
&\left\{E\left(\frac{\mathbf{P}}{2}+\mathbf{k}\right)+E\left(\frac{\mathbf{P}}{2}-\mathbf{k}\right)-2W\right\}\Psi\left(\frac{\mathbf{P}}{2}+\mathbf{k},\frac{\mathbf{P}}{2}-\mathbf{k}\right) \\
&=\frac{2G}{4\pi}g(k)\int g(k')Q\left(\frac{\mathbf{P}}{2}+\mathbf{k'},\frac{\mathbf{P}}{2}-\mathbf{k'}\right)\Psi\left(\frac{\mathbf{P}}{2}+\mathbf{k'},\frac{\mathbf{P}}{2}-\mathbf{k'}\right)d\mathbf{k'},
\end{aligned} (4\cdot1)$$

where k is the relative momentum and 2W is the eigenvalue, the sum of the centre of mass energy and internal energy of the deuteron, and

$$E\left(\frac{\mathbf{P}}{2} \pm \mathbf{k}\right) = \hbar^2 \left(\frac{\mathbf{P}}{2} \pm \mathbf{k}\right)^2 / 2M^* \left(\left|\frac{\mathbf{P}}{2} \pm \mathbf{k}\right|\right), \tag{4.2}$$

$$Q\left(\frac{\mathbf{P}}{2} + \mathbf{k}, \frac{\mathbf{P}}{2} - \mathbf{k}\right) = 1 - \rho\left(\left|\frac{\mathbf{P}}{2} + \mathbf{k}\right|\right) - \rho\left(\left|\frac{\mathbf{P}}{2} - \mathbf{k}\right|\right). \tag{4.3}$$

Here Q is the projection operator which inhibits the nucleons of the deuteron to occupy the states already occupied by the nuclear matter, and the occupation density of nuclear matter is $\rho(k)$. We take into account only the relative S-state interaction and so take the average value for $E(|P/2\pm k|)$ and $\rho(|P/2\pm k|)$ over the angle between P and k. Then Eq. (4·1) becomes as follows:

$$(E(k,P)-W)\Psi(k,P) = Gg(k) \int_{0}^{\infty} g(k')Q(k',P)\Psi(k',P)k'^{2}dk', \qquad (4\cdot 4)$$

$$E(k,P) = \frac{1}{2} \int_{-1}^{1} E\left(\sqrt{\frac{P^2}{4} + k^2 + Pkx}\right) dx, \qquad (4.5)$$

$$Q(k,P) = 1 - 2\rho(k,P) \tag{4.6}$$

$$\rho(k, P) = \frac{1}{2} \int_{-1}^{1} \rho\left(\sqrt{\frac{P^2}{4} + k^2 + Pkx}\right) dx.$$
 (4.7)

Equation $(4 \cdot 4)$ is easily rewritten in the form of dispersion:

$$G \int_{0}^{\infty} \frac{g(k)^{2}Q(k,P)}{E(k,P)-W} k^{2}dk = 1.$$
 (4.8)

We cannot separate the energy E(k,P) into the centre of mass and internal energy, because the effective mass M^* depends on P and k. But we may put E(k=0,P) as the (half of) kinetic energy of the centre of mass. Then the eigenvalue is separated as follows:

$$W = E(k=0, P) + w(P),$$
 (4.9)

where w(P) is the internal energy of the deuteron with c.m. momentum P. If the deuteron is bound, then w(P) will be negative. $\rho(k,P)$ is interpreted as the occupation density at the state k of nuclear matter felt by the deuteron moving with c.m. momentum P.

It is obvious that for very large P w(P) gives the binding energy of free deuteron, because $\rho(k,P)$ becomes very small and Q(k,P) almost unity for not so large k. By solving Eqs. $(4\cdot 1) \sim (4\cdot 3)$ over the values of P we can see the change of the deuteron. In Figs. 5(a) and 5(b) we give the results of w(P) and $\rho(k,P)$ calculated using the values of $\rho(k)$ in §2. As seen from Fig. 5(a), the deuteron is the more deeply bound for a c.m. momentum P, the lower the density of nuclear matter is. Figure 5(b) shows that $\rho(k,P)$ decreases for small k as P increases and this leads to gain in w(P). In Fig. 5(b), at $P=1.9 \, \mathrm{fm^{-1}}$ and $k_F=0.7 \, \mathrm{fm^{-1}}$ the occupation density $\rho(k,P)$ is about 0.1 in the range of interaction. This is very small, never-

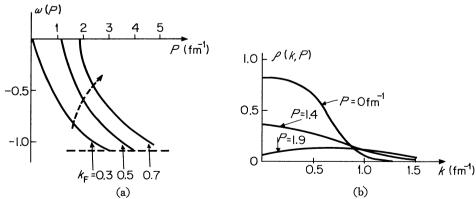


Fig. 5. (a) Internal energy w(P) of a deuteron moving in the nuclear matter with c.m. momentum P. The broken arrow is the path of a deuteron coming into the matter with momentum $P_0=1 \,\mathrm{fm}^{-1}$. (b) Occupation density $\rho(k,P)$ of nuclear matter with $k_F=0.7 \,\mathrm{fm}^{-1}$ felt by a nucleon in a deuteron moving with momentum P.

theless w(P) is nearly zero (Fig. 5(a)) and so the deuteron can be hardly bound. This fact shows that the Pauli effect is very severe. From Eq. (4·7) the relation $\rho(0,P) = \rho(P/2,0)$ is found. From this relation we see that if the Fermi surface of nuclear matter is diffused, $\rho(k,P)$ has non-vanishing value for small k even for large P. This suggests that the less correlated the nuclear matter is, the more easily bound a cluster moving in the nuclear matter is.

The condition that the state with a cluster moving in the nuclear matter is energetically more favourable than the state with the cluster melted into the matter is intuitively given by

$$E(k=0,P)+w(P)-\lambda+\Delta(k_{\rm F})<0, \qquad (4\cdot10)$$

where λ and $\Delta(k_{\rm F})$ are the Fermi energy and energy gap when a pair of nucleons is picked up out of the nuclear matter. This condition can be satisfied, if $\Delta(k_{\rm F})$ is small and w(P) tends to the binding energy of free deuteron rapidly with P. The condition is, however, very severe and in our calculation it is impossible to find such a P satisfying this condition for any value of $k_{\rm F}$.

We consider such a case as a deuteron passing through the nuclear matter. In the sense of local density approximation the deuteron with incident momentum P_0 has a momentum $P \geq (P_0 + 2k_F)$ when it passes the matter with density k_F . Therefore to see the behaviour of the deuteron passing through the matter, we follow the path of $w(P_0 + 2k_F)$ from $k_F = 0$ to $k_F = 1.4 \text{ fm}^{-1}$. In Fig. 5(a) the path is given for the case of $P_0 = 1 \text{ fm}^{-1}$ ($\hbar^2 P_0^2 / 4M \approx 10 \text{ MeV}$) by the broken arrow and it is expected that the deuteron is already dissociated at the normal density.

We would like to express our sincere thanks to Professor R. Tamagaki and Professor S. Nagata for valuable discussions.

References

- 1a) K. Sekine, M. Taketani and T. Toyoda, Soryushiron Kenkyu (mimeographed circular in Japaneses) 12 (1956), 498.
- b) K. Wildermuth and W. McClure, Springer Tracts in Modern Physics, Vol. 41; "Cluster Representation of Nuclei" (Springer Verlag, Berlin, 1966).
 See this for further references.
- c) I. Shimodaya, R. Tamagaki and H. Tanaka, Prog. Theor. Phys. 27 (1962), 793.
 Y. Abe, S. Saito, O. Endo and J. Hiura, I.C.N.S. (1967), Tokyo.
- d) H. Horiuchi and K. Ikeda, Prog. Theor. Phys. 40 (1968), 277.
- e) T. Marumori and K. Suzuki, Nucl. Phys. A106 (1968), 610.
- 2) H. Horie, JAERI **531** (1962), 108.
- 3) K. A. Brueckner and J. L. Gammel, Phys. Rev. 110 (1958), 431.
- 4a) K. A. Brueckner, T. Soda, P. W. Anderson and P. Morel, Phys. Rev. 118 (1960), 1442.

- b) V. J. Emery and A. M. Sessler, Phys. Rev. 119 (1960), 248.
- c) T. Ishihara, R. Tamagaki, H. Tanaka and M. Yasuno, Prog. Theor. Phys. 30 (1963), 601.
- 5) Y. Yamaguchi, Phys. Rev. 95 (1954), 1628.
- 6) M. Baranger, Phys. Rev. 120 (1960), 957.